MATHEMATICAL EXERCISES.

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MATHEMATICAL EXERCISES:

COMPRISING 4500 EXAMPLES IN THE VARIOUS BRANCHES OF PURE MATHEMATICS, STATICS, DYNAMICS, AND HYDROSTATICS, TAKEN FROM MILITARY, CIVIL SERVICE, AND OTHER EXAMINATION PAPERS; COLLECTED AND ARRANGED IN SETS, FOR THE USE OF STUDENTS PREPARING FOR EXAMINATION.

WITH TABLES, FORMULÆ, ANSWERS, AND REFERENCES.

BY

SAMUEL H. WINTER

MILITARY TUTOR.

NEW EDITION,

CORRECTED AND ENLARGED, WITH 900 ADDITIONAL QUESTIONS.

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PREFACE TO THE FIRST EDITION.

The sources from which the following Exercises have been taken are sufficiently indicated by the Table of Contents. This collection differs, in two respects, from any with which the author is acquainted; in the first place, the questions are arranged in such a manner that most of the papers embrace various subjects; secondly, 'book-work' is introduced, in order to assimilate the papers to those set at Examinations.

Woolwich Common: *August* 1864.

PREFACE TO THE SECOND EDITION.

This Edition contains about 900 additional Examples, selected from Examination Papers which have been set since the publication of the first Edition. The greater part of the new matter will be found in the Appendix; but the fresh questions in Mixed Mathematics commence at page 287. Considerable pains having been taken in correcting errors, it is hoped that they now amount to a comparatively small number.

Woolwich Common: *January* 1867.

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Erratum.

Page 329, line 1, *for* $\frac{1}{2}$ *read* $\frac{3}{2}$.

MATHEMATICAL EXERCISES.

TABLES, FORMULÆ, ETC.

TABLES OF WEIGHTS AND MEASURES.

TROY WEIGHT.

24 grains (grs.)	= 1 pennyweight (dwt.)
20 dwts.	= 1 ounce (oz.)
12 oz.	= 1 pound (lb.)

AVOIRDUPOIS WEIGHT.

16 drams (dr.)	= 1 ounce (oz.)
16 oz.	= 1 pound (lb.)
28 lbs.	= 1 quarter (qr.)
4 qrs. or 112 lbs.	= 1 hundredweight (cwt.)
20 cwt.	= 1 ton.
1 stone (st.)	= 14 lbs.

1 lb. avoirdupois = 7000 grs. troy.
1 lb. troy = 5760 grs. troy.

CLOTH MEASURE.

$2\frac{1}{4}$ in.	= 1 nail (n.)
4 n.	= 1 quarter (qr.)
4 qrs.	= 1 yd.
3 qrs.	= 1 Flemish ell.
5 qrs.	= 1 English ell.
6 qrs.	= 1 French ell.

LINEAL MEASURE.

12 inches (in.)	= 1 foot (ft.)
3 ft.	= 1 yard (yd.)
2 yds. or 6 ft.	= 1 fathom.
$5\frac{1}{2}$ yds. or $16\frac{1}{2}$ ft.	= 1 pole (po.)
40 po. or 220 yds.	= 1 furlong (fur.)
8 fur. or 1760 yds.	= 1 mile (mi.)
3 mi.	= 1 league (lea.)
$69\frac{1}{9}$ mi.	= 1 degree (1°).

1 chain = 22 yds. = 66 ft. = 100 links.
1 hand = 4 inches.
1 line = $\frac{1}{12}$ of an inch.

SUPERFICIAL MEASURE.

144 square inches = 1 square foot (sq. ft.)
9 sq. ft. = 1 square yard (sq. yd.)
$30\frac{1}{4}$ sq. yds. = 1 sq. pole (sq. po.)
40 sq. po. = 1 rood (ro.)
4 roods, or 4840 sq. yds. = 1 acre (ac.)

1 square chain = 10000 square links.

1 ac. = 10 ch. = 100000 sq. li.

1 sq. mi. = $(1760)^2$ sq. yds. = 640 acres.

1 rod of brickwork = $272\frac{1}{4}$ sq. ft. $1\frac{1}{2}$ bricks thick.

1 square of flooring = 100 sq. ft.

CUBIC MEASURE.

1728 cubic in.	= 1 cubic foot.
27 cub. ft.	= 1 cub. yd.

CORN MEASURE.

2 qts.	= 1 gal.
2 gal.	= 1 pk.
4 pecks	= 1 bushel (bu.)
8 bu.	= 1 quarter (qr.)

LIQUID MEASURE.

2 pints (pt.)		= 1 quart (qt.)
4 qts.		= 1 gallon (gal.)
9 gal.	Beer.	= 1 firkin.
18 gal.	Beer.	= 1 kilderkin (fir.)
36 gal.	Beer.	= 1 barrel (bar.)
54 gal.	Beer.	= 1 hogshead (hhd.)
42 gal.		= 1 tierce.
63 gal.		= 1 hhd.
126 gal.		= 1 pipe.
252 gal.		= 1 tun.

TIME.

60 seconds (″)	= 1 minute (′)
60′	= 1 hour (hr.)
24 hrs.	= 1 day (da.)
7 da.	= 1 week (1 wk.)

NUMBERS OF FREQUENT OCCURRENCE.

π { = Circumference of circle to diameter 1; = Surface of sphere to diameter 1; = Area of circle to radius 1 } $= 3{\cdot}1415926$; $\log \pi = 0{\cdot}4971499$; $\log \frac{1}{\pi} = \bar{1}{\cdot}5028501$.

$\frac{\pi}{4} = {\cdot}7853986$; $\log \frac{\pi}{4} = \bar{1}{\cdot}8950899$; $\frac{\pi}{6} = {\cdot}5235988$; $\log \frac{\pi}{6} = \bar{1}{\cdot}7189986$

Unit of Circular Measure = $57^\circ{\cdot}2957795$;
$\log 57{\cdot}2957795 = 1{\cdot}7581226$.

Do. do. = $206264{\cdot}''8$;
$\log 206264{\cdot}8 = 5{\cdot}3144251$.

$M_{10} = \frac{1}{\log_e 10} = \frac{1}{2{\cdot}3025851} = {\cdot}43429448$; $\log_{10} M = \bar{1}{\cdot}6377843$.

e = base of Napierian system of logarithms = $2{\cdot}718281828$;
$\log_{10} e = {\cdot}4342945$.

g = force of gravity = $32{\cdot}19084$ ft.; $\log g = 1{\cdot}5077323$;
$\log \frac{1}{g} = \bar{2}{\cdot}4922677$.

Length of seconds' pendulum = 39·1393 inches; log 39·1393 = 1·5926130.

No. of cubic in. in 1 gal. = 277·274; log 277·274 = 2·4429091; wt. of 1 cub. in. of water = 252·458 grs.; log. 252·458 = 2·4021892.

Table of the Squares, Cubes, Square and Cube Roots, Reciprocals, Logarithms and Logarithms of Reciprocals of the natural numbers from 1 to 20.

No.	Sq.	Cube	Square Root	Cube Root	Reciprocal	Logarithm	Log. of Reciprocal
1	1	1	1·000000	1·000000	1	0·0000000	0·0000000
2	4	8	1·414213	1·259921	·5	0·3010300	$\bar{1}$·6989700
3	9	27	1·732050	1·442249	·3333333	0·4771213	$\bar{1}$·5228787
4	16	64	2·000000	1·587401	·25	0·6020600	$\bar{1}$·3979400
5	25	125	2·236068	1·709976	·2	0·6989700	$\bar{1}$·3010300
6	36	216	2·449489	1·817121	·1666666	0·7781513	$\bar{1}$·2218487
7	49	343	2·645751	1·912931	·$\dot{1}42857\dot{1}$	0·8450980	$\bar{1}$·1549020
8	64	512	2·828427	2·000000	·125	0·9030900	$\bar{1}$·0969100
9	81	729	3·000000	2·080083	·1111111	0·9542425	$\bar{1}$·0457575
10	100	1000	3·162277	2·154434	·1	1·0000000	$\bar{1}$·0000000
11	121	1331	3·316624	2·223980	·0909090	1·0413927	$\bar{2}$·9586073
12	144	1728	3·464101	2·289428	·0833333	1·0791812	$\bar{2}$·9208198
13	169	2197	3·605551	2·351335	·0769230	1·1139434	$\bar{2}$·8860566
14	196	2744	3·741657	2·410142	·0714285	1·1461280	$\bar{2}$·8538720
15	225	3375	3·872983	2·466212	·0666666	1·1760913	$\bar{2}$·8239087
16	256	4096	4·000000	2·519842	·0625	1·2041200	$\bar{2}$·7958800
17	289	4913	4·123105	2·571281	·0588235	1·2304489	$\bar{2}$·7695511
18	324	5832	4·242640	2·620741	·0555555	1·2552725	$\bar{2}$·7447275
19	361	6859	4·358898	2·668402	·0526315	1·2787536	$\bar{2}$·7212464
20	400	8000	4·472136	2·714417	·05	1·3010300	$\bar{2}$·6989700

USEFUL ALGEBRAICAL FORMULÆ.

$(x \pm y)^2 = x^2 \pm 2xy + y^2$; $(x \pm y)^3 = x^3 \pm y^3 \pm 3xy(x + y)$.

$(x+y)(x-y) = x^2 - y^2$ $\therefore (x^2 + xy + y^2)(x^2 - xy + y^2)$
$= x^4 + x^2y^2 + y^4$.

$(x^2 - xy + y^2)(x + y) = x^3 + y^3$.

$(x^2 + xy + y^2)(x - y) = x^3 - y^3$.

$(x + a)(x + b) = x^2 + (a + b)x + ab$.

$\dfrac{x^n - y^n}{x \pm y}$ is integral when n is an even number.

$\dfrac{x^n + y^n}{x + y}$ and $\dfrac{x^n - y^n}{x - y}$ are integral when n is an odd number.

If $ax^2 + bx + c = 0$,

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

In an arithmetic progression,

$$l = a + (n-1)d;\ s = \frac{n}{2}(a+l) = \frac{n}{2}\left\{2a + (n-1)d\right\};$$
$$d = \frac{l-a}{n-1}.$$

In a geometric progression,

$$l = ar^{n-1};\ s = \frac{rl-a}{r-1} = \frac{a(r^n - 1)}{r-1};\ s_{\frac{1}{0}} = \frac{a}{1-r};\ r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}}.$$

If a and b be any two quantities their

$$\text{A.M.} = \tfrac{1}{2}(a + b);\ \text{G.M.} = \sqrt{ab};\ \text{H.M.} = \frac{2ab}{a+b}.$$

No. of shot in a square pile of n courses $=$

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{1.2.3}.$$

No. of shot in a triangular pile of n courses $=$

$$1+\overline{1+2}+\overline{1+2+3}+\ldots\ldots+\frac{n(n+1)}{2}=$$
$$\frac{n(n+1)(n+2)}{1\,.\,2\,.\,3}.$$

No. of shot in a rectangular pile of n courses $=$

$$(1+m)+2(2+m)+3(3+m)+\ldots\ldots n(n+m)=$$
$$\frac{n(n+1)(2n+1+3m)}{1\,.\,2\,.\,3}.$$

No. of permutations of n things taken r at a time $=$

$$\frac{\underline{|n}}{\underline{|n-r}}.$$

No. of combinations of n things taken r at a time $=$

$$\frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}.$$

$$(a\pm x)^n=a^n\pm\frac{n}{1}a^{n-1}x+\frac{n(n-1)}{\underline{|2}}a^{n-2}x^2$$
$$\pm\frac{n(n-1)(n-2)}{\underline{|3}}a^{n-3}x^3+\text{ \&c. \&c.; where}$$

the $(r+1)$th term is $\dfrac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}\,.\,a^{n-r}x^r$.

General term of the expansion of

$(a_0+a_1x+a_2x^2+a_3x^3+\ldots.)^n$ is

$$\frac{\underline{|n}}{\underline{|p}\,\underline{|q}\,\underline{|r}\,\underline{|s}\,\underline{|t\ldots}}\;a_0^p\,a_1^q\,a_2^r\,a_3^s\,a_4^t\ldots.\;x^{q+2r+3s+4t\ldots.}$$

where $p+q+r+s+\ldots\ldots=n$.

$$a^x=1+(\log_e a)\,x+\frac{(\log_e a)^2x^2}{\underline{|2}}+\frac{(\log_e a)^3x^3}{\underline{|3}}+\text{ \&c.}$$

$$\log_e a=a-1-\frac{(a-1)^2}{2}+\frac{(a-1)^3}{3}-\frac{(a-1)^4}{4}+\text{ \&c.}$$

$$\log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\text{ \&c. \&c.}$$

$$\log_e(n+1) = \log_e n + 2\left\{\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \text{\&c. \&c.}\right\}$$

$$\log_a(n+1) = \log_a n + 2\text{M}_a\left\{\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \text{\&c. \&c.}\right\};\quad \text{M}_a = \frac{1}{\log_e a}.$$

$$\log \text{MN} = \log \text{M} + \log \text{N};\quad \log \text{M} \div \text{N} = \log \text{M} - \log \text{N};$$

$$\log \text{N}^p = p \log \text{N};\quad \log \sqrt[p]{\text{N}} = \frac{1}{p}\log \text{N};$$

$$\log_a \text{N} : \log_b \text{N} :: \log b : \log a$$

$$\therefore \log_b \text{N} = \log_a \text{N} \times \frac{\log_a a}{\log_a b} = \log_a \text{N} \div \log_a b.$$

$$\log_a a = 1;\ \log_a 1 = 0;\ \log_a 0 = -\infty.$$

Compound Interest. $\text{A}=\text{P}(1+r)^t = \text{P}\left(1+\frac{r}{n}\right)^{nt}$, when interest is payable n times a year.

Amount of an annuity of a £ in n years $= \dfrac{(1+r)^n - 1}{r} \cdot a.$

Present value of same annuity $= \dfrac{1-(1+r)^{-n}}{r} \cdot a.$

FORMULÆ IN PLANE TRIGONOMETRY.

Unit of circular measure $= \dfrac{180°}{\pi} = 57°{\cdot}2957795$

$= 206264''{\cdot}8.$

$$\text{A}° = \frac{180°}{\pi} \times \frac{\text{arc}}{\text{radius}}.$$

$\text{Sin}(-\text{A}) = -\sin \text{A};\ \cos(-\text{A}) = \cos \text{A};$

$\tan(-\text{A}) = -\tan \text{A}.$

$\text{Sin}\,(90° + A) = \cos A$; $\cos\,(90° + A) = -\sin A$;
$\tan\,(90° + A) = -\tan A$.

$\text{Sin}\,(180° - A) = \sin A$; $\cos\,(180° - A) = -\cos A$;
$\tan\,(180° - A) = -\tan A$.

$\text{Sin}\,(180° + A) = -\sin A$; $\cos\,(180° + A) = -\cos A$;
$\tan\,(180° + A) = \tan A$.

$\text{Sin}\,(360° - A) = -\sin A$; $\cos\,(360° - A) = \cos A$;
$\tan\,(360° - A) = -\tan A$.

$\text{Sin}\,A = \sin\,\{n\pi + (-1)^n A\}$; $\cos A = \cos\,\{2\,n\pi \pm A\}$;
$\tan A = \tan\,(n\pi + A)$.

$$\text{Sin}\,A = \sqrt{1 - \cos^2 A} = \frac{1}{\operatorname{cosec} A} = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A = \sqrt{\frac{1 - \cos 2A}{2}} = \sin\,(60° + A) - \sin\,(60° - A).$$

$$\text{Cos}\,A = \sqrt{1 - \sin^2 A} = \frac{1}{\sec A} = \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A = 1 - 2 \sin^2 \tfrac{1}{2} A = 2 \cos^2 \tfrac{1}{2} A - 1 = \sqrt{\frac{1 + \cos 2A}{2}}.$$

$$\text{Tan}\,A = \frac{\sin A}{\cos A} = \frac{1}{\cot A} = \sqrt{\sec^2 A - 1} = \frac{2 \tan \frac{1}{2} A}{1 - \tan^2 \frac{1}{2} A} = \frac{1 - \cos 2A}{\sin 2A} = \sqrt{\frac{1 - \cos 2A}{1 + \cos 2A}}.$$

$\text{Sin}\,(A \pm B) = \sin A \cos B \pm \cos A \sin B$;
$\text{Cos}\,(A \pm B) = \cos A \cos B \mp \sin A \sin B$.

$$\text{Tan}\,(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B};$$

$$\text{Tan}\,(45° \pm B) = \frac{1 \pm \tan B}{1 \mp \tan B}.$$

$\text{Sin}\,A + \sin B = 2 \sin \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$;
$\text{Cos}\,A + \cos B = 2 \cos \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$;

$\text{Sin}\,A - \sin B = 2 \cos \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$;
$\text{Cos}\,A - \cos B = -2 \sin \tfrac{1}{2}(A + B) \sin \tfrac{1}{2}(A - B)$.

Sin (A + B) sin (A − B) = $\sin^2 A - \sin^2 B$
$= (\sin A + \sin B)(\sin A - \sin B)$.

Cos (A + B) cos (A − B) = $\cos^2 A - \sin^2 B$;

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}.$$

Sin 3 A = 3 sin A − 4 $\sin^3 A$; cos 3 A = 4 $\cos^3 A$ − 3 cos A;

$$\text{Tan } 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}.$$

Cosec A − cot A = tan $\frac{1}{2}$ A; cosec A + cot A = cot $\frac{1}{2}$ A.

If A + B + C = 90°;
Tan A tan B + tan A tan C + tan B tan C = 1;

Cot A + cot B + cot C = cot A cot B cot C;
Sin 2 A + sin 2 B + sin 2 C = 4 cos A cos B cos C.

If A + B + C = 180°;
Sin A + sin B + sin C = 4 cos $\frac{1}{2}$ A cos $\frac{1}{2}$ B cos $\frac{1}{2}$ C;

Cos A + cos B + cos C = 1 + 4 sin $\frac{1}{2}$ A sin $\frac{1}{2}$ B sin $\frac{1}{2}$ C;

Tan A + tan B + tan C = tan A tan B tan C;

Cot A cot B + cot A cot C + cot B cot C = 1.

$2 \sin A = \pm \sqrt{1 + \sin 2A} \pm \sqrt{1 - \sin 2A}$.

$2 \cos A = \pm \sqrt{1 + \sin 2A} \mp \sqrt{1 - \sin 2A}$.

Numerical Values of Functions of some Angles.

Sin 0° = 0 = cos 90°; cos 0° = 1 = sin 90°;
tan 0° = 0 = cot 90°; tan 90° = ∞ = cot 0°;

Sin 15° = $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ = cos 75°; sin 45° = $\frac{1}{\sqrt{2}}$ = cos 45°;

Sin 18° = $\frac{\sqrt{5} - 1}{4}$ = cos 72°; sin 60° = $\frac{\sqrt{3}}{2}$ = cos 30°;

Sin 30° = $\frac{1}{2}$ = cos 60°; sin 75° = $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ = cos 15°;

$$\text{Tan } 15^\circ = 2 - \sqrt{3} = \cot 75^\circ;\ \tan 60^\circ = \sqrt{3} = \cot 30^\circ;$$

$$\text{Tan } 30^\circ = \frac{1}{\sqrt{3}} = \cot 60^\circ;\ \tan 75^\circ = 2 + \sqrt{3} = \cot 15^\circ;$$

$$\text{Tan } 45^\circ = 1 = \cot 45^\circ.$$

In any Plane Triangle ABC

$$\text{Sin A} = \frac{a \sin \text{B}}{b} = \frac{a \sin \text{C}}{c} = \sin(\text{B} + \text{C}) =$$
$$\frac{2}{bc}\left\{s(s-a)(s-b)(s-c)\right\}^{\frac{1}{2}};\ s = \frac{a+b+c}{2}.$$

$$\text{Cos A} = \pm \frac{\sqrt{c^2 - a^2 \sin^2 \text{C}}}{c} = \frac{b^2 + c^2 - a^2}{2bc} =$$
$$\frac{c \sin^2 \text{B} \mp \cos \text{B} \sqrt{b^2 - c^2 \sin^2 \text{B}}}{b}.$$

$$\text{Sin } \tfrac{1}{2}\text{A} = \sqrt{\frac{(s-b)(s-c)}{bc}};\ \cos \tfrac{1}{2}\text{A} = \sqrt{\frac{s(s-a)}{bc}};$$
$$\tan \tfrac{1}{2}\text{A} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}:$$
$$\tan \tfrac{1}{2}(\text{A}-\text{B}) = \frac{a-b}{a+b}\cot \tfrac{1}{2}\text{C}.$$

$$a = \frac{b \sin \text{A}}{\sin \text{B}} = \frac{c \sin \text{A}}{\sin \text{C}} = b \cos \text{C} + c \cos \text{B} =$$
$$\sqrt{b^2 + c^2 - 2bc \cos \text{A}} = b \cos \text{C} \pm \sqrt{c^2 - b^2 \sin^2 \text{C}}.$$

If S = area of triangle; $\text{S} = \sqrt{s(s-a)(s-b)(s-c)} =$
$$\frac{ab}{2}\cdot \sin c = \frac{c^2}{2}\cdot \frac{\sin \text{A} \sin \text{B}}{\sin(\text{A}+\text{B})}.$$

Radius of circumscribed circle $= \dfrac{abc}{4\,\text{S}} = \dfrac{a}{2 \sin \text{A}}.$

Radius of inscribed circle $= \frac{S}{s} = \frac{a \sin \frac{1}{2} B \sin \frac{1}{2} C}{\cos \frac{1}{2} A}$.

Radius of escribed circle touching the side $a = \frac{S}{s-a}$.

Area of quadrilateral inscribed in circle $=$

$$\sqrt{(s-a)(s-b)(s-c)(s-d)}\ ;\ s = \frac{a+b+c+d}{2}.$$

Area of polygon of n sides inscribed in circle whose radius is $r = \frac{1}{2} nr^2 \sin \frac{360^\circ}{n}$; perimeter $= 2nr \sin \frac{180^\circ}{n}$.

Area of polygon circumscribing the same circle $= nr^2 \tan \frac{180^\circ}{n}$; perimeter $= 2nr \tan \frac{180^\circ}{n}$.

FORMULÆ IN SPHERICAL TRIGONOMETRY.

The sides and angles of the polar triangle are the supplements respectively of the angles and sides of the primitive triangle.

$A + B + C > \pi < 3\pi$.

In any spherical triangle ;

$\operatorname{Cos} a = \cos b \cos c + \sin b \sin c \cos A$.

$\operatorname{Sin} A : \sin B : \sin C :: \sin a : \sin b : \sin c$.

$\operatorname{Cot} a \sin b = \cot A \sin C + \cos b \cos C$; $\cos A = - \cos B \cos C + \sin B \sin C \cos a$.

$\operatorname{Sin} \frac{1}{2} A = \sqrt{\left\{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}\right\}}$; $\cos \frac{1}{2} A = \sqrt{\left\{\frac{\sin s \sin (s-a)}{\sin b \sin c}\right\}}$; $\tan \frac{1}{2} A = \sqrt{\left\{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}\right\}}$

$$\operatorname{Sin} A = \frac{2}{\sin b \sin c}\left\{\sin s \sin (s-a) \sin (s-b) \sin (s-c)\right\}^{\frac{1}{2}}$$

$$\text{Sin } \frac{a}{2} = \sqrt{-\frac{\cos S \cos (S-A)}{\sin B \sin C}};$$

$$\cos \frac{a}{2} = \sqrt{\frac{\cos (S-B) \cos (S-C)}{\sin B \sin C}},$$

$$\tan \frac{a}{2} = \sqrt{-\frac{\cos S \cos (S-A)}{\cos (S-B) \cos (S-C)}}.$$

Napier's Analogies.

$$\text{Tan } \tfrac{1}{2} (A + B) = \frac{\cos \frac{1}{2} (a-b)}{\cos \frac{1}{2} (a+b)} \cot \tfrac{1}{2} C;$$

$$\text{Tan } \tfrac{1}{2} (A - B) = \frac{\sin \frac{1}{2} (a-b)}{\sin \frac{1}{2} (a+b)} \cot \tfrac{1}{2} C.$$

$$\text{Tan } \tfrac{1}{2} (a + b) = \frac{\cos \frac{1}{2} (A-B)}{\cos \frac{1}{2} (A+B)} \tan \tfrac{1}{2} c;$$

$$\text{Tan } \tfrac{1}{2} (a - b) = \frac{\sin \frac{1}{2} (A-B)}{\sin \frac{1}{2} (A+B)} \tan \frac{c}{2}.$$

Area of lune $= \dfrac{A}{2\pi} \cdot 4\pi r^2 = 2Ar^2$.

Area of spherical triangle $= \dfrac{A+B+C-\pi}{2\pi} \cdot 2\pi r^2 =$

$\dfrac{\text{spherical excess}}{\text{4 right angles}} \times$ area of hemisphere.

FORMULÆ IN MENSURATION.

Area of rectangle $= ab$; a and b being adjacent sides.

Area of parallelogram $= ab \sin \theta$, a and b being adjacent sides including an angle θ.

Area of triangle $= \dfrac{pc}{2} = \dfrac{ab}{2} \sin C =$

$$\sqrt{\{s(s-a)(s-b)(s-c)\}} = \frac{c^2}{2} \cdot \frac{\sin A \sin B}{\sin (A+B)}$$

p being the $\perp$r drawn from $\angle$ A to side c; $s = \frac{1}{2}(a+b+c)$.

Area of trapezoid $= \dfrac{(a+b)\,h}{2}$;

h being the $\perp$r distance between the parallel sides a, b.

Area of trapezium inscribed in circle $=$

$\sqrt{\{(s-a)(s-b)(s-c)(s-d)\}}$, $s = \frac{1}{2}(a+b+c+d)$.

Area of regular polygon $= \dfrac{na^2}{4} \times \cot \dfrac{180°}{n}$;

$n =$ number of sides each equal to a.

Area of regular polygon of n sides inscribed in circle, radius $= r$, $= \dfrac{nr^2}{2} \sin \dfrac{360°}{n}$.

Area of regular polygon of n sides circumscribing same circle $= nr^2 \,.\, \tan \dfrac{180°}{n}$.

Circumference of circle $= 2\pi r$; $\pi = 3{\cdot}14159265$ etc. $= 3{\cdot}1416$ nearly.

Length of arc subtending $\angle$ A° at centre $= \dfrac{\pi r \mathrm{A}°}{180°}$.

Area of circle $= \pi r^2$; area of sector $= \pi r^2 \times \dfrac{\mathrm{A}°}{360°}$.

Area of segment of circle $= \dfrac{r}{2}(a-p)$,

p being the perpendicular drawn from the end of the arc a to radius through its origin.

Area of ring between two concentric circles $= \pi\,(r_1{}^2 - r_2{}^2)$ $= \pi\,(r_1 + r_2)\,(r_1 - r_2)$.

r_1 and r_2 being the radii of the outer and inner circles.

Surface of solid bounded by rectilineal figures $=$ sum of areas of bounding surfaces.

Curve surface of cylinder $= 2\pi rh$; $r =$ rad. of base, $h = \perp$ height;

Whole surface of cylinder $= 2\pi r\,(h+r)$.

Curve surface of cone $= \pi r l$; $l =$ slant height.

Curve surface of frustum of cone $= \pi\ (r_1 + r_2) l$
r_1, r_2, radii of ends; $l =$ slant height.

Surface of sphere $= 4\pi r^2$.

Curve surface of segment of sphere $= 2\pi r h$; $h =$ height of segment.

Curve surface of spherical zone $= 2\pi r h$; $h =$ height of zone.

Volume of cylinder $= \pi r^2 h$.

Volume of frustum of pyramid or cone

$$= \frac{h}{3}\left\{\Delta + \sqrt{\Delta\,\delta} + \delta\right\}$$

h, perpendicular height; Δ, δ areas of ends.

Volume of sphere $= \frac{4}{3}\pi r^3$.

Volume of segment of sphere $= \frac{\pi h}{6}\ (h^2 + 3r^2)$, r being radius of base, h, the height, of the segment; or volume $= \pi h^2\ (r - \frac{h}{3})$

r being radius of sphere; h, height of segment.

Volume of zone of sphere $= \frac{\pi h}{6}\left\{h^2 + 3\,(p^2 + q^2)\right\}$

p, q being radii of ends.

FORMULÆ IN CO-ORDINATE GEOMETRY.

Equation to straight line, axes rectangular, $y = mx + c$.

Equation to straight line, axes rectangular, through (x', y'), $y-y' = m\,(x-x')$.

Equation to straight line, axes rectangular, through (x', y') and (x'', y''); $y-y' = \frac{y''-y'}{x''-x'}(x-x')$.

Equation to straight line, axes rectangular, through (x', y') & $\perp$ to $y=mx+c$; $y-y' = -\frac{1}{m}(x-x')$.

Equation in terms of the intercepts of the axes, $\frac{x}{a} + \frac{y}{b} = 1$.

Equation in terms of perpendicular from origin, $x\cos\alpha + y\sin\alpha - p = 0$.

Equation when the axes are oblique, $y = \frac{\sin\alpha}{\sin(\omega-\alpha)}.\,x+c$.

Condition of perpendicularity of $y = mx + c$ and $y = m_1 x + c_1$; $mm_1 = -1$.

Length of perpendicular, from (x', y') upon $y = mx + c = \pm \frac{y'-mx'-c}{\sqrt{1+m^2}}$.

Length of perpendicular, from (x', y') upon $x\cos\alpha + y\sin\alpha - p, = x'\cos\alpha + y'\sin\alpha - p$.

Equation to circle, origin anywhere, $(x-a)^2 + (y-b)^2 = r^2$.

Equation to circle, origin at centre, $x^2 + y^2 = r^2$.

Equation to tangent at (x', y'), $xx' + yy' = r^2$; $y = mx + r\sqrt{1+m^2}$.

Equation to normal at (x', y'), $y = \frac{y'}{x'}.x$.

Equation to chord of contact, $xh + yk = r^2$.

Equation to parabola, origin at vertex, $y^2 = 4ax$.

Equation to tangent at (x', y'), $yy' = 2a(x + x')$; $y = mx + \frac{a}{m}$

Equation to normal at (x', y'), $y - y' = -\frac{y'}{2a}(x - x')$;
or $y = mx - 2am - am^3$.

Equation to ellipse, origin at centre, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Equation to tangent at (x', y'), $a^2yy' + b^2xx' = a^2b^2$,
or $y = mx + \sqrt{a^2m^2 + b^2}$.

Equation to normal at (x', y'), $y - y' = \frac{a^2y'}{b^2x'}(x - x')$,
or $y = mx - \frac{(a^2 - b^2)\,m}{\sqrt{b^2m^2 + a^2}}$.

Equation to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Equation to tangent at (x', y'), $a^2yy' - b^2xx' = -a^2b^2$,
or $y = mx + \sqrt{m^2a^2 - b^2}$.

Equation to normal at (x', y'), $y - y' = -\frac{a^2y'}{b^2x'}(x - x')$,
or $y = mx - \frac{(a^2 + b^2)\,m}{\sqrt{a^2 - b^2m^2}}$.

Polar equation to straight line, $p = r\cos(\theta - a)$.

Polar equation to circle, $c^2 = r^2 + l^2 - 2lr\cos(\theta - a)$.

Polar equation to parabola, focus, the pole, $r = \frac{2a}{1 + \cos\theta}$;
vertex, the pole, $r = \frac{4a\cos\theta}{\sin^2\theta}$.

Polar equation to ellipse, focus, the pole, $r = \dfrac{a(1-e^2)}{1+e\cos\theta}$;
centre, the pole, $r^2(a^2\sin^2\theta + b^2\cos^2\theta) = a^2b^2$.

Polar equation to hyperbola, focus, the pole, $r = \dfrac{a(e^2-1)}{1+e\cos\theta}$;
centre, the pole, $r^2(a^2\sin^2\theta - b^2\cos^2\theta) = -a^2b^2$.

FORMULÆ IN DIFFERENTIAL CALCULUS.

$u = x^n,\ \dfrac{du}{dx} = nx^{n-1}$; $u = \log_a x,\ \dfrac{du}{dx} = \dfrac{1}{x\log_e a}$; if $a = e$,
$\dfrac{d(\log_e x)}{dx} = \dfrac{1}{x}$.

$u = a^x,\ \dfrac{du}{dx} = a^x\log_e a$; if $a = e$, $\dfrac{d(e^x)}{dx} = e^x$.

$u = z^n$, z a function of x, $\dfrac{du}{dx} = az^{n-1}\dfrac{dz}{dx}$.

$u = a^z$, z a function of x, $\dfrac{du}{dx} = a^z\log_e a\,\dfrac{dz}{dx}$.

$u = z^v$, z and v functions of x, $\dfrac{du}{dx} = z^v\left\{\dfrac{dv}{dx}\log_e z + \dfrac{v}{z}\cdot\dfrac{dz}{dx}\right\}$

$u = f(z)$, z a function of x, $\dfrac{du}{dx} = \dfrac{du}{dz}\cdot\dfrac{dz}{dx}$.

$u = \sin\dfrac{x}{a},\ \dfrac{du}{dx} = \dfrac{1}{a}\cos\dfrac{x}{a}$; $u = \cos\dfrac{x}{a},\ \dfrac{du}{dx} = -\dfrac{1}{a}\sin\dfrac{x}{a}$.

$u = \tan\dfrac{x}{a},\ \dfrac{du}{dx} = \dfrac{1}{a}\sec^2\dfrac{x}{a}$; $u = \cot\dfrac{x}{a},\ \dfrac{du}{dx} = -\dfrac{1}{a}\operatorname{cosec}^2\dfrac{x}{a}$.

$u = \sec\dfrac{x}{a},\ \dfrac{du}{dx} = \dfrac{1}{a}\sin\dfrac{x}{a}\sec^2\dfrac{x}{a}$;

$u = \operatorname{cosec}\dfrac{x}{a},\ \dfrac{du}{dx} = -\dfrac{1}{a}\cos\dfrac{x}{a}\operatorname{cosec}^2\dfrac{x}{a}$.

$u = \sin^{-1}\frac{x}{a}$; $\frac{du}{dx} = \frac{1}{\sqrt{a^2 - x^2}}$; $u = \cos^{-1}\frac{x}{a}$, $\frac{du}{dx} = -\frac{1}{\sqrt{a^2 - x^2}}$.

$u = \tan^{-1}\frac{x}{a}$, $\frac{du}{dx} = \frac{a}{a^2 + x^2}$; $u = \cot^{-1}\frac{x}{a}$, $\frac{du}{dx} = -\frac{a}{a^2 + x^2}$.

$u = \sec^{-1}\frac{x}{a}$, $\frac{du}{dx} = \frac{a}{x\sqrt{x^2 - a^2}}$;

$u = \operatorname{cosec}^{-1}\frac{x}{a}$, $\frac{du}{dx} = -\frac{a}{x\sqrt{x^2 - a^2}}$.

$u = \operatorname{vers}^{-1}\frac{x}{a}$, $\frac{du}{dx} = \frac{1}{\sqrt{2ax - x^2}}$.

Leibnitz's Theorem.

If $u = yz$, y and z functions of x, $\frac{d^n u}{dx^n} = y\frac{d^n z}{dx^n} + n\frac{dy}{dx}$.

$$\frac{d^{n-1}z}{dx^{n-1}} + \frac{n(n-1)}{\lfloor 2} \cdot \frac{d^2y}{dx^2} \cdot \frac{d^{n-2}z}{dx^{n-2}} + \dots + \frac{\lfloor n}{\lfloor r \, \lfloor n-r} \cdot$$

$$\frac{d^r y}{dx^r} \cdot \frac{d^{n-r}z}{dx^{n-r}} + \frac{\lfloor n}{\lfloor r+1 \, \lfloor n-r-1} \cdot \frac{d^{r+1}y}{dx^{r+1}} \cdot \frac{d^{n-r-1}z}{dx^{n-r-1}} \dots$$

$$+ \frac{d^n y}{dx^n} z.$$

Maclaurin's Theorem.

$$f(x) = f(0) + xf'(0) + \frac{x^2}{\lfloor 2} f''(0) + \dots$$

$$+ \frac{x^n}{\lfloor n} f^n(0) + \frac{x^{n+1}}{\lfloor n+1} f^{n+1}(\theta x).$$

Taylor's Theorem.

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{\lfloor 2} f''(x) + \dots$$
$$+ \frac{h^n}{\lfloor n} f^n(x) + \frac{h^{n+1}}{\lfloor n+1} f^{n+1}(x+\theta h).$$

Equation to tangent, $y' - y = \frac{dy}{dx}(x' - x)$.

Equation to normal, $y' - y = -\frac{1}{\frac{dy}{dx}}(x' - x)$.

Subtangent $= y\frac{dx}{dy}$; subnormal $= y\frac{dy}{dx}$.

Differential coefficient of an arc, $\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$\therefore\ 1 = \left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2$.

Differential coefficient of an area, $\frac{dA}{dx} = y$.

Differential coefficient of volume of solid of revolution, $\frac{dV}{dx} = \pi y^2$.

Differential coefficient of surface of solid of revolution, $\frac{dS}{dx} = 2\pi y \cdot \frac{ds}{dx}$.

Radius of curvature, $\rho = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$

Parabola, $\rho = \frac{2(a+x)^{\frac{3}{2}}}{\sqrt{a}}$; Ellipse, $\rho = \frac{(a^2 - e^2x^2)^{\frac{3}{2}}}{ab}$;

Hyperbola, $\rho = \frac{(e^2x^2 - a^2)^{\frac{3}{2}}}{ab}$.

Cycloid, $\rho = 2\sqrt{2ay}$.

FORMULÆ IN INTEGRAL CALCULUS.

$$\int ax^m dx = \frac{ax^{m+1}}{m+1}; \int \frac{dx}{x} = \log x; \int a^x dx = \frac{a^x}{\log_e a};$$

$$\int e^x dx = e^x.$$

$$\int \sin x\, dx = -\cos x; \int \cos x\, dx = \sin x;$$

$$\int \sec^2 x\, dx = \tan x; \int \operatorname{cosec}^2 x\, dx = -\cot x.$$

$$\int \sin mx\, dx = -\frac{1}{m}\cos mx; \int \cos mx\, dx = \frac{1}{m}\sin mx;$$

$$\int \frac{dx}{\sin x} = \log \tan \frac{x}{2}.$$

$$\int \frac{dx}{\cos x} = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right); \int \frac{dx}{\sin x \cos x} = \log \tan x;$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}; \quad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}.$$

$$\int \frac{-dx}{\sqrt{a^2 - x^2}} = \cos^{-1}\frac{x}{a}; \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a};$$

$$\int \frac{dx}{x\sqrt{2ax - x^2}} = \text{vers}^{-1}\frac{x}{a}.$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \cdot \log \frac{x-a}{x+a};$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log \left\{x + \sqrt{x^2 \pm a^2}\right\};$$

$$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = \frac{1}{a} \cdot \frac{\log x}{a + \sqrt{a^2 \pm x^2}}.$$

Lengths of Curves.

$$s = \int \sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}} dx.$$

Parabola, $s = \sqrt{ax + x^2} + a \log \frac{\sqrt{x} + \sqrt{a+x}}{\sqrt{a}}$;

Cycloid, $s = \sqrt{8ax}$; Catenary, $s = \frac{c}{2}\left(e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right)$.

Ellipse, $s = 2\pi a\left\{1 - \frac{e^2}{2^2} - \frac{1 \cdot 3e^4}{2^2 \cdot 4^2} - \text{etc.}\right\}$

Areas of Curves.

$$A = \int f(x)dx.$$

Circle, $A = \pi r^2$; Sector, $A = \frac{ra}{2}$; Ellipse, $A = \pi ab$;

Cycloid, $A = 3\pi r^2$.

Parabola, $A = \frac{2}{3}$ of circumscribing rectangle.

Surfaces of Solids.

$$S = \int 2\pi y ds = \int 2\pi y \frac{ds}{dy} dy = \int 2\pi y \frac{ds}{dx} dx.$$

Sphere, $S = 4\pi r^2$; Cylinder, $S = 2\pi r (x_2 - x_1)$;

Frustum of cone $S = \pi \tan \alpha \sec \alpha (x_2^2 - x_1^2)$.

Cone, $S = \pi \operatorname{cosec} \alpha r^2$.

Paraboloid, $S = \frac{8\pi\sqrt{a}}{3}\left\{(x+a)^{\frac{3}{2}} - a^{\frac{3}{2}}\right\}$.

Oblate spheroid, $s = 2\pi a^2\left\{1 + \frac{1-e^2}{2e}\log\frac{1+e}{1-e}\right\}$;

Prolate spheroid, $s = 2\pi a^2\left\{\sqrt{1-e^2}\ \frac{\sin^{-1}e}{e} + 1 - e^2\right\}$

Volumes of Solids.

$$v = \int \pi y^2 dx + c;\ v_2 - v_1 = \pi\int_{x_1}^{x_2} y^2 dx.$$

Sphere, $v = \frac{4\pi r^3}{3}$; Pyramid, $v = \frac{1}{3}a^2 h$; Cone, $v = \frac{1}{3}\pi r^2 h$.

Paraboloid $v = \frac{1}{2}$ circumscribing cylinder;

Prolate spheroid, $v = \frac{4}{3}\pi ab^2$;

Oblate spheroid, $v = \frac{4}{3}\pi a^2 b$; Solid generated by revolution of cycloid about its axis, $v = \pi a^3\left\{\frac{3\pi^2}{2} - \frac{8}{3}\right\}$.

CIVIL SERVICE PAPERS.

I.

1. Write in figures one million four hundred and eighty-seven thousand six hundred and twenty-three; and express in words 70600804016003.

2. From £4768954 13*s.* 7$\frac{1}{2}$*d.* take £2989767 18*s.* 11$\frac{3}{4}$*d.*

3. (1) Multiply £2487 14*s.* 2$\frac{1}{4}$*d.* by 3076: (2) divide £7652194 0*s.* 9*d.* by 3076.

4. (1) Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$: (2) divide $\frac{3}{8}$ of $\frac{8}{9}$ by $\frac{7}{8}$ of $\frac{3}{4}$: (3) Multiply 332·1 by 44·1: (4) divide 2·5 by ·32.

5. Calculate the duty at £15 per cent. on £963 15*s.*

6. What is the income of a person who pays £22 7*s.* 5*d.* for income-tax at the rate of 7*d.* in the pound?

7. Find the value of 136 at £1 7*s.* 2*d.*

8. What is the expense of digging a ditch of which the solid content is 5755 cub. yds. at 15*s.* 7$\frac{3}{4}$*d.* per yard.

9. At what rate will the interest on £4127 10*s.* amount to £92 17*s.* 4$\frac{1}{2}$*d.* in a year?

10. Find the compound interest on £364 for 3 years at 3$\frac{1}{4}$ per cent.

11. Assuming that 32 bricks will make 9 sq. ft. of pavement, how many will be required to pave a yard 63 ft. long by 36 ft. wide?

12. How much time, in the course of 33 yrs., will a person, who rises at 5 o'clock in the morning, gain over another, who rises at 8 o'clock? supposing both to go to bed at 10 P.M. and computing 365$\frac{1}{4}$ days to the year.

II.

1. Express in figures (1) Nine millions ten thousand and nineteen; (2) Five thousand and six million eight *thousand and ninety*-four.

2. Divide £89075 12*s.* 6*d.* by 9; and £713202 12*s.* by 576.

3. Reduce (1) £527 16*s.* $8\frac{1}{2}$*d.* to farthings: (2.) 47 cwts. 1 qr. 16 lbs. 2 oz. to drams.

4. (1) Reduce $\frac{7040}{7392}$ to its lowest terms: (2) add $\frac{1}{4}$, $\frac{5}{12}$, $\frac{7}{15}$, $\frac{3}{20}$.

5. (1) Add 35·2176, 201·00541, 3·1482, ·054, 7543·4: (2) take 5·7124 from 3017·215: (3) multiply 5412·384 by 1·0023: (4.) divide 21546·8306 by ·0542.

6. Find the income-tax on £875 15*s.* at 14*d.* in the pound.

7. If a rental of £8050 a year be charged with land-tax at £11 5*s.* per £100, find the net income.

8. Find the value (1) of 373 at 9*s.* $7\frac{3}{4}$*d.*; (2.) of 821 at 13*s.* $10\frac{3}{4}$*d.* by practice.

9. Find the interest on £1158 17*s.* 6*d.* for 1 yr. 115 da. at $2\frac{1}{2}$ per cent.

10. What sum of money put out at interest, for 6 yrs., at $3\frac{1}{2}$ per cent., will amount to £259 2*s.* 10*d.*?

11. A ship is worth £16,000, and a person possessing $\frac{3}{8}$ of her, sells $\frac{5}{18}$ of his share, how much is the remainder of his share worth?

12. How many yards of paper 29 inches wide, will paper a room which is 22 ft. 9 in. long, 17 ft. 3 in. wide, and 9 ft. 8 in. high?

III.

1. Multiply 53786417276 by 5846, and divide 632799181 by 7243.

2. (1) Multiply £3001 6*s.* $4\frac{1}{2}$*d.* by 351, and (2) divide £20954 17*s.* $2\frac{1}{4}$*d.* by 9.

3. How many lbs. of silver are there in 270 spoons, each weighing 1 oz. 13 dwts. 8 grs.? and how many seconds in 35 d. 8 h. 48 m. 29 sec.?

4. Add $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{11}$ and $\frac{2}{13}$ of $7\frac{4}{5}$, and subtract the result from $3\frac{3}{35}$.

5. (1) Subtract 239·89759 from 1314·9 ; and (2) multiply 95·376 by ·0283.

6. If a tax on an income of £1132 amount to £124 19*s.* 10*d.*, what is that in the pound ?

7. If the carriage of 107 cwts. 10 lbs. cost £37 9*s.* 7½*d.*, what would the carriage of a ton cost?

8. Find the price of 329 yds. 3 qrs. 2 nls. at 5*s.* 2½*d.* per quarter.

9. Find the discount on £517 10*s.* due 8 months hence at 5¼ per cent. simple interest.

10. If telegraph posts are 66 yards apart, and a train passes one every 3″, how fast is the train moving?

11. If $\frac{6}{7}$ of 1 qr. cost 54*s.*, find the cost of $\frac{4}{5}$ of 1 bushel.

12. A coach goes 9 mi. an hour, and a train goes 21 mi. whilst the coach goes 11 ; how much time will be saved on a journey of 378 miles, by taking the train instead of the coach ?

IV.

1. Multiply 567039884276 by 8, and 403692158 by 396.

2. Divide 5802476484 by 6, and 294683010 by 865.

3. From £40132 7*s.* 2¼*d.* take £37849 15*s.* 7½*d.*

4. Multiply £25841 16*s.* 5¼*d.* by 7, and £1854 7*s.* 7½*d.* by 465.

5. Divide £85432 19*s.* 5½*d.* by 7, and £27543 12*s.* 8*d* by 352.

6. A person having £5704 18*s.* 4½*d.* lays out ⅓ in goods which he sells for £2316 5*s.* 10*d.* ; how much has he gained?

7. Reduce (1) 375 cwts. 2 qrs. 1 st. 13 lbs. to oz., and (2) 36 mi. 3 fur. 36 po. 5 yds. to feet.

8. If £59 10*s.* will buy 25 cwts. 2 qrs. of sugar, what weight will £6 14*s.* 2*d.* buy ?

9. If the 1*d.* loaf weighs 5 oz. when wheat is 62*s.* per qr., *what ought it to* weigh when wheat is 64*s.* per quarter ?

10. Find, by Practice, the value of; (1) 373 articles at £2 16*s.* 10$\frac{1}{4}$*d.* each; (2) the rent of 67 acres of land for 2 yrs. 9 mo. 11 da., at £2 5*s.* an acre per annum.

11. Find the interest on £1275 15*s.* for 5$\frac{3}{4}$ years at 4 per cent. per annum.

12. At what rate per cent., simple interest, will £951 7*s.* 6*d.* amount to £1141 13*s.* in 5 years ?

V.

1. The Norman Conquest took place in 1066 A.D., the accession of Queen Victoria in 1837; how many years elapsed between these events?

2. (1) Multiply £36758 17*s.* 5$\frac{1}{4}$*d.* by 7, and (2) £1563 17*s.* 9$\frac{3}{4}$*d.* by 357.

3. Divide £13854 9*s.* 2$\frac{1}{2}$*d.* by 158.

4. (1) How many solid yards, feet and inches, are there in 175893 solid inches?. And (2) how many square yds., ft. and in. are there in the top of a table, the sides of which are 21 ft. 7$\frac{1}{2}$ in. and 19 ft. 5 in. ?

5. (1) Multiply $\frac{3}{5}$ of $\frac{7}{43}$ of 15 by $\frac{14}{15}$ of 12$\frac{2}{7}$; (2) what number multiplied by $\frac{3}{5}$ will produce 15$\frac{3}{5}$?

6. What fraction of half-a-crown is $\frac{2}{9}$ of 6*s.* 8*d.* ?

7. If 24 men, working 8 hrs. a day, can build a house in 70 days, in how many days might it be built by 42 men, working 10 hrs. a day ?

8. Find, by Practice, (1) the price of 329 yds. 3 qrs. 2 nls. at 5*s.* 2$\frac{1}{2}$*d.* per quarter; (2) the dividend on £2734 16*s.* 8*d.* at 9*s.* 4$\frac{1}{2}$*d.* in the pound.

9. (1) Express $\frac{241}{15424}$ as a decimal; (2) reduce 8*s.* 11$\frac{1}{4}$*d.* to the decimal of 1 guinea.

10. Find the sum of ·3211 guin. $\frac{3}{8}$ cr., ·6925*s.*, and reduce the result to the decimal of £1.

11. Find the interest on £4000, in 6 months, at 5 per cent., less income-tax at 14*d.* in the £.

12. What sum will amount to £798 15*s.* in 1 year at 6$\frac{1}{2}$ per cent.?

VI.

1. (1) Reduce 6 ac. 1 ro. 4 po. to the fraction of $2\frac{1}{2}$ roods; (2) if $\frac{3}{5}$ lb. cost 8*s*. 2*d*. find the cost of $5\frac{1}{7}$ lb.

2. Multiply 41·018 by 200, and 1·02 by 3·067; divide 69·814 by ·00521 and by 52100.

3. Reduce (1) $\frac{7}{8}$ hf. cr. to the decimal of 1 guin.; (2) 2·0445, (3) ·000625, (4) $\cdot 1\dot{4}\dot{4}$, (5) $\cdot 34\dot{2}1\dot{8}$ to vulgar fractions; (6) $\frac{3}{32}$, $\frac{40}{33}$ to decimal fractions.

4. The shares of a railway are at $59\frac{1}{2}$ when consols are at $93\frac{1}{2}$; what should be their value when consols are at $71\frac{1}{2}$?

5. If 49 men can do a piece of work in 130 days of 8 hrs. each, how many hrs. a day must 196 men work to do as much in 26 days?

6. Convert 22 lbs. 5 oz. 15 dwts. 20 grs. troy into avoirdupois weight, 1 lb. avoirdupois being to 1 lb. troy as 175 to 144.

7. Find, by Practice, the value of 5 lbs. 9 oz. 7 dwts. 12 grs. of gold at £3 17*s*. 10*d*. an ounce.

8. What sum of money put out at interest for 3 years at $4\frac{1}{2}$ per cent. will amount to £1602 $1\frac{1}{4}$*d*.?

9. In what time will £1337 12*s*. 1*d*. amount to £1698 15*s*. $1\frac{3}{4}$*d*., at 6 per cent. simple interest?

10. If £3 = 20 thalers, 25 thalers = 93 francs, 62 francs =25 gulden: how many gulden are equal in value to £1?

11. If 20 horses and 196 sheep can be kept 18 days for £151 10*s*., what sum will keep 15 horses and 72 sheep for 8 days, if 5 horses eat as much as 76 sheep?

12. Find the amount of £13333 6*s*. 8*d*. for 4 years at 5 per cent. compound interest.

VII.

1. Reduce $\frac{3432}{25574}$ to its lowest terms and divide the result *by* $1\frac{1}{2}$.

2. Simplify (1) $\frac{3}{4} + \frac{2}{3} + \frac{7}{12} + \frac{5}{96} + \frac{11}{100}$ and take the result from $2\frac{1}{6}$; (2) $3\frac{4}{7}$ of $\frac{47}{95}$ of $\dfrac{38}{20\frac{1}{7}} \times 6\frac{2}{3}$; (3) $\frac{5}{8}$£ + $\frac{7}{9}$ of £2 10*s.* 3*d.* + $3\frac{1}{3}$ of 14*s.*

3. Take the sum of ·004, 2·08, 314·6073, ·19874, 1000, from 21118·0116.

4. (1) Multiply ·671 by ·42 and by ·0420; (2) divide 738·952716 by ·07 and by 70.

5. How many francs at $9\frac{1}{2}$*d.* each are there in £693 14*s.* 9*d.* ?

6. The area of a rectangular plot is 101 yds. 0 ft. 81 in.; its length is 36 ft. 9 in. Find the breadth.

7. If when wheat is 11*s.* 3*d.* a bushel, the sixpenny loaf weighs 28 oz., how much should it weigh when wheat is 84*s.* a quarter?

8. If the carriage of 65 lbs. for 120 miles costs 6*s.*, how much may I have carried 750 miles for a guinea?

9. Find, by Practice, the value of 2 roods 19 poles 12 yards of land at £80 13*s.* 4*d.* an acre.

10. What sum of money put out at interest for $3\frac{1}{2}$ years at 4 per cent. will amount to £296 8*s.* ?

11. In what time will £171 7*s.* 6*d.* amount to £217 1*s.* 6*d.* at $3\frac{1}{3}$ per cent.?

12. Find the discount on £1511 7*s.* 6*d.*, payable $3\frac{1}{4}$ years hence, at 4 per cent. simple interest.

VIII.

1. Reduce £125 10*s.* into francs and centimes, exchange at 23 francs 6 centimes per £1 (1 franc = 100 centimes).

2. What salary is due for 95 days at £32 17*s.* a year?

3. An officer's pay is 12*s.* 3*d.* a day; how much is that a year?

4. A banker owes £24680, and can pay 15*s.* 6*d.* in the pound, what is the value of his assets?

5. What is the amount of the weekly wages of 377 men at 1*s.* 8*d.* a day?

6. A coach goes from London to Liverpool, at the rate of 9 miles an hour, in 24 hrs.; in what time would the same distance be traversed by a train moving at the rate of 24 miles an hour?

7. A besieged town containing 22400 inhabitants has provisions for 3 weeks; how many must leave in order that the provisions may last 7 weeks?

8. If 5 men receive £18 15*s.* for 12 weeks' wages, what will be the wages of 16 men for 20 weeks?

9. If 6 iron bars 4 ft. long, 3 in. broad, and 2 in. thick, weigh 288 lbs., how much will 15 weigh each $6\frac{1}{2}$ ft. long, 4 in. broad, and 3 in. thick?

10. Find the value of 345 tuns 2 hhds. 42 gals. at £45 10*s.* per tun.

11. Reduce (1) $\cdot 00175$ and (2) $\cdot 13\dot{0}\dot{9}$ to fractions in their lowest terms.

12. $44\frac{1}{2}$ guineas used to be coined out of 1 lb. of standard gold, how many sovereigns are now coined out of this weight?

IX.

1. Reduce (1) 13 lbs. 7 oz. 16 grs. to grains; (2) 1847638 in. to miles &c., (3) 138297 cubic inches to yards.

2. Find, by Practice, the value of 3 lbs. 5 oz. 14 dwts. 12 grs. at 17*s.* 6*d.* an oz.

3. Find the value of (1) $\frac{2}{7} \times \dfrac{\frac{5}{11} \text{ of } 5\frac{1}{2} \times 2\frac{1}{3}}{\frac{4}{9} \text{ of } \frac{3}{13} \times 8\frac{2}{3}}$;
(2) £$\frac{2}{3}$ + $\frac{3}{7}$ of £2 10*s.* 2*d.* + $2\frac{1}{3}$ of 16*s.*

4. Reduce $\frac{5}{64}$, $\frac{3}{14}$ to decimals; and express $23\cdot 038125$, $\cdot 0006875$ as vulgar fractions in their lowest terms.

5. Reduce $\frac{5}{8}$ of 1 cr. to the decimal of 21*s.*; and $6\frac{2}{3}$ cwt. to the dec. of 1 ton.

6. Reduce $\cdot 1\dot{3}\dot{5}$, $\cdot 012\dot{3}\dot{6}$, to their equivalent vulgar fractions.

7. A bankrupt's effects amount to 2548 guineas, and his debts to £3057 12*s.*, what will he pay in the £?

8. If 5 men can do a piece of work in 27 days, how long would 6 men be in doing double the quantity?

9. Find the interest on £260 10*s*. for $3\frac{1}{3}$ years, at $4\frac{1}{4}$ per cent.

10. At what rate per cent. will £320 amount to £360 16*s*. in 3 years?

11. Find how many yards of paper $\frac{7}{8}$ yd. wide, will paper a room whose length is 26 ft. 4 in., breadth 18 ft. 8 in., height 12 ft. 3 in.?

12. Extract the square root, (1) of 531441; (2) of $32\frac{1}{9}$.

X.

1. (1) Reduce $\frac{6398}{7312}$ to its lowest terms; (2) divide the sum of $\frac{2}{5}$, $\frac{4}{9}$, $\frac{3}{10}$, $\frac{7}{18}$ by $7\frac{5}{6}$; (3) find the value of ($\frac{2}{3}$ of $\frac{7}{8}$ + $\frac{3}{7}$ of $\frac{14}{15}$) of 1*s*.

2. (1) Multiply 3·05 by ·25 and ·32 by ·231: (2) divide 721·41885 by 21·9, and ·1 ÷ ·001 by ·2.

3. The price of consols is $88\frac{3}{8}$; how many railway shares, at £8 15*s*. each, can be purchased for £1000 stock?

4. The rates of the express and mail trains on a railway are 40 and 28 miles an hour respectively: what time is saved by taking the faster for a journey of 192 miles?

5. If 21 men take 8 days to dig 20 ac., how many acres will 16 men dig in 12 days?

6. Find, by Practice, the cost (1) of 11 mi. 3 fur. 55 yds. of railway at £32500 a mile; (2) of 716 ac. at £44 11*s*. $3\frac{1}{4}$*d*. an acre.

7. Find the amount of £322 13*s*. for $10\frac{1}{4}$ years at $3\frac{1}{2}$ per cent. simple interest.

8. Find the discount on £1452, paid 2 months before it is due, at 5 per cent.

9. Find the amount of £240 in 4 years at 3 per cent., compound interest.

10. If 23 lbs. at 2*s*. are mixed with 27 lbs. at 3*s*., and the mixture is sold at 2*s*. 9*d*. per lb., what is the total gain; and what the gain per cent.?

11. Extract the square root of 106929 ; also of 803·7 to 3 places of decimals.

12. Explain the meaning of the term *local* value. Have Roman numerals any local value ?

XI.

1. Multiply (1) 85001997 by 5, and (2) 73299863 by 497.

2. Divide 2975069895 by 35, and 628010082036 by 709.

3. From £19813 7*s.* $7\frac{1}{2}$*d.* take £2009 6*s.* $10\frac{3}{4}$*d.*

4. Multiply (1) £35087 17*s.* $7\frac{1}{2}$*d.* by 9, and (2) £9005 19*s.* $1\frac{1}{2}$*d.* by 39.

5. How many yards of paper 1 yd. wide are required to paper a room 15 ft. long, 12 ft. wide, and 10 ft. high ?

6. How often will a wheel $3\frac{1}{4}$ yds. in circumference turn in a distance of 208 miles ?

7. If by paying down £70 17*s.* 6*d.* a person becomes entitled to £2 9*s.* 7*d.* a year, what income should he gain by paying down £2126 5*s.* ?

8. Find the value of ; (1) 9 tons 4 cwts. 3 qrs. 21 lbs. at £14 5*s.* 9*d.* a ton ; (2) of $217\frac{1}{4}$ yards of lace at £2 17*s.* $7\frac{1}{2}$*d.* a yard.

9. Find the interest on £194, at 5 per cent. per annum, from the 3rd of March to the 20th of December following.

10. When the 3 per cents. are at £85 17*s.* 6*d.*, what is the rate of interest?

11. Find the sum of (1) £$3\frac{5}{8}$, $9\frac{3}{7}$*s.* and $2\frac{4}{5}$*d.* : (2) of ·130055, 900, 57·1, 13·34 and ·00000397.

12. Divide the quotient of 16 ÷ 2 by ·0002.

XII.

1. (1) Multiply £3467 5*s.* $7\frac{1}{2}$*d.* by 346 ; (2) divide 2665 miles 2 furl. 31 poles $\frac{1}{2}$ yard by 58.

2. A train is travelling at the rate of $32\frac{1}{2}$ miles per hour ; over how many feet does it pass per second ?

3. If 84 men can do a piece of work in two months, how long will it take 12 men to do the same?

4. If the carriage of 18 tons 13 cwts. 3 qrs. 14 lbs. cost 18*s.* $7\frac{3}{4}$*d.*, what will be the cost of carriage of 255 tons, 9 cwts. 2 qrs. 14 lbs.?

5. What is the value of $\dfrac{\frac{2}{5} \text{ of } 5\frac{3}{4} - \frac{3}{8} \text{ of } 4\frac{1}{4}}{\frac{7}{8} \text{ of } \frac{3}{7} \text{ of } 5\frac{1}{4} - \frac{5}{8} \text{ of } \frac{4}{15} \text{ of } 3\frac{2}{3}}$

6. Find, by Practice, the value of 1389 tons of coal at £1 13*s.* 8*d.* per ton.

7. What decimal of a pound troy is 16 dwts.?

8. What are the values of ·346 of a pound sterling, and of ·058 of a year?

9. (1) Multiply 32·47 by ·0033; (2) divide 1·0226816 by ·0436.

10. What sum placed out at interest at 4 per cent. would amount to £560 in 3 years, allowing simple interest.

11. Divide £350496 among 4 persons in the proportion of 3, 7, 9, and 11.

12. A sum of £250 17*s.* 6*d.* is transmitted through Paris to New York. One sovereign English is worth 24 francs 79 cent. at Paris, and 6 francs 20 cent. French are worth 2 dollars 25 cents American. What is the value of the sum at New York in American currency?

COLLEGE OF PRECEPTORS.

XIII.

1. What sum added to £43 14*s.* $3\frac{3}{4}$*d.* will make 48000 farthings?

2. Multiply 3 tons 14 cwts. 2 qrs. 14 lbs. 12 oz. by 38, and prove the result.

3. A person bought 240 yards of cloth at 14*s.* 9*d.* a yard, and sold it at 15*s.* 7*d.* a yard; how much did he gain?

4. How long a time would it take a column of men, extending half a mile in length, to pass by, at the rate of 75 steps a minute, reckoning each step as 2 feet 8 inches ?

5. Explain clearly the meaning of a vulgar fraction. State the rule for the addition of vulgar fractions, and prove it by an example.

6. (1) Add together $\frac{3}{4} + \frac{5}{6} + \frac{11}{12} + \frac{14}{15}$.
   (2) From $2\frac{1}{3}$ of $4\frac{1}{19}$ take $2\frac{1}{3} + 4\frac{1}{19}$.
   (3) Divide $5\frac{1}{4} + 3\frac{9}{11}$ by $(\frac{3}{7} \div 1\frac{4}{7})$.

7. How often is $\frac{3}{5}$ of £1 11*s*. 8*d*. contained in 19 guineas ?

8. If $5\frac{1}{2}$ yards make one pole, show that there are $30\frac{1}{4}$ square yards in a square pole.

9. A man working 10 hours a day completes a piece of work in 12 days ; in how many days could he do it, working 8 hours a day ?

10. If a garrison of 1500 men, having provisions for 5 weeks, be reinforced with 500 men, how long will the provisions last ?

11. Find, by Practice, the rent of $209\frac{1}{4}$ acres, at £2 17*s*. 6*d*. per acre.

12. Find the area of a path 6 feet wide, which surrounds a court 30 yards long and 24 yards broad.

XIV.

1. Multiply 591863 by 907 ; and divide 10546279 by 84 by short division, (two factors).

2. What will be the cost of relieving 165000 poor persons at 1*s*. $3\frac{1}{2}$*d*. per head ?

3. If £1279 13*s*. $8\frac{3}{4}$*d*. be divided into sums of £55 12*s*. $9\frac{1}{4}$*d*. each, how many such sums will there be ?

4. A building society purchases three estates containing 35 a. 2 r. 23 p. $12\frac{1}{2}$ square yds., 9 a. 2 r. 36 p. $27\frac{1}{4}$ sq. yds., and 40 a. 0 r. 35 p. $20\frac{3}{4}$ sq. yds. respectively. If the property be divided into 128 equal parts, of how much does each part consist ?

5. Reduce to their simplest forms—

(1) $4\frac{3}{4} + \frac{3}{7}$ of $10\frac{1}{2} + 3\frac{3}{10}$. (2) $\dfrac{2\frac{1}{2}-\frac{1}{3}}{2\frac{1}{3}-1\frac{1}{4}}$. (3) $\dfrac{13667}{14186}$.

Is $\dfrac{13}{77}$ greater or less than $\dfrac{14\frac{1}{2}}{88}$, and by how much ?

6. Reduce $\frac{3}{10}$ of £2 0*s.* $3\frac{3}{4}$*d.* to the fraction of £1 4*s.* $2\frac{1}{4}$*d.*; and 2 qrs. 3 lbs. 1 oz. to the fraction of 1 cwt. 3 qrs. 0 lbs. 14 oz.

7. Find, by Practice, the price of 1298 yards at 17*s.* $9\frac{3}{4}$*d.* per yard, and the price of 15 yds. 2 ft. 8 in. at £1 19*s.* per yard.

8. Find the value of a wedge of gold weighing 14 lbs. 3 oz., if 2 oz. 3 dwts. be worth £6 9*s.*

9. If the carriage of 169 cwts. 2 qrs. for 130 miles cost £34, what weight may I have carried 78 miles for the same money ?

10. What length of paper 1 ft. 9 in. wide will be required to cover a wall 17 ft. 3 in. long, and 11 ft. 8 in. high ?

11. Divide £154 10*s.* among 3 railway companies in the proportions of 27, 31, 45 ; and show that the result is correct.

12. Extract the square root of 8450649.

XV.

1. Add together 72 tons 5 cwts. 15 lbs., 18 tons 6 cwts. 3 qrs., 14 cwts. 18 lbs., and multiply the sum by 56.

2. How many days, hours, minutes, and seconds are there in 1213459 seconds ?

3. In 1000 guineas how many crowns, florins, and three-penny pieces are there ?

4. If 144 tons of coals cost £181 16*s.*, find the price of one ton, and of 245 tons.

5. How many half-crowns, shillings, and farthings, an equal number of each, are there in £169 ?

6. What is meant by the expression $\frac{7}{8}$?

Give the rules for reducing whole numbers and mixed numbers to improper fractions; and for multiplying and dividing a fraction by a whole number. Give examples in each case.

7. (1) Add together $2\frac{15}{16}$, $3\frac{17}{18}$, $\frac{26}{27}$, and 4.
 (2) Subtract $\frac{5}{6} + \frac{7}{12} + \frac{3}{4}$ from $\frac{2}{3} + \frac{3}{4} + \frac{11}{12}$.
 (3) Multiply $2\frac{1}{4}$ of $10\frac{2}{3}$ of 15 by $7\frac{1}{5}$ of $\frac{1}{144}$.
 (4) Divide $2\frac{1}{5}$ of $3\frac{1}{4}$ by $5\frac{1}{2}$ of $4\frac{1}{3}$.

8. Find the value of $12\frac{4}{15}$ of £3 15*s*. $3\frac{3}{4}$*d*.

9. How much money will pay for 196 qrs. of wheat at £2 15*s*. 6*d*. per quarter, and $109\frac{1}{2}$ qrs. of oats at £1 17*s*. 11*d*. per quarter?

10. If 161 lbs. of tea are given in exchange for 12 cwts. 21 lbs. of sugar at $5\frac{3}{4}$*d*. per lb., what is the price of tea per lb.?

11. If a staff 5 feet high casts a shadow 7 ft. 2 in. long, what is the height of a steeple which casts a shadow 215 ft. long?

12. How many planks $15\frac{1}{4}$ feet long $1\frac{1}{4}$ feet wide will floor a room 60 feet long by $30\frac{1}{2}$ feet wide?

XVI.

1. Reduce 53 tons 4 cwts. 77 lbs. to ounces; and 1 mile 5 furl. 13 poles 2 inches, to inches.

2. What is the value of 687 Napoleons, a Napoleon being worth 15*s*. $11\frac{1}{2}$*d*.?

3. A person, after paying 10*d*. in the pound income-tax upon a yearly income of £400, laid by £85 17*s*.; what was his average weekly expenditure?

4. What sum must be added to £31 14*s*. $6\frac{1}{2}$*d*. to make it exactly divisible by £2 11*s*. 4*d*.?

5. What is meant by the Least Common Multiple of two or more numbers? Find that of 18, 51, 34, 7, 30.

6. Reduce $\frac{2160}{2295}$, $\frac{285714}{857142}$, to their lowest terms.

7. *Add together* (1) $5\frac{14}{15}$, $3\frac{19}{20}$, $11\frac{44}{45}$. (2) $\frac{7}{10}$, $\frac{11}{15}$, $\frac{13}{18}$, $\frac{5}{36}$.

8. Simplify $\dfrac{6\frac{13}{19}-4\frac{11}{18}}{\frac{2}{3}\div 15\frac{6}{11}}$

9. Find the value of $14\frac{14}{15}$ times £2 6*s*. $4\frac{3}{4}$*d.*

10. What is the cost of 31 oz. 15 dwts. 13 grs., at 7*s*. $7\frac{1}{2}$*d.* per oz.?

11. If the carriage of 3 cwts. 50 lbs. for 135 miles cost 15*s*., how far ought 5 cwts. 19 lbs. to be carried for the same money?

12. Divide 241 sq. yds. 8 sq. ft. 112 sq. inches by 46 sq. ft 8 sq. in.

XVII.

1. If 177 be the quotient, 379 the divisor, and 85 the remainder, what is the dividend?

2. Divide 6287644 by 121 by short division (two factors); and find the least number which added to 152181255 will make it exactly divisible by 3854.

3. Find the total weight of a luggage train of 23 carriages, the average weight of each carriage being 3 tons 5 cwts. 1 qr. 23 lbs.

4. From £1076 4*s*. $3\frac{3}{4}$*d*., 5*s*. $1\frac{3}{4}$*d*. is subtracted, and the remainder is divided equally among 527 people: how much will each have?

5. Reduce $27\frac{3}{4}$ guineas to three-pennies, and 1234567 inches to miles.

6. What is meant by the G. C. M. of two or more numbers? Find the G. C. M. of 13536 and 23148.

7. Simplify (1) $\frac{501}{1837}$. (2) $\frac{2}{3}+\frac{5}{6}+\frac{11}{12}+\frac{17}{18}$.
(3) $[(2\frac{1}{3}+\frac{1}{6})\times(3\frac{1}{2}+\frac{3}{8})]\div(2-\frac{1}{16})$.
(4) $\dfrac{1}{2+\dfrac{3}{4+\frac{5}{6}}}$

8. Express 15 sq. yds. 6 sq. ft., as the fraction of 1 acre 6 poles; and find the value of $\frac{3}{4}$ of a guinea $+\frac{3}{8}$ of a crown $-\frac{3}{8}$ of 7*s*. 6*d.*

9. Find, by Practice, the rent of 75 acres 2 roods 12 poles, at £1 11*s.* 6*d.* per acre.

10. If 5 pieces of cloth, each containing 12 yards, cost £17 10*s.*, what will 22 yards cost?

11. If 15 men in 9 days can reap a field, how long will 10 men require to reap a field of twice the area?

12. A cistern 8 ft. long, 3 ft. wide, and 9 inches deep, is filled with pulp for making paper. How long a sheet 2 feet 6 inches wide can be made from it, the thickness of the paper being $\frac{1}{250}$ of an inch, supposing $\frac{1}{2}$ the volume of the pulp to be lost in the process of drying?

XVIII.

1. Reduce 4 tons 5 cwts. 3 qrs. 27 lbs. to ounces, and 495 yards to ells. Prove the results.

2. Divide £395 14*s.* $0\frac{3}{4}$*d.* by 125, and prove the result.

3. Divide £20 among 3 persons, giving to the first £2 more than to the second, and to the second £1 more than to the third.

4. Find the G. C. M. of 17255 and 14161; and the L. C. M. of 40, 42, 44, 48. Give reasons for the method employed in finding the L. C. M.

5. Show that the value of a fraction is not altered by multiplying the numerator and denominator by the same number. Reduce $\frac{13455}{11385}$ to its lowest terms.

6. Reduce to simple fractions—

(1) $3\frac{2}{5} + 4\frac{3}{8} + \frac{1}{12} + 2\frac{19}{20}$.

(2) $5\frac{1}{11}$ of $1\frac{1}{14}$ of $4\frac{8}{9}$ of 3.

(3) $10\frac{1}{2} \times 3\frac{1}{7} - 4\frac{2}{5} \div 3\frac{1}{2}$.

7. Find the value of $\frac{3}{16}$ of a guinea + $1\frac{1}{6}$ of a crown + $\frac{2}{15}$ of half-a-crown, and reduce the result to the fraction of £1.

8. Find, by Practice, the value of—

(1) 397 things, at £3 2*s.* $9\frac{3}{4}$*d.* each.

(2) 135 sq. yds. 6 ft. 72 in., at 2*s.* 3*d.* per yard.

9. A grocer buys 200 lbs. of sugar at $5\frac{1}{4}d.$ per lb.; what does it cost him? At what price per lb. must he sell it to clear £1 9*s.* 2*d.*?

10. If 1 lb. 2 oz. 9 dwts. of silver cost £5 8*s.* $4\frac{1}{2}d.$, what is the price of an ounce?

11. In what time will 40 men do a piece of work which 36 men can do in 30 days?

12. If 11,000 copies of the *Times* be issued daily, each copy consisting of 2 sheets, and each sheet being 4 ft. by 3 ft., how many acres will one edition cover?

XIX.

1. How many half-guineas are there in a thousand half-crowns; and in 28624 inches, how many furlongs, poles, yards, feet, and inches?

2. How many steps of 2 ft. 9 in. each must be taken in a minute, in order to walk at the rate of four miles an hour?

3. Simplify (1) $\frac{1}{39}$ of $6\frac{1}{2}$ of $114-3\frac{8}{13}$.

(2) $\dfrac{7\frac{1}{2}\times\frac{1}{5}\text{ of }\frac{4}{9}}{13\frac{5}{12}-12\frac{7}{9}}$.

(3) Compare the values of $\dfrac{2\frac{1}{2}}{27}$, $\dfrac{8}{81}$, and $\dfrac{20}{243}$; and resolve 30 and 42 into their prime factors.

4. Express the difference between $\frac{1}{8}$ of 18*s.* 4*d.*, and $\frac{3}{8}$ of a crown, as a fraction of 2*s.* 6*d.*; and express $2\frac{2}{3}$ of 5 a. $3\frac{5}{8}$ r. as the fraction of 19 a. $2\frac{3}{4}$ r.

5. A bankrupt's debts amount to £869 10*s.*, and he can pay only 11*s.* 6*d.* in the pound. What is his estate worth? and how much will a creditor, to whom he owes £57 6*s.* 8*d.*, receive?

6. Find, by Practice, the rent of a farm of 225 a. 1 r. 19 p., at £2 12*s.* 10*d.* per acre per annum.

7. Divide the product of ·035 and ·0056 by ·00007. Also divide 5·04 by ·012, and 420 by ·420.

8. Find the difference between ·70323 of a pound, and 3·5646 of a shilling; and reduce 14 hrs. 15 min. to the decimal of $3\frac{1}{2}$ days.

9. What will a piece of plate cost, which weighs 3 lbs. 4 oz. 5 dwts., if the price of silver be £3 6*s.* per lb., and the charge for workmanship be 2*s.* 8*d.* per ounce?

10. How many hours a day must 42 boys work, to do in 45 days what 27 men can do in 28 days of 10 hours each, the work of a boy being half that of a man?

11. If 8 guineas be expended in purchasing Brussels carpet $\frac{3}{4}$ yd. wide, at 3*s.* 6*d.* per yard, for a room 20 ft. long and 16 ft. 9 in. broad; how much of the floor will remain uncovered?

12. At what rate per cent. will the interest on £375 amount to £48 2*s.* 6*d.* in 3 years 8 months?

XX.

1. How many acres, roods, poles, yards, feet, and inches are there in 40253798 square inches?

2. Simplify (1) $\frac{5}{7}-1\frac{3}{5}+2\frac{6}{45}+\frac{4}{7}$ of $2\frac{1}{5}-2\frac{3}{7}$.

(2) $819\frac{13}{561}-815\frac{7}{187}$.

3. Find the value of £17 2*s.* 9*d.* $\div 11\frac{5}{12}$ + £1 16*s.* 9*d.* $\div \frac{63}{698}$.

What part of half-a-crown is $\frac{1}{7}$ of $\dfrac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{6}}{\frac{1}{5}\times 3\frac{1}{3}+\frac{13}{36}}$ of $\frac{2}{11}$ of a guinea?

4. What is the value of $\frac{1}{11}$ of $\frac{1}{12}$ of a vessel, if a person who owns $\frac{3}{11}$ of it sells $\frac{1}{9}$ of $\frac{7}{8}$ of his share for £350?

5. Standard gold consists of 22 parts pure gold with two parts copper, and from a pound Troy of this metal are coined $46\frac{29}{40}$ sovereigns. Find the quantity of pure gold in 100 sovereigns.

6. Divide ·027 by 14·4; 1632 by ·0008; and 7 by ·091.

7. Reduce 7*s.* $8\frac{1942}{10000}$*d.* to the decimal of half a guinea; and express the difference between 2·778125 of 6*s.* 8*d.* and $\cdot 06\dot{3}$ of £13 2*s.* 6*d.* as the decimal of 2*s.* 3*d.*

8. If an ounce of gold be worth £4·$\dot{0}09\dot{9}$, what is the value of ·56$\dot{1}$ of a lb. ?

9. If 12 men can dig a trench 40 yards long and 4 feet wide, in 10 days of 8 hours each, how many hours a day must 55 men work, in order to dig a trench of the same depth, 220 yards long and 5 feet wide, in 18 days ?

10. When the three per cents. are at 92⅝, what must be given for £533 6*s.* 8*d.* stock ? What rate of interest will be obtained by purchasing three per cents. at 91¼ ?

11. Define simple and compound interest. Find the present value of £252 19*s.* 3*d.* due a year hence at 3¼ per cent. ; and the amount of £850 in 3 years at 6 per cent. compound interest.

12. The content of a solid block of stone 12 ft. 6 in broad, and 3 ft. 9 in. thick, is 27 cu. yds. 1 cu. ft. 810 cu. in. Required its length, and its price at 6*d.* per cubic foot.

XX (*a*).

1. (1) Add together 369 tons 13 cwts. 1 qr. 14 lbs. ; 127 tons 1 cwt. 3 qrs. 17 lbs.; 258 tons 19 cwts. 2 qrs. 23 lbs.; and 1 ton 5 cwts. 3 qrs. 2 lbs. (2) Multiply £628 17*s.* 4½*d.* by 1407. (3) Divide 375 miles 2 fur. 7 po. 2 yds. 1 ft. by 39.

2. (1) How many seconds are there in a leap year ? (2) Divide £6 16*s.* 9*d.* between two persons so that one may have double the sum which the other receives. (3) How many times will a coach wheel of 18½ ft. in circumference turn between London and York, being 197 miles ?

3. (1) If £4 15*s.* 7¼*d.* purchase 3 cwts. 17 lbs., what must be given for 16 cwts. 3 qrs. 12 lbs. ? (2) A owes B £56 5*s.*, but B consents to remit 15 per cent. for immediate payment ; what sum does A pay B ? (3) If 4 cwts. 1 qr. 7 lbs. cost £5 per cwt., what will be the cost per lb. when the cost of the whole is reduced by £5 9*s.* 3*d.* ?

4. Divide 1⅜ of 2⅓ by 1¾ of 5½.

5. Reduce $\frac{667}{1102}$ to its lowest terms.

6. Reduce $\frac{3}{4}$ of $\frac{1}{2}$ guinea to the fraction of £1.

7. Multiply 5·6 by ·00457.

8. Divide 816 by ·0004.

9. Reduce 7*s.* 4½*d.* to the decimal of £1.

10. A man after doing $\frac{3}{8}$ of a piece of work in 30 days is assisted by another, with whom he completes it in 6 days; how long would each be about it separately?

11. What number is that from which if there be taken $\frac{2}{7}$ of $\frac{3}{8}$, and to the remainder $\frac{8}{15}$ of $\frac{5}{16}$ be added, the sum will be 10?

12. How many 3-inch cubes can be cut from a 12-inch cube? (Royal Naval College.)

SANDHURST AND LINE PAPERS.

XXI.

1. Find (1) the number of sixpences, (2) the number of half-guineas, contained in £9095 12*s.* 6*d.*

2. Find, by the rule of Practice, the cost of clothing a regiment of 835 men, at the rate of £3 17*s.* 6½*d.* for each man.

3. Add together $\frac{1}{3}$, $\frac{9}{16}$, $\frac{16}{27}$, $\frac{5}{108}$, $\frac{19}{144}$, and reduce the result to its lowest terms.

   Explain why the numerator and denominator of a fraction may be both multiplied by the same number without altering the value of the fraction.

4. Multiply 1·25 (1) by ·007; (2) by ·008; verify the latter result by vulgar fractions.

5. What length of matting, $\frac{3}{4}$ yd. wide, will be required to cover a room 39 ft. 6 in. long by 25 ft. 6 in. wide? What is the cost of the same at 4*s.* 6*d.* a yard?

   Express the length of the room (1) as the fraction; (2) as the decimal of 100 yds.

6. Find (1) the cube of 2·07; and the square root of (2) 32·764176; (3) ·694$\dot{4}$.

7. If 10 men receive £28 10*s.* for 6 weeks' work, how many weeks must 8 men work to receive £38?

8. Find the amount of £2375 15*s.* in 6 years at $3\frac{1}{2}$ per cent., simple interest.

9. Reduce to their simplest forms :—

$$(1)\ 2+3x+4x^2-(1+x-4x^2);\ (2)\ \frac{a-3x}{a+3x}+\frac{a+3x}{a-3x}$$

10. Multiply $3x^4-21x^3y+8x^2y^2-6xy^3+y^4$ by $x^2+7xy-y^2$.

11. (1) Divide $4x^2-28xy+49y^2$ by $2x-7y$;

$$(2)\ \text{prove}\ \frac{a^6+2a^3b^3+b^6}{a^2+2ab+b^2}=(a^2-ab+b^2)^2.$$

12. Solve the equations:—(1) $3x-9=6-2x$;

$$(2)\ 3x-\frac{x-4}{4}=\frac{5x+14}{3}+\frac{47}{12};\ (3)\ \frac{x}{2}+\frac{y}{3}=7 \text{ and } \frac{x}{3}+\frac{y}{2}=8.$$

13. What fraction is that to the numerator of which if 4 be added, the value is $\frac{1}{2}$, and if 7 be added to the denominator, the value is $\frac{1}{5}$?

XXII.

1. 37 labourers are to receive £2 15*s.* $9\frac{1}{2}$*d.* each; how much money will be required to pay them? Amongst how many labourers will £64 be divided when they each receive £2 13*s.* 4*d.*?

2. If £12 19*s.* $1\frac{1}{2}$*d.* be paid as income-tax on an income of £345 10*s.*, what ought to be paid at the same rate on an income of £1745 6*s.* 8*d.*, and at what rate in the pound is it levied?

3. If 800 soldiers consume 10 sacks of flour in 12 days, how many soldiers will consume 30 sacks in 4 days?

4. Which is the greater fraction, $\frac{22}{7}$ or $\frac{355}{113}$? Find their difference.

Express the sum of $\frac{13}{14}$, $\frac{11}{15}$, $\frac{17}{21}$, $\frac{23}{30}$, as a mixed number.

5. Explain the principle on which fractions, to be added, are reduced to a common denominator.

6. A steamer which has to complete a voyage of 15,450 miles, has been 15 days at sea, steaming 180 miles daily; what fraction of her voyage has the steamer performed? How long will she be completing $\frac{3}{5}$ of her whole voyage?

7. Express as decimal fractions (1) $\frac{13}{5}$, (2) $\frac{13}{64}$; (3) Multiply 2·574 by ·00005; (4) Divide 5·646784 by ·4576.

8. Find the square $\frac{11}{12}$, and the square root of 532·2249.

Can a whole number with the digit 2 in the units' place be a perfect square?

9. Express, by means of algebraical symbols, the result of the following operations:—

Three times the square of a multiplied by b is to be added to the difference between the cubes of a and b.

Find the value of $(a+b)^2$ when $a=\frac{1}{2}$, $b=\frac{3}{2}$.

10. Perform the operations indicated in the following examples:—

(1) $(7x^2+5xy-\frac{1}{2}y^2)+(2x^2-5xy+\frac{1}{3}y^2)-(9x^2-3xy+\frac{1}{4}y^2)$;

(2) $\dfrac{1+7x}{1-7x}-\dfrac{1-7x}{1+7x}$;

(3) $(1+3x+3x^2+x^3)\times(1-3x+3x^2-x^3)$;

(4) $(49a^3-72ab^2+27b^3)\div(7a-3b)$;

(5) $(a+b+c)^2-(a-b+c)^2$.

11. Show that (1) $\dfrac{ax+2x^2}{a+x}=x+\dfrac{x^2}{a+x}$;

(2) $\dfrac{6x^2+x-2}{4x^2-1}=\dfrac{3x+2}{2x+1}$.

12. What is meant by a "simple equation"? Solve the equations:—

(1) $x+\dfrac{x}{2}=30$; (2) $\dfrac{x-2}{3}-\dfrac{x-3}{5}=\dfrac{x+40}{12}$;

(3) $\dfrac{x}{y}=3$ and $5x-4y=44$.

13. An uncle is older than his nephew by 10 years; and 15 years ago the uncle was twice as old as his nephew. *What are their* respective ages?

XXIII.

1. Subtract $\frac{2}{3}$ of £252 15*s.* 6*d.* from $\frac{1}{2}$ of £368 7*s.* 4*d.*, and multiply the remainder by 29.

2. Find, by the rule of Practice, the cost of 5 cwts. 2 qrs. 15 lbs. of goods, at £27 for 1 cwt.

3. If in a camp 12 men occupy one tent, and each tent cost £3 17*s.* 6*d.*, what will be the cost of providing tents for 9600 men, at the same rate?

4. Reduce $\frac{3465}{7920}$ to its lowest terms. Find the value of $\frac{3}{11}$ of $(\frac{7}{9} + 1\frac{2}{3})$, and prove it equal to $\frac{1}{3}$ of $20\frac{3}{4} \div 10\frac{3}{8}$.

5. Reduce $\frac{11}{16}$ to a decimal that terminates. Express $\frac{16}{11}$ in the form of a recurring decimal. Add together 13·57, ·044 and $3\frac{1}{4}$; (2) Divide 6·25 by ·00005; prove the result by vulgar fractions.

6. If 1 mètre French=39·371 inches English, show that 32 mètres = 35 yards nearly; (2) Express by means of decimals £7 12*s.* 6*d.* in pounds, and find the value of £3·5.

7. Find the interest on £1552 10*s.* for 18 months at $5\frac{1}{2}$ pei cent., simple interest; (2) If I buy into the 3 per cent. stock at 75, what rate of interest do I receive for the money I invest?

8. If the army and navy of a country together cost £26000000, and the cost of the army exceeds that of the navy by $\frac{3}{8}$ of the cost of the navy, find the cost of each.

9. What signs are used in Algebra to express addition, subtraction, multiplication, division? How does it appear that $ab=ba$, and $a \div b = \frac{a}{b}$?

10. Subtract $(a-\frac{b}{2}+c)$ from the sum of $a+\frac{b}{2}$ and $\frac{a}{3}+c$;

(2) Reduce $\frac{a}{x}+\frac{2a^2-x^2}{a^2-x^2}-\frac{a}{a+x}$;

(3) Multiply $(a^2-ab-ac-bc+b^2+c^2)$ by $a+b+c$;

(4) Divide $49xy^2$ by $-7x^{\frac{1}{2}}y^{\frac{3}{2}}$; and

(5) $6x^4-13ax^3+13a^2x^2-13a^3x-5a^4$ by $2x^2-3ax-a^2$.

11. If $x = \dfrac{a+b}{a-b}$ and $y = \dfrac{a-b}{a+b}$, prove that $\dfrac{x+y}{x-y} = \dfrac{a}{2b} + \dfrac{b}{2a}$.

Reduce $\dfrac{3x^2-x-4}{6x^2+7x-20}$ to its lowest terms.

12. What is the rule for transferring an algebraical quantity from one side of an equation to another? Solve the equations :—

(1) $\dfrac{2(x-4)}{3} + 5x = x - \frac{1}{3}$; (2) $\dfrac{2x-72}{9} + \dfrac{x}{15} - \dfrac{x-25}{5} = x - 44$;

(3) $22 = \dfrac{3x-1}{4} + \dfrac{7y-2}{3}$ and $x = 3y$.

13. There are two sorts of wine, one worth 5*s.* a quart, the other, 3*s.*; how much of each must be taken to mix a quart worth 3*s.* 6*d.*?

XXIV.

1. Add together the following sums of money :— £10 11.*s* $7\frac{1}{4}$*d.*; £237 18*s.* $3\frac{1}{2}$*d.*; £4234 2*s.* $11\frac{3}{4}$*d.*; £12347 14*s.* $9\frac{1}{4}$*d.*

2. Multiply £2 15*s.* $7\frac{1}{2}$*d.* by 153, and divide £858 9*s.* $3\frac{1}{4}$*d.* by 1735.

3. How many farthings are there in the sum of 200 guas., £123, 50 hf. crs. and 75*s.*?

4. If $35\frac{1}{2}$ lbs. of sugar cost £1 2*s.* $2\frac{1}{4}$*d.*, how much will 2 cwt. 51 lbs. cost?

5. Find the simple interest on £198 3*s.* 4*d.* for 2 years, at $2\frac{1}{2}$ per cent. per annum.

6. Reduce $\frac{2}{3} + \frac{5}{8} + \frac{1}{16} - 1\frac{1}{6}$ to a single fraction; and convert that fraction into a decimal.

7. Find the value of £·334375, and express 2 ro. 28 po. as the decimal of 1 acre.

8. Extract the square root of 3915380329, and also of $41\frac{184}{225}$.

9. Add together $a^2-3ab+b^2+a+b-1$; $2a^2+4ab-3b^2-2a-2b+3$; $3a^2-5ab-4b^2+3a+4b-2$; and $6a^2+$

$10ab+5b^2+a+b$; and find the numerical value of the result when $a=1, b=2$.

10. Subtract $3x^3-x^2-x-7$ from $4x^3-2x^2+x+1$, and from the remainder take x^3-4x^2+2x+8.

11. (1) Multiply

$x^4-2x^3+3x^2-2x+1$ by $x^4+2x^3+3x^2+2x+1$;

(2) Divide $6a^4-a^3b+2a^2b^2+13ab^3+4b^4$ by $2a^2-3ab+4b^2$;

and (3) Reduce $\dfrac{x^3+3x^2+4x+12}{x^3+4x^2+4x+3}$ to its lowest terms.

12. Solve the equations :—(1) $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=\frac{13}{14}$;

(2) $(x-5)(x-2)-(x-5)(2x-5)+(x+7)(x-2)=0$;

(3) $\frac{x+y}{2}-\frac{x-y}{3}=8$ and $\frac{x+y}{3}+\frac{x-y}{4}=11$.

13. Two shepherds, A and B, owning a flock of sheep, agree to divide it. A takes 144 sheep, and B takes 184 sheep, paying £70 to A. Required the value of a sheep.

XXV.

1. The sum of £55 6*s.* 10½*d.* is to be equally divided among 35 persons; how much will each receive?

2. If a soldier step ¾ yd., how many steps will he take in 3 miles?

3. If with a capital of £500 a tradesman gain £50 in 7 months, in what time will he gain £60 10*s.* with a capital of £385?

4. If $\frac{7}{24}$ of the cargo of a ship be worth £714 14*s.*, what is the value of the cargo?

5. Find the simple interest on £4968 15*s.* for 3 years, at 4½ per cent. per annum.

6. (1) Add together $2\frac{1}{5}$, $3\frac{2}{9}$, $5\frac{3}{8}$; and explain why fractions are reduced to a common denominator in addition. (2) Express 12*s.* 6¾*d.* and £4 12*s.* 6¾*d.* as decimals of £1.

7. A quantity of matting 37 ft. 9 in. long, and 7 ft. 6 in. wide, will just cover a room; what width of matting 75½ ft. long will cover the same room?

8. Extract the square root of (1) 154·157056; (2) of 13 to 4 decimal places.

9. Account for $(a-b)^2$ and $(b-a)^2$ having the same value when $a=7$, $b=3$; what is the value of $\frac{a^2-b^2}{a+b}$ if $a=7$, $b=3$.

10. (1) From $a^3+3a^2b+3ab^2+b^3$ take $a^3-3a^2b+3ab^2-b^3$;
(2) Simplify $3x^2+2xy-(4x^2+y^2)-(xy-x^2)$.

11. (1) Multiply $2x^3-3x^2y+4xy^2-3y^3$ by $2x+3y$;
(2) Divide the product by $2x^2-xy+3y^2$;
(3) Reduce $\frac{2x+3y}{2x-3y}-\frac{2x-3y}{2x+3y}$; and prove that

$$(a+b-c)^2-(a-b+c)^2=4a(b-c).$$

12. Solve the equations :—

$$(1)\ \frac{2x-7}{3}-\frac{x-3}{5}=x-\frac{62}{5};$$

$$(2)\ \frac{x-2}{x+4}=\frac{x-6}{x-3};$$

$$(3)\ \frac{x+y}{2}-\frac{x-y}{3}=13 \text{ and } \frac{x+y}{3}-\frac{x-y}{4}=8\tfrac{1}{2}.$$

13. A general, after losing a battle, found that he had only $\frac{2}{3}$ of his army fit for action; $\frac{1}{9}$ of the army were wounded, and the remainder 2000 men, were either killed or missing: of how many men did his army consist at first?

XXVI.

1. (1) Add together the following sums of money: £234 13*s.* 7½*d.*, £1278 4*s.* 11¾*d.*, £1009 18*s.* 2¼*d.*, £25344 7*s.* 6½*d.*, £347 14*s.* 4¼*d.*; (2) Multiply £14 13*s.* 7½*d.* by 237.

2. (1) Divide 606 tons 1 cwt. 2 qrs. 27 lbs. by 145; (2) Find the value of 3 cwts. 2 qrs. 16 lbs. at £3 7*s.* 8*d.* per cwt.

3. If the carriage of 41 cwts. 1 lb. 49 miles cost £20 9*s.* 6*d.*, what must be paid for the carriage of 13 cwts. 2 qrs. 19 lbs. *35 miles?*

4. (1) Add together $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, ·09375 and 2·46; (2) Divide ·0048 by 1·2; and (3) 213·419596 by 1·00103.

5. Add $\frac{5}{7}$ of $\frac{1}{4}$ of 6*s*. 5*d*. to $\frac{1}{6}$ of $\frac{2}{5}$ of 2*s*. 6*d*.; and express the result as a decimal of £1.

6. Find the interest on £237 10*s*. for 4 months, at 4 per cent. per annum.

7. Extract the square root of ·191810713444.

8. Add together $2a^2-6ab+2b^2+2a+2b-2$, $4a^2+8ab-6b^2-4a-4b+6$, $6a^2-10ab-8b^2+6a+8b+4$, and $12a^2+20ab+10b^2+2a+2b$; and find the value of the result when $a=2$, $b=3$.

9. Divide the product of a^2+ax+x^2 and a^3+x^3 by $a^4+a^2x^2+x^4$.

10. Reduce $\dfrac{6x^3-5x^2+4}{2x^3-x^2-x+2}$ to its lowest terms.

11. Simplify the expression—

$$\frac{a^2-bc}{(a+b)(a+c)}+\frac{b^2-ca}{(b+c)(b+a)}+\frac{c^2-ab}{(c+a)(c+b)}$$

12. Solve the equations :—

$$(1)\ \frac{3x-7}{5}+\frac{25-4x}{9}=\frac{5x-14}{3}.$$

(2) $x+1=5y$, and $\frac{1}{5}(2x+7y)-1=\frac{2}{3}(2x-6y+1)$.

13. There is a fraction which becomes equal to $\frac{1}{3}$, if 1 be added to its numerator, and becomes equal to $\frac{1}{4}$ if 1 be added to its denominator; determine the fraction.

XXVII.

1. (1) Add together the following sums of money :— £432 11*s*.7½*d*., £17 16*s*. 4*d*., £3427 2*s*. 11¼*d*., £10121 19*s*.3¾*d*. (2) Divide £425 10*s*. 7½*d*. by 153; and verify the result by Multiplication.

2. Reduce 5813456 lbs. to tons; and find how many grains of gold are contained in 4 lbs. 11 oz. 16 dwts. 22 grs.

3. A person's weekly income is £7, and his quarterly expenditure is £64 5*s*. How much will he have saved at

the end of 4 years, supposing a year to consist of 52 weeks?

4. Find the cost of 1735 lbs. at 19*s.* 9½*d.* per lb.

5. (1) Reduce $\dfrac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}-2\frac{1}{7}}$ to a simple fraction. (2) Divide 1·8619375 by 12·0125; and (3) ·04032 by ·0048.

6. If $\frac{3}{7}$ of an estate be worth £450, what is the worth of $\frac{11}{24}$ of the estate?

7. Find the amount of £226 5*s.* in 10 years, at 4¼ per cent., simple interest.

8. Extract the square root of 13277·9529.

9. Add together $x^2-3xy+y^2+x+y-1$, $2x^2+4xy-3y^2-2x-2y+3$, $3x^2-5xy-4y^2+3x+4y-2$, and $6x^2+10xy+5y^2+x+y$; and find the numerical value of the result when $x=2$, $y=5$.

10. (1) Multiply together x^3-2x+1 and x^3-3x+2; (2) Divide the product by x^3-3x^2+3x-1; (3) Reduce $\dfrac{x^3-6x^2+11x-6}{x^3-2x^2-x+2}$ to its lowest terms.

11. Simplify $\dfrac{a}{a+b}+\dfrac{ab}{a^2-b^2}-\dfrac{a^2}{a^2+b^2}-\dfrac{a^2b^2}{a^4-b^4}$; and find the numerical value of the result when $a=3b$.

12. Solve the equations:—

(1) $\dfrac{3x-7}{5}+\dfrac{25-4x}{9}=\dfrac{5x-14}{3}$;

(2) $\dfrac{3x-5y}{2}+3=\dfrac{2x+y}{5}$ and $8-\dfrac{x-2y}{4}=\dfrac{x}{2}+\dfrac{y}{3}$;

(3) $\dfrac{a+b}{x+b}=\dfrac{a}{x-a}+\dfrac{b}{x-b}$.

13. A workman was employed for 60 days, on condition that for every day he worked he should receive 2*s.* 6*d.*, and for every day he was absent he should forfeit 10*d.*: at the end of the time he had to receive £2. How many days did he work?

XXVIII.

1. (1) Add together the following sums of money :— £144 3*s*. 10½*d*., £5 18*s*. 9*d*., £1142 7*s*. 7¾*d*., £3373 19*s*. 9¼*d*. (2) Multiply £1 7*s*. 9¾*d*. by 306, and verify by division.

2. How many grains of gold are contained in 4 lbs. 10 oz. 16 dwts. 22 grs.?

3. Find the value of 319 cwts. 3 qrs. 16 lbs. at £2 12*s*. 6*d*. per cwt.

4. A person buys 4 cwts. 3 qrs. 14 lbs. of sugar at £2 16*s*. 8*d*. per cwt., and sells it at 8½*d*. per lb. How much does he gain or lose ?

5. Reduce to its simplest form $\frac{\frac{5}{8}-\frac{2}{5}}{\frac{4}{3}-\frac{1}{7}}$ and convert the result into a decimal.

6. (1) Multiply 16·02 by ·0007; (2) Divide ·0006594 by ·0021 ; (3) Express 3*s*. 1½*d*. as a fraction of 1 guinea, and also as the decimal of £1.

7. A reservoir is 24 ft. 8 in. long, by 12 ft. 9 in. wide ; how many cubic feet of water must be drawn off to make the surface sink 1 ft. ?

8. A person, after paying 7*d*. in the £ for income-tax on his income, has £1632 18*s*. 10*d*. remaining ; what had he at first ?

9. (1) Simplify
$4x^3-2x^2+x+1-(3x^3-x^2-x-7)-(x^3-4x^2+2x+8)$;
(2) Subtract $(a-b)\,x-(b-c)\,y$ from $(a+b)\,x+(b+c)\,y$.

10. (1) Divide $x^3+y^3+3xy-1$ by $x+y-1$;

(2) Reduce $\dfrac{x^3+x^2+x-3}{x^3+3x^2+5x+3}$ to its lowest terms.

11. Simplify the expression $\dfrac{3+2x}{2-x}-\dfrac{2-3x}{2+x}+\dfrac{16x-x^2}{x^2-4}$.

12. Solve the equations :—

(1) $\dfrac{2x-6}{5}-\dfrac{x-4}{9}=\dfrac{3x}{13}$;

(2) $\dfrac{6x+7}{15}-\dfrac{2x-2}{7x-6}=\dfrac{2x+1}{5}$;

(3) $\dfrac{x-a}{x-b}=\left(\dfrac{2x-a}{2x-b}\right)^2$.

13. A person bought some yards of cloth for £6. If there had been 6 yds. more, each yard would have cost 1*s.* less. Required the number of yards, and the price of each.

XXIX.

1. (1) What weight must be added to 15 cwts. 3 qrs. 19 lbs. to make up 3 tons? (2) Multiply £947 10s. $6\frac{1}{2}$*d.* by 251.

2. If the value of a rupee be 1*s.* $11\frac{1}{2}$*d.*, how many rupees are there in £88 4*s.* $5\frac{1}{2}$*d.*?

3. If a train travel 215 miles in 10 hrs. 45 min., what distance will it travel in $24\frac{1}{4}$ hrs. at the same rate?

4. If 7 men mow 22 acres in 8 days, working 11 hrs. a day, in how many days, working 10 hrs. a day, will 12 men mow 360 acres?

5. Find the expense of lining with tin the whole of a cubical box, one edge of which is 4 ft. 6 in., at 1*s.* 8*d.* a square yard?

6. If £11187 10*s.* be invested in the purchase of land, what income will be derived from the investment at $2\frac{1}{4}$ per cent.?

7. (1) Reduce $\dfrac{\frac{2}{3}+\frac{5}{6}+\frac{1}{3}\text{ of }2\frac{1}{3}+\frac{7}{12}}{\frac{2}{3}-\frac{5}{6}+\frac{7}{9}-\frac{1}{4}\times\frac{7}{3}}$; (2) Divide 26·531 by 2·15, and prove the result by vulgar fractions; (3) Reduce half-a-guinea to the decimal of £2 12*s.* 6*d.*

8. Extract the square root of ·00015625.

9. Find the value of $a^3+3a^2b+3ab^2+b^3$, and determine which is greater,

$$5a-2b-a \text{ or } 5a-(2b-a), \text{ when } a=1,\ b=2.$$

10. Perform the operations indicated in the following examples :—

(1) $\left(\frac{a}{2}+\frac{b}{3}\right)-\left(\frac{a}{3}-\frac{b}{2}\right)+\left(\frac{a}{2}-b\right)$;

(2) $(1+2x)\times(1-3x)\times(1+4x)$;

(3) $(-25x^3y^2z)\div(-5x^2y)$.

11. Simplify

(1) $(a^2-2ax+x^2)\times(a^3+3a^2x+3ax^2+x^3)\div(a^2-x^2)$;

(2) $\frac{x^2-a^2}{2x+a}\div\frac{x-a}{4x^2-a^2}$.

12. Solve the equations :—

(1) $\frac{x+1}{8}=\frac{x}{7}-1$;

(2) $\frac{3x+4}{7}-\frac{5x-39}{6}=\frac{27-x}{12}$;

(3) $\frac{x}{2y}=\frac{2}{5}$ and $\frac{y-x}{3}+\frac{2x+y}{13}=\frac{y+1}{4}$;

13. A luggage train leaves a station, and travels at the rate of 10 miles an hour ; after 4 hrs., another train follows from the same station, travelling $16\frac{2}{3}$ miles an hour. How far must the second train travel before it comes up with the first ?

XXX.

1. (1) Add together £17 11*s.* $7\frac{1}{2}$*d.*, £128 15*s.* $8\frac{1}{4}$*d.*, £372 4*s.* 3*d.*, £1234 9*s.* $4\frac{3}{4}$*d.*, and £2579 2*s.* $2\frac{1}{2}$*d.* (2) Multiply £19 13*s.* $7\frac{1}{2}$*d.* by 135, and prove by Division.

2. Reduce (1) 17 tons 11 cwts. 1 qr. 8 lbs. 11 oz. to ounces; and (2) 237458 inches to miles.

3. Find the value of 28 cwts. 0 qrs. 14 lbs. at 12*s.* 2*d.* per cwt.

4. If the wages of a labourer for $17\frac{3}{4}$ days amount to £2 4*s.* $4\frac{1}{2}$*d.*, what will be the amount of his wages for $68\frac{3}{4}$ days ?

5. (1) Required the sum of $\frac{2}{3}$ of $3\frac{3}{10}$, $\dfrac{1\frac{3}{4}}{2\frac{5}{8}}$ of 17 and $\frac{3}{5}$ of $5\frac{3}{4}$ of $\frac{34}{51}$. (2) Divide ·0078125 by ·00125; and (3) ·000123123 by ·0041.

6. Express $\frac{3}{16}$ of ·375 of 10*s.* + $\frac{4}{5}$ of 2*s.* 6*d.* − $\frac{3}{4}$ of 1*s.* as the decimal of £1.

7. Find the interest on £1049 16*s.* 6*d.* for 6 yrs. 4 mo. at $4\frac{1}{2}$ per cent., simple interest.

8. Reduce $\frac{1}{64}$ to a decimal fraction, and extract the square root of that decimal.

9. Add together $10a-8b+8c-2d$, $3a+3b-7c+11d$, $3a-17b+9c+5d$, and $3a+9b-c-12d$; and find the numerical value of the result when $a=5$, $b=4$, $c=2$, $d=1$.

10. (1) Multiply

$$x^4-4x^3a+6x^2a^2-4xa^3+a^4 \text{ by } x^2+2xa+a^2;$$

(2) divide the product by $x^4-2x^3a+2xa^3-a^4$;

(3) reduce $\dfrac{2x^5-11x^2-9}{4x^5+11x^4+81}$ to its lowest terms.

11. Simplify the expression:—

$$\left(\frac{b}{c}+\frac{c}{b}\right)^2+\left(\frac{c}{a}+\frac{a}{c}\right)^2+\left(\frac{a}{b}+\frac{b}{a}\right)^2-\left(\frac{b}{c}+\frac{c}{b}\right)\left(\frac{a}{c}+\frac{c}{a}\right)\left(\frac{a}{b}+\frac{b}{a}\right).$$

12. Solve the equations:—

(1) $\dfrac{5x-14}{4}-\dfrac{13-7x}{8}=3x-7\frac{3}{4}$;

(2) $\dfrac{2x+y}{9}+\dfrac{7y+6x+11}{18}=\dfrac{19}{2}-\dfrac{5x-17}{6}$ and

$$\tfrac{6}{7}(5x+3y+2)=9y+6.$$

13. A pound of tea and three pounds of sugar cost 4*s.* 6*d.*, but if tea were to rise 20 per cent. in price, and sugar 25 per cent., the cost would be 5*s.* 6*d.* Find the price per lb. of tea and sugar.

XXXI.

1. A ship and its cargo are worth £21456 8*s.* 9*d.*; the value of the ship is £4978 19*s.* 10*d.*, find the value of the cargo.

2. Divide £691 2*s.* $1\frac{1}{2}$*d.* by 93.

3. At the International Exhibition, £2857 15*s.* was received in one day by 1*s.* admissions; how many people must be admitted by payments on a half-crown day to make up the same amount?

4. Find the value of 25 tons 3 cwts. 14 lbs. of powder at £73 6*s.* 8*d.* per ton.

5. If 24000 yds. of cotton cloth $\frac{3}{4}$ yd. wide be worth £400, when raw cotton is at $4\frac{1}{2}$*d.* per lb., what is the value of 36000 yards of cotton cloth $1\frac{1}{4}$ yds. wide, when raw cotton is at 9*d.* per lb.?

6. Find the amount of £3968 15*s.* in 4 yrs. at $3\frac{1}{4}$ per cent., simple interest.

7. (1) Reduce to a simple fraction

$$\frac{15}{17} \text{ of } \frac{6\frac{4}{5}}{2\frac{8}{11}} \text{ of } \left(\frac{25}{9} \div \frac{35}{9}\right);$$

(2) Express $\frac{1}{5}$ of 13*s.* 4*d.* as the fraction of £5;

(3) Divide 76·11 by 21·5;

(4) Prove that $\frac{46\cdot2}{92\cdot4} = \cdot75 \times \cdot\dot{6}$;

(5) Reduce $(\frac{3}{4})^2$ to a decimal.

8. Extract the square root of 4·20291001.

9. If $a=1$, $b=2$, $c=3$, $d=4$, $n=4$, prove that

$(a+b+c+d)^2 = \frac{n^2(n+1)^2}{4}$; for the same values, find $(b+c)^2$.

10. Reduce the following expressions to their simplest forms :—

(1) $5x^2-4a^2-(9-3x^2)+(3a^2-7)-(7x^2-16)$;

(2) $\frac{x^2-a^2}{a+b} \times \frac{(a+b)^2}{x-a} \times \frac{a}{x+a}$;

(3) $\frac{x^2-7x+10}{2x^2-x-6}$.

11. (1) Multiply together $x-2$, $2x+3$, and $2x^2-x-6$;
(2) Divide x^6y^6-64 by $xy-2$.

12. Solve the equations :—

(1) $\frac{2x-3}{5}-\frac{x-5}{7}=5$;

(2) $\frac{x+6}{2x+5}=\frac{x}{2x-5}$;

(3) $\frac{x+8}{7}-\frac{2x-y}{9}=2(y-16)$; $\frac{5}{7}(y-3)-\frac{x+8}{21}=x-4$.

13. On an average, a postman in London delivers on a Monday double the number of letters he delivers on any of the other 5 working days in a week, and his whole weekly delivery exceeds by 3000 the average delivery on any day except Monday. Find the average number of letters he delivers on a Monday.

XXXII.

1. Add together the following sums of money :—
£23567 18*s.* 7½*d.*, 11¼*d.*, £948 11*s.* 4*d.*, 13*s.* 8¼*d.*, £4002 15*s.* 9¼*d.*, and £20 17*s.* 6¾*d.*

2. Multiply 75125 by 2056, and divide the product by 514.

3. How much money will be required to pay £3 15*s.* 6*d.* each to 455 men?

4. If 754 tons 5 cwts. 1 qr. 23 lbs. of cheese cost £21119 12*s.* 9*d.*, what is the cost of one ton, one cwt., one pound, respectively?

5. Find the cost of 43 ac. 3 ro. 7 po. of ground at £110 6*s.* 8*d.* an acre.

6. A person after paying 10*d.* in the £ as income-tax, has remaining an income of £862 10*s.*; what would his income have been had there been no income-tax?

7. (1) Find the simple interest on £378 2*s.* 6*d.* for 7 yrs. at 6½ per cent. (2) If 3*s.* 5½*d.* be paid as yearly interest on £3 2*s.* 6*d.*, what is the rate per cent.?

8. (1) Reduce $\frac{5}{8}+\frac{5}{16}+\frac{5}{32}$, and express the result as a decimal. (2) Express ·225 as a vulgar fraction in its lowest terms. (3) Extract the square root of 49084036 and of $90\frac{1}{4}$.

9. (1) Add together $7a+5b-3c+4d$, $3a-5b-4c-d$, $3b-5a-3c-3d$, $a+b-c-d$, and $a-b+c-d$;

(2) From $3x^3-2x^2y+3xy^2-4y^3$ take $2x^3-x^2y-4xy^2+y^3$;

(3) Simplify $\frac{a+b-c}{2}-\left(\frac{a}{3}-\frac{2b}{5}-\frac{4c}{7}\right)$.

10. (1) Multiply

$x^4-4x^3y+6x^2y^2-4xy^3+y^4$ by $x^3-3x^2y+3xy^2-y^3$;

(2) Divide $a^5-5a^3b^2-5a^2b^3+b^5$ by $a^2-3ab+b^2$.

11. Reduce (1) $\frac{x}{x-4}-\frac{x}{x+4}-\frac{6x}{x^2-16}$;

(2) $\frac{2x^2-x-15}{6x^2+19x+10}$ to their simplest forms.

12. Solve the equations :—

(1) $\frac{x}{2}+\frac{3x}{4}=25$;

(2) $\frac{2x-1}{5}-\frac{x+3}{7}=x-10-\frac{2}{7}$;

(3) $\frac{x}{5}-\frac{y}{9}=16$, and $\frac{x+5}{13}=2+\frac{y-1}{10}$.

13. Find the fraction to the numerator of which if 16 be added, the fraction becomes equal to 4, and if 11 be added to the denominator, the fraction becomes $\frac{1}{4}$.

XXXIII.

1. How long will a person be in saving £100, if he lay by $7\frac{1}{4}d.$ a day?

2. Find the value of 75 cwts. 3 qrs. 21 lbs. at £5 6*s.* 8*d.* per cwt.

3. If 6 horses can in 2 days plough 17 acres, how many acres will 93 horses plough in $4\frac{1}{2}$ days?

4. (1) Multiply $\frac{5}{8}$ of $\frac{5\frac{1}{3}}{7\frac{1}{2}}$ by $\frac{1}{9}$ of $4\frac{1}{8}$; (2) divide $17\frac{1}{4}$ by $6\frac{3}{4}$.

5. Divide (1) 7·44775 by 48·05 ; (2) 2·0164 by ·071.

6. (1) Find the value of ·046775 cwts.; (2) reduce 7*s.* $10\frac{1}{2}$*d.* to the decimal of a guinea ; (3) extract the square root of ·48 to 6 places.

7. Required the simple interest of £118 15*s.* for 5 yrs. at $2\frac{1}{2}$ per cent. per annum.

8. What will be the expense of carpeting a room $23\frac{3}{4}$ ft. long and $16\frac{1}{4}$ ft. wide, at 2*s.* 9*d.* per square yard?

9. (1) Find, by substitution, the values of; (1) $(a-b)^2$, (2) $a^2-2ab+b^2$, when $a=2$, $b=\frac{1}{2}$, and account for the values being the same.

(2) Multiply $x^3-6ax^2+12a^2x-8a^3$ by $x^2-4ax+4a^2$.

10. Divide $a^2-2ab+b^2-c^2+2cd-d^2$ by $a-b+c-d$.

11. Reduce to their simplest forms :

(1) $5x^2-7y^2-(3x^2-7xy)-(5xy-4y^2)$;

(2) $\dfrac{x^2-xy}{x^2-y^2}-\dfrac{xy-y^2}{x^2-y^2}$;

(3) Prove that $\dfrac{15a^2+5ab}{6a^2-ab-b^2}=\dfrac{5a}{2a-b}$;

(4) That $\dfrac{a-c}{(a-b)(x-a)}-\dfrac{b-c}{(a-b)(x-b)}=\dfrac{x-c}{(x-a)(x-b)}$.

12. Solve the equations :—

(1) $ax-b=x$;

(2) $\dfrac{3x-1}{5}-\dfrac{x-4}{3}=\dfrac{x-1}{2}$;

(3) $\dfrac{x+y}{2}-\dfrac{x-y}{3}=35$ and $\dfrac{x+y}{3}+\dfrac{x-y}{4}-\frac{1}{2}=37$.

13. A man engages to perform, at a uniform rate, a certain distance on horseback in 10 hours, but when he is half way he increases his speed by two miles an hour, and thus arrives at his journey's end one hour and a quarter before the time stipulated. Required the distance he performed and the rate at which he started.

XXXIV.

1. Write down in words the number 300030000, also the difference between a hundred and a million.

2. After purchasing 13 articles at 2*s*. 3*d*. each, 59 at 1*s*. 8*d*., and 19 at 3*s*. 4*d*., how much will remain out of £10.

3. If 3 cwts. 3 qrs. 12 lbs. cost £9, what is the price of 6 lbs.

4. In standard gold 11 parts out of 12 are pure gold. What weight of alloy is there in 3 oz. 5 dwts. of standard gold?

5. (1) Add together $\frac{5}{7}$, $\frac{1}{2}$, $\frac{1}{8}$ and $\frac{3}{28}$ of $3\frac{1}{3}$.
   (2) Find the value of £·36875.
   (3) Divide 27·5264 by ·0374.

6. In how many days will the interest on £4600, at $1\frac{1}{2}d.$ per cent. daily, amount to £18 8*s*. ?

7. If the corn of 13 horses for 63 days cost £17 6*s*. 8*d*., when corn is 4*s*. a bushel, how many horses will cost £10 13*s*. 4*d*. for corn in 56 days, when corn is 4*s*. 6*d*. a bushel?

8. Extract the square root of (1) 8, (2) ·0002, to three places of decimals, and multiply the square roots together. Explain why the product is not the square root of the product of 8 and ·0002.

9. (1) Multiply $b+c-a$, $a+c-b$, and $a+b-c$; and find the value of the product when $a=b=c=\cdot01$; (2) find the continued product of $x-3$, $x+7$, and x^2-2x+5 ; (3) square $a-1+\frac{1}{2}a$.

10. Divide $x^2+(a+c)x+ac$ by $a+x$.

11. Simplify

   (1) $(a+b+c)(b+c-a)+(a+c-b)(a+b-c)$.

   (2) $\dfrac{2x^2+xy-y^2}{x^3+x^2y-x-y}$; (3) $\dfrac{(x^4-a^4)(x-a)}{x^2-2ax+a^2}\times\dfrac{1}{x^2+ax}$.

12. Solve the equation $\dfrac{4x+3}{9}=\dfrac{8x+19}{18}-\dfrac{7x-29}{5x-12}$.

13. (1) The difference of the ages of two brothers three

years ago was $\frac{1}{6}$ of the age of the elder, whereas it is now $\frac{1}{7}$. What are their ages?

(2) At the election of a member of parliament the successful candidate was returned by a majority of 88, but if 1 out of every 7 of his supporters had voted for his opponent, he would have been in a minority of 18. How many votes were recorded for each candidate?

XXXV.

1. (1) Divide £2 2*s.* 6*d.* into 17 equal parts. (2) By what number must 4 cwts. 2 qrs. 8 lbs. be multiplied to produce 40 tons?

2. (1) Reduce 760320 inches to miles. (2) Find the highest number that will divide both 12499 and 14790 without a remainder. What is that highest number called?

3. A railway train travels $\frac{1}{4}$ of a mile in 18 seconds, how many miles an hour does it travel at this rate?

4. If 40 panes of glass, each 2 ft. long and $1\frac{1}{2}$ ft. wide, cost £3 8*s.*, what will be the price of 34 panes, each measuring $2\frac{1}{2}$ ft. long by 1 ft. wide?

5. The interest on £11587 10*s.* for 1 year is £434 10*s.* $7\frac{1}{2}$*d.*; find the rate per cent.

6. (1) Take any two vulgar fractions and explain the rule for their multiplication.

(2) Reduce $(\frac{1}{2}-\frac{3}{10}+\frac{1}{7}+\frac{2}{35})\times(4\frac{2}{7}+\frac{9}{8}-\frac{5}{14}-\frac{3}{56})$.

7. (1) Divide ·0088266 by ·000235, and express 17*s.* 6*d.*, (2) as the vulgar fraction, (3) as the decimal of £100.

8. (1) Prove $(\cdot 02)^2\times(\cdot 005)^2=(\cdot 0001)^2$; (2) extract the square root of 13, to 4 decimal places.

9. Show that $\frac{x^3-a^3}{x-a}$ and x^2+ax+a^2 have the same numerical value when $x=3$, $a=2$. Will they always have the same value whatever be the values of x and a?

10. Perform the operations indicated in the following examples :—

(1) $\frac{a^2+b^2-c^2}{2}-\frac{a^2-b^2}{3}+\frac{3c^2}{4}-\frac{5b^2}{6}$;

(2) $(a^4-7a^3b+6a^2b^2+8ab^3-2b^4)\times(a^2-3ab+2b^2)$;

(3) $(a^4+b^4-c^4-2a^2b^2)\div(a^2-b^2-c^2)$;

(4) $(x^2-y^2)^3\div(x-y)^3$.

11. Prove that (1) $\frac{15x^2+xy-2y^2}{9x^2+3xy-2y^2}=\frac{5x+2y}{3x+2y}$;

$$(2)\ \frac{\frac{a}{a-b}-\frac{a}{a+b}}{\frac{b}{a-b}-\frac{b}{a+b}}=\frac{a}{b}.$$

12. Solve the equations :—

(1) $\frac{x-3}{x-4}=\frac{4}{5}$;

(2) $\frac{2x-13}{9}-\frac{x-1}{11}=\frac{x}{8}+\frac{x}{7}-9$;

(3) $\frac{x+y}{2}+\frac{3x-5y}{4}=2$, and $\frac{x}{7}+\frac{y}{9}=2$.

13. Two merchants, A and B, start with the same capital in trade; at the end of a year A has gained £3000 and B has lost £1000; it is then found that A's capital is double the capital of B. What did they each start with ?

XXXVI.

1. Write in figures the number seventy millions, four thousands, and three hundreds; and write in words the number 18004760.

2. Find the cost of 219 articles at £8 5*s.* 3*d.* each.

3. What is the cost of 1 dozen silver spoons, weighing 2 oz. 7 dwts. each, at 6*s.* 8*d.* an ounce ?

4. If 18 bushels cost 12 guineas, what will be the cost of 13 bu. 2 pks.?

5. How many square yards are there in 9 ac. 3 ro. 12 p. of land ?

6. (1) Add together $\frac{5}{8}$ of 10 guineas, and $\frac{3}{4}$ of $\frac{7}{9}$ of 10s.;
(2) Express 1s. 5d. as the decimal of £13 6s. 8d.;
(3) Divide 750·35 by ·000215.

7. What sum will amount to £591 12s. 4d. in 4 years at $2\frac{1}{2}$ per cent. simple interest?

8. Extract the square root of 502681.

9. (1) Multiply $\frac{x}{2}-\frac{y}{3}+\frac{z}{4}$ by $\frac{x}{3}+\frac{y}{2}-\frac{z}{4}$;
(2) Divide $2a^3+7a^2b-9b^2(a+b)$ by $2a-3b$.
(3) Find the continued product of

$$\frac{x}{x-y}+\frac{y}{x+y},\ \frac{x}{x-y}-\frac{y}{x+y}\text{ and }\frac{(x+y)^2}{x+y}.$$

10. Simplify the expressions:—

(1) $\frac{3-2x}{1-2x}-\frac{5-2x}{7-2x}+\frac{4x^2-8x-9}{7-16x+4x^2}$;

(2) $\frac{x^3-3x-2}{x^4-x^2-x-10}$;

(3) $\frac{x^4+ax^3-9a^2x^2+11a^3x-4a^4}{x^4-ax^3-3a^2x^2+5a^3x-2a^4}$.

11. Solve the equations :—

(1) $\frac{x}{4}-\frac{2x-1}{2\frac{1}{2}}-\frac{6x-4}{15}=\frac{x}{6}$;

(2) $\frac{2x+1}{3}-\frac{2}{2x+1}=\frac{2}{3}$;

(3) $\frac{x}{2}-\frac{y}{4}=20$, and $\frac{x+y}{2}-23=\frac{2y+20}{5}$.

12. A has £3, and B has £2 8s. How much shall A give to B, that B may then have 3 times as much as A?

13. In a school of 600 children there are 18 more boys than girls, and the numbers of boys and girls are together double the number of infants. How many boys are there?

XXXVII.

(1) Multiply 17 yds. 2 ft. 11 in. by 137; (2) how many *times* will £7 18s. 9d. be contained in £928 13s. 9d.?

2. Find the rent of 107 acres 3 roods 24 perches at £2 9*s*. 6*d*. per acre?

3. If one steamer sail 6408 miles in 24 days, how far will another sail in 10 days, the latter steamer travelling 8 miles in the same time as the former travels $7\frac{1}{2}$ miles?

4. In how many years will £1937 10*s*. amount to £2547 16*s*. 3*d*. at $4\frac{1}{2}$ per cent., simple interest?

5. Three contractors A, B, C, undertake to build a wall. A employs 30 men for 8 weeks, B, 60 men for 7 weeks, C, 70 men for 5 weeks; how much ought each to receive when £5167 16*s*. 8*d*. is paid for the whole work?

6. What is represented (1) by the numerator, (2) by the denominator of a vulgar fraction? Show from definition that $\frac{5}{8}$ is three times as great as $\frac{5}{24}$; (3) find the fraction equal to $\frac{5}{8}$, and having 10 for the greatest common measure of its terms.

7. (1) Reduce $\frac{1}{2}-\frac{3}{5}+\frac{7}{8}-\frac{11}{25}+\frac{117}{125}$, and express the result as a decimal; (2) divide 32·035 by ·86; (3) reduce ·45 of £5 10*s*. to the decimal of £3.

8. (1) A plot of ground required for building a warehouse in a city is a square of 90 ft. 9 in. side: find its cost at £1 12*s*. per square yard; (2) Extract the square root of ·0031843449.

9. Find the value of a^n+b^n when $a=5$, $b=4$, $n=3$; and of $\dfrac{x^3-a^3}{x-a}$ when $x=4$, $a=-2$.

10. Reduce the following algebraical expressions:—

(1) $\dfrac{a^2+3ab-5b^2}{2}-\dfrac{2a^2+4ab-10b^2}{5}$;

(2) $(x^4-x^3y+x^2y^2-xy^3+y^4)\times(x+y)$;

(3) $(2a^4+a^3b-13a^2b^2-3ab^3+b^4)\div(a^2-2ab-b^2)$.

11. (1) Find the greatest common measure of

$6x^2+x-35$ and $10x^2+21x-10$;

(2) Prove that $\dfrac{a+bx}{a-bx}-\dfrac{a-bx}{a+bx}=\dfrac{4abx}{a^2-b^2x^2}$.

12. Solve the equations :—

(1) $\frac{2x+5}{2x-5}=\frac{5}{3}$;

(2) $\frac{x+7}{4}=\frac{2x+7}{14}-\frac{2x-7}{21}+\frac{27}{14}$;

(3) $\left.\begin{array}{l} ax+by=c \\ bx-ay=d \end{array}\right\}$

13. An express train, which travels at the rate of 70 miles an hour, starts half an hour after a luggage train, and comes up with it in 20 minutes : find the speed of the luggage train.

XXXVIII.

1. Reduce 15 tons 13 cwts. 2 qrs. 17 lbs. 5 oz. to ounces.

2. Telegraphic posts are to be placed at the distance of 22 yards from each other; how many posts will there be in a distance of 125 miles ?

3. If 5 balls of lead weigh as much as 9 of iron, and 3 of iron weigh as much as 7 of marble, how many balls of marble will be equal in weight to 35 balls of lead ?

4. If 140 horses eat 560 bushels of oats in 16 days, how many horses may be kept for 24 days upon 1200 bushels ?

5. (1) Find the simple interest on £1500 for 4 yrs. at $5\frac{3}{8}$ per cent. per annum; (2) find the time in which £1500 will amount to £1822 10*s.* at the same rate per cent.

6. (1) Express in its simplest form $\frac{15}{16}-\frac{14}{15}+\frac{13}{14}-\frac{11}{12}$.

(2) Express $2\frac{1}{2}$ guineas as the decimal of £$2\frac{1}{2}$.

7. (1) Reduce $\frac{17}{256}$ to a decimal; (2) divide ·010875 by ·00625, and verify the result by means of vulgar fractions.

8. (1) Extract the square roots of 4928·742025 and

(2) $\frac{16{\cdot}9}{22{\cdot}5}$.

(3) Compare the values of $(\cdot 001)^2$ and $(\cdot 01)^3$.

9. If $a=4$, $b=3$, $c=2$, prove :—

(1) $(a-b)(a-c)=ac-bc$;

2) $\frac{a^2-ab+b^2}{a+b}=\frac{a+b+3c}{2b+1}$.

10. (1) Multiply $x^4-7x^2y^2+6xy^3-y^4$ by $x^3-2xy^2+y^3$.

(2) Divide $(a+2b)^2-(c-3d)^2$ by $a+2b+c-3d$.

11. Reduce to their simplest forms :—

(1) $\dfrac{x^2+a^2}{x^2-a^2}-\dfrac{x^2-a^2}{x^2+a^2}$; (2) $\dfrac{x^2-(a+b)x+ab}{x^2+(a-b)x-ab}$.

(3) Prove that

$$\frac{1}{x(x-a)(x-b)}=\frac{1}{abx}+\frac{1}{a(a-b)(x-a)}-\frac{1}{b(a-b)(x-b)}.$$

12. Solve the equations :—

(1) $\dfrac{x}{3}-\dfrac{2x-50}{25}=\dfrac{2x}{5}-6-\dfrac{x+15}{30}$;

(2) $\dfrac{x}{a}-\dfrac{y}{b}=1$ and $\dfrac{x}{c}-\dfrac{y}{d}=1$.

13. In a regiment 1200 strong, the number of English and Scotch together was double the number of the Irish, and the number of English and Irish together was 5 times the number of Scotch. How many were there of each nation in the regiment?

XXXIX.

1. Reduce 11763256 half-pence to pounds, shillings, and pence.

2. (1) How many fourpenny pieces are contained in 11763256 half-pence? (2) What will be the date as to hour and day of the month 1123200 seconds after 3 o'clock P.M. January 2nd?

3. If a contractor buy 100 cwts. of meat at £2 4*s.* 4*d.* per cwt., what will he gain by supplying it at $5\frac{1}{2}$*d.* per lb.?

4. If 24 men can mow a field 400 yards square in 20 days, how many men can mow a field 500 yds. long and 20 yds. wide in 5 days?

5. What sum of money, at $4\frac{1}{2}$ per cent., will produce the same income that £2700 produces at $5\frac{1}{2}$ per cent.? Find the income.

6. (1) Add together $\frac{2}{3}$ of $\frac{4}{5}$. $\dfrac{7\frac{1}{2}}{5\frac{1}{2}+\frac{1}{4}}$, $\dfrac{\frac{1}{2}-\frac{1}{4}}{7\frac{1}{2}}$;

(2) What fraction of £1 together with 3*s.* 4*d.* = 10*s.*?

7. (1) Multiply 12·345 by ·00014, and explain the rule for pointing the product of two decimals. (2) Find the value of the recurring decimal $\cdot 2\dot{3}1\dot{5}$ in the form of a vulgar fraction; (3) find $\left(\frac{\cdot 004}{\cdot 02}\right)^3$; (4) determine the square root of 906·67834321.

8. Gunpowder being composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts, find what weight of each ingredient is required for 1 ton 10 cwts. of powder.

9. (1) Show that, for any numerical value of x, $\frac{x^2-4}{x-2}=x+2$.

(2) For what value of x is $\frac{x^2-4}{x-2}=-x$?

10. Perform the operations indicated in the following examples :—

(1) $3a+5b-2c-\{3a-2b-(7b-a+3c)\}$;

(2) $(x^2+y^2+z^2-xy-yz-xz)\times(x+y+z)$;

(3) $(x-4x^3+12x^4-9x^5)\div(1+2x-3x^2)$;

(4) $(xy-a)\div(x^{\frac{1}{3}}y^{\frac{1}{3}}-a^{\frac{1}{3}})$.

11. Prove (1) $\frac{3x^3+2x^2-8x}{9x^3-12x^2-36x+48}=\frac{x}{3(x-2)}$;

(2) $\frac{1}{(x+2)(x+3)(x+4)}=\frac{1}{2(x+2)}-\frac{1}{x+3}+\frac{1}{2(x+4)}$.

12. Solve the equations :—

(1) $\frac{x}{7}+\frac{5x}{9}+\frac{4x}{11}-\frac{6x+1}{99}=x$;

(2) $(2x+5)(3y-4)=6xy+4y+3$, and $3x-2y=2$.

13. A wine merchant has two sorts of wine, one sort worth 12*s.* a gallon, the other sort, 9*s.* a gallon; he wants to make a mixture of 36 gallons to be sold at 10*s.* a gallon: how many gallons of each sort must he take?

XL.

1. (1) Multiply 4 tons 15 cwts. 2 qrs. 27 lbs. by 29;
   (2) Divide £163 17*s.* 6*d.* by 57.

2. A contractor purchases 46 tons 7 cwts. of potatoes for £145, and sells them at $5\frac{1}{2}d.$ per stone of 14 lbs.; calculate his profit.

3. If 9 horses can plough 184 acres in 23 days, how many acres can 54 horses plough in 7 days?

4. Find the simple interest on £8333 6*s.* 8*d.* for 5 years, at $3\frac{1}{2}$ per cent.

5. A yacht is bought for £360; at what price must it be sold to gain $12\frac{1}{2}$ per cent. on the purchase money?

6. (1) Reduce $\frac{2436}{3045}$ to its lowest terms, and find the value of $\frac{2436}{3045}$ of £210.
   (2) Multiply 1·25 by ·0016, and prove the result by vulgar fractions.
   (3) Reduce $\frac{17}{16}$ to a decimal.

7. A piece of cloth 5 times as long as it is wide cost £38, find its length and width, supposing its price to be 9*s.* 6*d.* a square yard?

8. Compare $(\cdot001)^6$ with $(\cdot01)^9$, and find the square root of 3587·890201.

9. (1) How are the squares and cubes of numbers represented in Algebra? Show that $a^3 \times a^2 = a^5$.
   (2) Find the value of $\dfrac{a^3 - b^3}{a - b}$ when $a=3$, $b=2$.

10. Perform the operations indicated in the following examples:—
   (1) $\dfrac{xy - 2x^2}{3} - \dfrac{5xy - x^2}{4} + \dfrac{11xy + 6x^2}{12}$;
   (2) $(a^2 - ab + b^2)(a + b) - (a^2 + ab + b^2)(a - b)$;
   (3) $(x^4 + 4x^2y^2 + 16y^4) \div (x^2 - 2xy + 4y^2)$.

11. Prove that
   (1) $(1 + 2x + 3x^2 + 4x^3) \times (1 - 2x + 3x^2 - 4x^3) =$
   $1 + 2x^2 - 7x^4 - 16x^6$;

(2) $\dfrac{a^2-b^2-2bc-c^2}{a^2+2ab+b^2-c^2}=\dfrac{a-b-c}{a+b-c}$.

12. Solve the equations :

(1) $\dfrac{10x}{9}-\dfrac{5x}{7}=8+\dfrac{x}{63}$;

(2) $\dfrac{3x-14}{4}-5x-\dfrac{2x-6}{11}=\dfrac{x}{2}-72$;

(3) $3x+2\frac{2}{3}=4(y-1)=2(x+y-1)$.

13. Two persons, A and B, have the same annual income; A lays by $\frac{1}{8}$ of his income, and B, spending £800 a year more than A, at the end of 4 years finds himself £2200 in debt. Find the annual income and expenditure of each.

XL. (*a*.)

1. Find the value in pounds, shillings, and pence of eighty millions and fourteen thousand farthings.

2. (1) If 107 articles cost £8 4*s*. 11½*d*., what will 21 cost?
(2) If 3 horses are kept 4 weeks for £10 10*s*., how long can 14 horses be kept for £168?

3. (1) Add together $\frac{1}{8}$ gua., $\frac{3}{8}$ of 5*s*. and $\frac{7}{16}$ of £7 5*s*.
(2) Divide 226·05924 by ·008354.

4. (1) The French mètre being 3·28 ft., find the number of miles and yards in 10000 mètres. (2) Express 3 pks. as a fraction and as a decimal of 1 quarter.

5. (1) Find the number of square feet in 3·7 acres;
(1) Extract the square root of ·0006780816.

6. What is the present value of £4785 due 2½ years hence, money bearing the simple interest of 3½ per cent. yearly?

7. (1) Multiply $bc+ac+ab$ by $a-b+c$.
(2) Square $\dfrac{2x}{y}+3+\dfrac{y}{2x}$.

8. Prove that $(2x^2-1)\left(\dfrac{x}{1-x}+\dfrac{1+x}{x}\right)=\dfrac{x}{1-x}-\dfrac{1+x}{x}$.

9. Divide $x^4+4x^2y^2-32y^4$ by $x-2y$.

10. Solve the equations:—

(1) $\frac{6x-1}{2}-\frac{7x-2}{10}=\frac{33}{5}$.

(2) $\frac{x-3}{x-4}=\frac{x+3}{x-1}$;

(3) $\left.\begin{array}{l}\frac{3x-1}{4}-\frac{3y-5}{7}=1. \\ \frac{2x+1}{7}+\frac{2y+1}{3}=4.\end{array}\right\}$

(4) $(3x-8)(2x-7)=(x+2)(x-2)$.

11. Find a number, which, after being multiplied by 4, will exceed 50 as much as it is now short of 50.

12. A man buys a case of oranges at 6*d.* a dozen. He finds fifty spoiled, and selling the remainder 3 for 2*d.* has 10*s.* 9*d.* for profit. How many oranges were there in the case?

13. A man has to travel 678 miles in 12 days, and in each day he goes 3 miles less than in the preceding day. What is the length of his first day's journey?

XLI.

1. Find the value in £. *s.* *d.* of six millions, fourteen thousands, and eight farthings.

2. (1) Find the cost of 365 articles at £7 15*s.* $7\frac{3}{4}$*d.* each.
   (2) If a silver spoon weighs 2 oz. 4 dwts., what is the weight of 15 such spoons?

3. Add together 3 ac. 3 ro. 32 po. and 5 ac. 1 ro. 12 po.; and express the result in square yards.

4. If 125 francs are worth £5, how many francs are equivalent to £2 8*s.*?

5. If 16 men can mow 65 acres in 15 days, how many men are required to mow 104 acres in 12 days?

6. (1 Find the sum of $\frac{4}{5}$ of half a guinea, $\frac{3}{8}$ of half a crown, and ·28125 of a florin.
   (2) Square ·00625 and express the result as a vulgar fraction.
   (3) Extract the square root of ·034596.

7. (1) Cube $a-2b+3c$;

(2) Divide $1-2x^4+x^8$ by x^2+2x+1.

8. Add the fractions $\frac{x}{a^2-ax+x^2}$, $\frac{1}{a-x}$, $\frac{a^2}{a^3+x^3}$.

9. Solve the equations:—

(1) $42\,(1-3x)=35\,(1-2x)$.

(2) $\frac{2x-3}{6}=\frac{4x+5}{12}-\frac{5x-13}{2x-9}$.

(3) $\left.\begin{aligned} x+2y&=34 \\ 2x+3y&=24 \end{aligned}\right\}$

10. Sum the series—

(1) $1-2+3-4+5-$&c. to 17 terms.

(2) $\frac{2}{3}+1+\frac{3}{2}+\frac{9}{4}+$&c. to n terms.

11. A person wishing to give some boys 1*s.* each, found his money too little by 1*s.*; and when he gave them 6*d.* each he had 5*s.* remaining. How much money had he?

12. Find a fraction whose denominator exceeds its numerator by 3, and which is equal to $\frac{1}{2}$ when 2 is added to the denominator.

13. A draper buys a piece of cloth for £12, and adds as his profit on each yard as many pence as there are yards in the piece. When he has sold 20 yards he finds that he has cleared the cost price of the whole. How many yards are there in the piece?

XLII.

1. If 4 cwts. 1 qr. 7 lbs. are bought at £5 per cwt., and sold at 8*d.* per lb., what is the total loss?

2. Divide £52 8*s.* 4*d.* into two parts which shall be in the proportion of 7 to 13.

3. Divide $\frac{2}{3}+\frac{3}{8}+\frac{7}{12}$ by $\frac{7}{8}-\frac{5}{6}$.

4. Divide 1·0125 by 3·375 and 13·375 by ·0125; and verify the work by converting the decimals into common fractions.

5. What would be the cost of painting the four walls of a room whose length is 24 feet 3 inches, breadth 15 feet 8 inches, and height 11 feet 6 inches, at 4*s.* per square foot?

6. If 5 men in 6 days can reap a field 1200 feet long and 800 feet broad, working 6 hours a day, what is the breadth of a field 1280 feet long which 6 men can reap in 8 days, working 5 hours a day?

7. Find the simple interest of £237 10*s*. for 5 years at 4 per cent.

8. Extract the square root of ·05368489.

9. Find the numerical value of—

(1) $a-b+c\times a-b+c$;

(2) $a-(b+c)\times a-(b+c)$; (3) $(a-b+c)\times(a-b)+c$; when $a=6$, $b=4$, $c=2$.

10. (1) Find the sum of $a^3+3a^2b-6ab^2+2b^3$, $3a^3+8a^2b+9ab^2+5b^3$ and $-4a^3-9a^2b+4ab^2-4b^3$;

(2) Take $\frac{x^3}{3}-\frac{x^2y}{4}+\frac{xy^2}{4}-\frac{y^3}{3}$ from $\frac{x^3}{2}+\frac{x^2y}{12}-\frac{xy^2}{12}+\frac{y^3}{2}$.

11. (1) Multiply $3x^py^2$ by $-4x^{p-2}y^3$;

(2) $7x^2-3xy+y^2$ by $2x^3-x+y$;

(3) Divide $x^4-2x^3y+x^2y^2-a^2x^2+2a^2xy-a^2y^2$ by $x-a$.

12. (1) Reduce $\dfrac{x^4+5x^3-2x^2-15x-3}{x^5+5x^4+x^3-2x^2-10x-2}$ to its lowest terms; (2) simplify the expression $\dfrac{2x-3}{x+3}-\dfrac{2x+3}{x-3}+\dfrac{4x^2-9}{x^2-9}$; (3) multiply $\dfrac{x^4+x^2y^2+y^4}{x^3+y^3}$ by $\dfrac{x^2-y^2}{x^3-y^3-2xy(x-y)}$.

13. Solve the equations :—

(1) $cx-q=bx+r$; (2) $9x-2y=37$, $3x-4y=-1$;

(3) $y(x^2+y^2)=4(x+y)^2$ and $xy=4(x+y)$;

(4) Find two numbers in the ratio of 7 to 8, the difference of their squares being 735.

MISCELLANEOUS PAPERS.

XLIII.

1. Show that a fraction is multiplied if its denominator is divided, and divided if its denominator is multiplied.

2. Simplify (1) $5\frac{2}{3}$ of $\frac{1}{15}\div(6\frac{1}{5}-\frac{1}{25})$; (2) $\frac{501}{1837}$.

3. (1) Multiply 6 by ·0005; (2) divide 17·25 by ·0023; (3) express $\frac{2}{15}$ as a decimal; and reduce (4) ·075 (5) $2\cdot3\dot{4}\dot{8}$ to vulgar fractions in their lowest terms.

4. (1) Reduce $5\frac{1}{2}$ gals. to the fraction of $1\frac{3}{8}$ pks.; (2) $4\frac{1}{2}$ oz. to the decimal of 4 lbs. 1 oz. 4 dr.; (3) find the value of ·6875 of 3*s.* 4*d.*

5. What will be the expense of paving an area which measures 35 ft. 10 in. by 18 ft. 6 in., at 6*s.* 3*d.* per square yard?

6. How much coffee at 1*s.* $10\frac{1}{2}$*d.* per lb. must be given in exchange for 75 lbs. of tea at 5*s.* $1\frac{1}{2}$*d.* per lb.?

7. If 8 men can build 2 roods of wall one brick in thickness in 3 days, how many men must be employed to build 5 roods, a brick and a half thick, in a week?

8. Find the difference between the simple and compound interest upon £383 5*s.* 8*d.* in $2\frac{1}{2}$ years at $3\frac{3}{4}$ per cent.

9. (1) State the rules for Multiplication in Algebra of monomials and of multinominals;

(2) Multiply $a^3-2a^2x+ax^2-\frac{1}{3}x^3$ by $3a^2-ax+3x^2$.

10. Divide $a^2+(ac-b^2)x^2+bcx^3$ by $a-bx+cx^2$.

11. Reduce to their simplest forms:—

$$(1)\ \frac{x^2+3x-4}{x^3+x^2-4x+2};\quad (2)\ \frac{a}{x(a-x)}-\frac{x}{a(a-x)};$$

$$(3)\ \frac{1}{1-\dfrac{1}{1-\dfrac{1}{x}}}.$$

12. Solve the equations:—

$$(1)\ \frac{2x-5}{6}+\frac{6x+3}{4}=5x-19\tfrac{1}{12};$$

$$(2)\ \frac{x-a}{a-b}-\frac{x+a}{a+b}=\frac{2ax}{a^2-b^2};$$

$$(3)\ \frac{x-1}{4}-\frac{y-2}{5}=1 \text{ and } x-\frac{2y-5}{3}=y-1.$$

13. A and B ran a race, which lasted 2 minutes; B had a start of 20 yards; A ran 3 yards whilst B was running

2, and won by 30 yards. What was the length of the course and the speed of each ?

XLIV.

1. Simplify (1) $\frac{1}{3}-\frac{4}{21}+\frac{1}{2}-\frac{7}{11}$;
(2) $\dfrac{2+\frac{1}{2}-\frac{1}{3}}{2\frac{1}{3}-1\frac{1}{4}}$; (3) $\dfrac{38}{39}\times 2\frac{1}{7}\times 2\dfrac{\frac{1}{4}}{4\frac{3}{4}}\div\frac{5}{14}$.

2. How is a decimal multiplied by any power of 10? Prove the truth of your answer.

3. Divide (1) ·169 by ·0013 ; (2) 49·7 by ·0025 ;
(3) ·00003 by ·001.

4. Explain the meanings of the terms Unit, Integer, Number, and Factor.

5. Find the value (1) of $\frac{12}{91}$ of 13 guas. ; and (2) of $\frac{17}{73}$ of £7 18*s.* 2*d.*

6. (1) Reduce $\frac{8}{1\cdot 25}d.$ to the decimal of £1 ; (2) Find the price of 63 cwts. 3 qrs. 18 lbs. 8 oz. at £8 12*s.* 8*d.* per cwt.

7. Find the value of (1) $\sqrt{2a}+\sqrt{2b}-3\sqrt{2c^2-d^2}$;
(2) $\dfrac{a-b}{a+b}-\dfrac{4(c-d)}{c+d}$, when $a=8$, $b=4$, $c=5$, $d=7$.

8. From
$3x-5y-6z+(2x+3y+4z)-(4x-3y-3z)$ take $3y-x+z$.

9. (1) Multiply $1-2x+x^2$ by $1-2y+y^2$;
(2) Divide $x^5-3x^4+4x^3+26x^2-92x+55$ by $x^2-3x+11$.

10. Find the L.C.M. of $-3x^3y^2z$, $-12x^2yz^3$, $42x^3yz^2$.

11. Simplify $\dfrac{a-b}{a}+\dfrac{a-b}{b}+\dfrac{a^2+b^2}{ab}$.

12. Solve the equations (1) $3x-5=2x+2$;
(2) $\dfrac{3x-9}{7}-\dfrac{x+1}{11}=\dfrac{3x-14}{8}$.

13. A garrison is provisioned for 180 days ; at the end of 30 days it is found that $\frac{1}{4}$ of the stores then remaining is unfit for use ; at the end of 30 days more, it is reinforced by 2000 men, and then the stores are made to last 110 days longer by putting the men on half rations. Find the number of the original garrison.

XLV.

1. Write down in figures six hundred and one millio fifteen thousand and seventy.

2. If 7 francs are worth 5*s.* $8\frac{1}{4}$*d.*, find the value of 2075 francs.

3. If 1 cwt. 2 qrs. 14 lbs. of sugar cost £3 11*s.* 6*d.*, wh will 7 lbs. cost?

4. Find the value of 10 oz. 15 dwts. 16 grs. of gold £3 15*s.* an ounce.

5. A bankrupt's estate pays 4*s.* $4\frac{1}{2}$*d.* in the pound; wh loss will be sustained by a creditor whose claim amounts £432 12*s.*?

6. Find the simple interest on £850 for 4 years at $3\frac{1}{2}$ p cent.

7. Take $b^2-\frac{3a^2}{5}-\left(\frac{8}{9}c^2-\frac{6}{7}d^2\right)$ from $a^2-\frac{b^2}{4}-\left(3d^2-\frac{c^2}{9}\right.$ and find the value of the remainder when $a=10, b=$ $c=3$, $d=7$.

8. Expand (1) $(\frac{1}{2}-x)^3$; (2) $(2x^2+\frac{3x}{2}-5)^2$.

9. Divide

$$x^5-\frac{23}{30}x^4+\frac{31}{10}x^3-\frac{7}{3}x^2-\frac{181}{18}x+\frac{5}{3} \text{ by } x^2-\frac{x}{6}+5.$$

10. Reduce to their simplest forms (1) $\dfrac{a^2-ab}{a^2b-abc-ab^2+b^2c}$

$$(2)\ \frac{\frac{x^2}{y^2}-2+\frac{y^2}{x^2}}{2\left(\frac{x^2}{y^2}-\frac{y^2}{x^2}\right)};\ (3)\ \frac{1}{x+4}+\frac{x+1}{x^2-4x+16}-\frac{x^2+x+24}{x^3+64};$$

$$(4)\ \frac{\frac{a}{x+m}}{y+\frac{n}{z}};$$

11. Reduce $\dfrac{x^5-2x^4+3x^3-3x^2+6x-9}{x^5+2x^4-3x^3-3x^2-6x+9}$ to its lowest *terms.*

12. Solve the equations :—

(1) $5(x+\cdot 5)-7x\left(\frac{1}{x}-\frac{1}{7}\right)=16\frac{1}{2}$.

(2) $\frac{a}{x}+\frac{b}{y}=p$ and $\frac{b}{x}+\frac{a}{y}=q$.

13: A can do a piece of work in 9 days, B in twice that time, C can only do $\frac{3}{4}$ as much work as A in a day; how long would A, B and C, working together, require to do the same piece of work ?

XLVI.

1. (1) Find the number of miles, furlongs, &c., in 78567 inches.

(2) Express 5 poles as the fraction of a mile.

2. What length of matting, $\frac{3}{4}$ yard wide, will cover the floor of a hospital 31 ft. 6 in. long by 18 ft. 9 in. wide? (2) What is the cost of the matting at 2*s.* 3*d.* a yard?

3. (1) Multiply $\frac{3}{10}$ of $\frac{5}{6}$ by $2\frac{7}{8}$, and divide the result by $\frac{1}{2}+\frac{3}{4}$.

(2) Show that the product of two proper fractions is less than either of them.

4. Reduce $\frac{113}{125}$ to a decimal. How is the number of decimal places indicated by the denominator of the fraction?

5. Find the vulgar fraction equivalent to the circulating decimal $\cdot 81\dot{8}\dot{1}$, and find the value of $\cdot 81\dot{8}\dot{1}$ of £1 2*s.*

6. Extract the square root of 25·7049; state why 2·57049 cannot be a perfect square.

7. Find the values of ; (1) $(x+y+2)^2$;

(2) $\frac{x^3-y^3}{x-y}$ when $x=3$, $y=2$.

8. Multiply (1) $3x^2+2xy+y^2$ by $3x^2+2xy-y^2$ and

(2) $\frac{a+2b+3c}{2}$ by $\frac{a-2b-3c}{2}$.

9. Divide, (1) $21a^4-16a^3b+16a^2b^2-5ab^3+2b^4$ by $3a^2-ab+b^2$, and

(2) $x-4y$ by $x^{\frac{1}{2}}-2y^{\frac{1}{2}}$.

10. Solve the equations: (1) $x-\frac{2x+1}{3}=\frac{x+3}{4}$.

(2) $9x+\frac{8y}{5}=70$ and $7y-\frac{13x}{3}=44$.

11. Extract the square root of $1-4x+10x^2-12x^3+9x^4$.

12. A contractor agreed to provide rations for a garrison at the rate of 8*d.* a head daily; after he had provisioned the garrison for 30 days, 600 more men were marched into the garrison, and at the end of 60 days from the commencement of his contract, the contractor received £2280. How many men were in garrison at first?

13. When m and n are whole numbers, and m greater than n, show that $a^m \div a^n = a^{m-n}$; and that $\frac{1}{a^n}$ is correctly symbolised by a^{-n}.

XLVII.

1. If a person's income be 400 guineas a year, and his daily expenditure £1 1*s.* 4½*d.*, how much can he save in 5 years?

2. A certain number of men can mow 4 acres of grass in 3 hrs., and a certain number of others can mow 8 ac. in 5 hrs. How long will they take to mow 22 ac. if they all work together?

3. If the carriage of 4 cwts. 3 qrs. for 20 miles cost 14*s.* 3*d.*, what will be the cost of the carriage of 5 cwts. 3 qrs. 21 lbs. for 100 miles?

4. (1) Divide $\frac{1}{4}+\frac{1}{8}+\frac{1}{12}+\frac{1}{16}+\frac{1}{20}$ by $\frac{1}{6}+\frac{1}{10}+\frac{1}{18}+\frac{1}{30}$, and reduce the result to a decimal; (2) Express $\frac{3}{8}$ of 15*s.* 9*d.* as the decimal of 1 guinea; and (3) find the value of ·01171875 of 1 ton.

5. Divide each of the following to 4 places of decimals: (1) 8·125 by 2·175; (2) ·005 by ·425; (3) ·25 by ·00325; (4) ·002346 by ·001825.

6. Find the simple interest on £237 10*s.* in 5 yrs. at 3 per cent.

7. (1) Find the sum of the squares of ·4, ·8, 1·2, and 1·72;

approximate as far as 4 places of decimals to the square roots of (2), $5\frac{1}{3}$ and (3), $4\frac{1}{4}$.

8. Perform the operations indicated in the following examples:—

(1) $2(a-b)-c+d-\{a-b-2(c-d)\}$.

(2) $(2x^4-4x^2-4x-1)\times(2x^4-4x^2-4x-1)$.

(3) $(x^3+y^3+3xy-1)\div(x+y-1)$.

9. Reduce $\dfrac{9x^3+53x^2-9x-18}{x^2+11x+30}$ to its lowest terms.

10. If $xy+yz+xz=1$, show that $\dfrac{x}{1-x^2}+\dfrac{y}{1-y^2}+\dfrac{z}{1-z^2}=\dfrac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}$.

11. Solve the equations:—

(1) $\dfrac{x}{7}-\dfrac{x-5}{11}+5=x-\left(\dfrac{2x}{77}+1\right)$

(2) $\dfrac{4}{7}\left(\dfrac{2x}{3}-\dfrac{5y}{12}\right)-\dfrac{2}{23}\left(\dfrac{3x}{2}-\dfrac{y}{3}\right)=2$, and $\dfrac{x-y}{x+y}=\dfrac{1}{5}$.

12. A person after paying a poor-rate, and also an income-tax of $7d.$ in the pound, has £486 remaining. The poor-rate amounts to £22 10$s.$ more than the income-tax. Find the original income; and the number of pence per £ in the poor-rate.

13. (1) Extract the square root of $x^4-2ax^3+5a^2x^2-4a^3x+4a^4$, and the cube roots of (2) $x^6-3x^5+9x^4+13x^3+18x^2-9x+8$; (3) 46656.

XLVIII.

1. How many hours are there in 1600 years, reckoning 97 leap years in 400 years?

2. Find the weight of 5 dozen spoons, each weighing 2 oz. 4 dwts.

3. How many yards, worth 4$s.$ $2\frac{1}{4}d.$ a yard, must be given in exchange for 402 yards at 3$s.$ $5\frac{1}{2}d.$ a yard?

4. State and explain the rule for division of fractions.

Simplify (1) $2\frac{1}{2} \div 3\frac{1}{4} + 4\frac{5}{13} \div 2\frac{5}{7}$.

(2) $13\frac{2}{15} - 5\frac{5}{27} - 6\frac{3}{45} + 5\frac{1}{12}$.

5. Reduce $\frac{1001}{1000}$, $\frac{19}{128}$, $\frac{1}{1111}$, to decimals; and $\cdot 015625$, $\cdot\dot{0}09\dot{9}$, to vulgar fractions.

6. If $x=5$, and $y=3$, find value of $x\sqrt{x^2-8y}+y\sqrt{x^2+8y}$.

7. (1) Add $a-(b-c)$, $2a-3(b+c)$, and $-3a-(-b+c)$;

(2) subtract $\frac{3}{4}x-\frac{7}{8}y$ from $\frac{2}{3}x+\frac{3}{5}y$.

8. Multiply (1) $x^3-4x^2+11x-24$ by x^2+4x+5.

(2) $a+b+c$, $a+b-c$, $a-b+c$, and $-a+b+c$.

9. Divide (1) $x^6-6x^4+9x^2-4$ by x^2-1.

(2) The product of $x^3-12x+16$ and $x^3-12x-16$ by x^2-16.

10. Find G. C. M. of $2x^4-12x^3+19x^2-6x+9$, and $4x^3-18x^2+19x-3$.

11. Simplify—

(1) $\dfrac{x^4+2x^2+9}{x^4+4x^3+10x^2+12x+9}$.

(2) $\dfrac{a}{a-x}+\dfrac{3a}{a+x}-\dfrac{2ax}{a^2-x^2}$.

12. Solve the equations—

(1) $3x-5=9x-7$.

(2) $(x-15)(x+1)+(x-9)(x-7)=(x-1)(x-7)-(x-15)(23-x)$.

(3) $\dfrac{x-5}{x-4}-\dfrac{x-6}{x-5}=\dfrac{x-2}{x-3}-\dfrac{x-3}{x-4}$.

13. The difference of two numbers is 14, their quotient is 8; find them.

XLIX.

1. The circumference of the earth being 25,000 miles, find the rate per minute at which a place on the equator moves.

2. In a million acres how many square miles are there?

3. A person's net income, after paying 7*d.* in the £ for

income-tax, was £339 15*s.* 10*d.*; what would it have been if the tax had been 10*d.* in the £?

4. Simplify (1) $10\frac{1}{21}-3\frac{2}{15}-1\frac{39}{70}+11\frac{21}{210}$.

(2) $(6\frac{1}{15}-4\frac{3}{20})\div(1\frac{2}{11}\times 1\frac{1}{10}\div 1\frac{1}{5})$.

5. If a rupee be worth 2*s.* $0\frac{1}{2}$*d.*, and a dollar 4*s.* $4\frac{1}{2}$*d.*, find the least number of rupees which makes an exact number of dollars.

6. Show, from the meaning of a^5 and a^3, that $a^5\times a^3=a^8$, and that $\frac{a^5}{a^3}=a^2$.

Multiply $3x^2-4x+7$ by $5x^2-x-4$.

7. Find the sum of $ax-by$, $(a-b)\,x-(a+b)\,y$, and $(a+b)\,x+(a-b)\,y$.

8. What is the meaning of the term *factor*? $x-1$ being one factor of x^2+x-2, what is the other?

9. Reduce $\dfrac{5x^3-14x^2+16}{3x^3-2x^2+16x-48}$ to its lowest terms.

10. What is the least common multiple of $12a^2bc$, $3ab^2c$, $15abc^2$?

11. Prove that $\dfrac{\dfrac{1+a}{1-a}-a}{a+\dfrac{a+a^2}{1-a}}=1$.

12. Solve the equations:—

(1) $\frac{1}{5}\,(3x-2)-\frac{1}{3}\,(5x-9)=\frac{1}{10}\,(x+2\frac{2}{3})$.

(2) $\dfrac{2x-11}{3x-5}=\dfrac{16}{95}+\dfrac{2x-7}{12x-1}$.

(3) $\dfrac{2}{x}+\dfrac{3}{y}=\dfrac{5}{6}$; $\dfrac{4}{x}+\dfrac{9}{y}=2$.

13. A plays at chess with B, winning 3 games out of 4; and afterwards with C, winning 2 out of 3; at the end of 21 games, he has won 15. How many did he play with each?

L.

1. Find the value of $\frac{4}{11}$ of 1*s.* 9*d.*$+\frac{2}{11}$ of 5*s.* 3*d.*$-\frac{2}{11}$ of 8*s.* 9*d.*

2. Find the value of—

(1) 2·125 of 17*s.* $5\frac{3}{4}$*d.*; (2) ·300694$\dot{4}$ of a day.

3. Reduce 5 cwts. 1 qr. 14 lbs. 14 oz. to the decimal of a ton.

4. Required the net weight of 29 barrels, each weighing 2 cwts. 3 qrs. 21 lbs., allowing $5\frac{3}{4}$ lbs. per barrel for tare.

5. How long will it take to empty a reservoir of water 15 feet long, 10 ft. 2 inches wide, and 10 ft. deep, at the rate of 10 gallons a minute, a gallon containing $277\frac{3}{11}$ inches?

6. Simplify $7a-[4a+(3a-b)]$.

7. Find the value of $a(b^2+c^2-a^2)+b(a^2+c^2-b^2)+c(a^2+b^2-c^2)$, when $a=1$, $b=2$, $c=-3$.

8. Multiply x^2-2x+4 by x^2+2x+4, and prove the result by division.

9. Extract the square root of $4a^4+12a^3b-7a^2b^2-24ab^3+16b^4$.

10. Find the G. C. M. and L. C. M. of $a^2-12a+32$, and $a^2-10a+16$.

11. What is meant by the root of an equation? Show that 1 and 2 are roots of the equation $x^3-7x+6=0$; and find the third root.

12. Solve the equations :—

(1) $6+3x=8-4x$.

(2) $2(x-1)+3(x-3)=4(x-2)$.

(3) $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=13$.

(4) $\frac{x-4}{x-1}+\frac{x-1}{x-4}=2+\frac{1}{x-4}$.

(5) $\left.\begin{array}{l}\frac{x-2}{3}+\frac{y-3}{2}=\frac{2}{3}\\(x-4)(y+3)=(x+4)(y-3)\end{array}\right\}$

13. A person, after paying away one-half of his money, and then one-third of the remainder, finds that he has remaining £2 more than its fourth part. What sum had he?

LI.

1. If 6 men can reap $4\frac{1}{2}$ acres in $3\frac{3}{4}$ days of 9 hours each, in how many days will 9 men reap a field of $22\frac{1}{2}$ acres, working 12 hours a day?

2. Find the interest of £1235 15*s.* 6*d.* for $4\frac{1}{2}$ years, at $3\frac{3}{4}$ per cent.

3. If a person purchase £4000 stock at $93\frac{5}{8}$, what loss will he sustain by selling out at $92\frac{7}{8}$?

4. How many square feet of board will be required to make a cubical box which shall contain $81\cdot\dot{3}7\dot{0}$ cubic feet?

5. If $a=\sqrt{2}$, $b=3$, $c=4$, $d=0$, find the value of

$$a^2b+cd-\sqrt{a^4+bc}+\sqrt[3]{2c+bd}.$$

6. Simplify $1-[2-(7-\overline{x-4})]+2-[3+(4-\overline{x-5})]$.

7. Find the product of—

(1) $a^2+b^2+c^2-ab-ac-bc$ by $a+b+c$.

(2) $2+x$, $x+3$, $2-x$, and $x-3$.

8. Divide (1) $x^5-5x^3+5x^2-1$ by x^2+3x+1.

(2) 1 by $1-2x+x^2$ to 5 terms in the quotient.

9. Find the G. C. M. of

$$2x^3-3x^2y-4xy^2+6y^3, \text{ and } 2x^3-x^2y+xy^2-6y^3.$$

10. Simplify (1) $\dfrac{1}{a-b}+\dfrac{1}{a+b}-\dfrac{2a}{a^2-b^2}$.

(2) $\dfrac{1+x^2+x^4}{1-x^4}\div\dfrac{1+x+x^2}{1+x^2}$.

11. Solve the equations:—

(1) $5-\dfrac{x+4}{11}=x-3$.

(2) $\dfrac{1}{x+2}-\dfrac{2}{x+2}=\dfrac{3}{x+3}-\dfrac{4}{x+4}$

(3) $\sqrt{12+x}=\sqrt{x}+6$. (4) $\left.\begin{aligned}3y-2x&=11\\13x-5y&=\ 1\end{aligned}\right\}$

12. A grocer has tea worth 3*s.* per lb. and other tea worth 5*s.* per lb. In what proportion must he mix 30 lbs. so that he may sell the mixture at 3*s.* 4*d.* per lb.?

13. There is a number of two digits, to which if 36 be added, the digits will be reversed, and the sum of the digits is 12; find the number.

LII.

1. How much water must be added to a cask containing 70 gallons of spirits, worth 13*s.* 4*d.* a gallon, to reduce the price to 11*s.* 8*d.* a gallon? What would be the gain per cent. if it were sold at 13*s.* 4*d.* a gallon?

2. Which is the better investment, the 3 per cents. at 90, or the $3\frac{1}{2}$ per cents. at 102? What income would be derived from the investment of £2499 15*s.* in the former?

3. (1) How many cubic feet of lead $\frac{1}{16}$ inch thick will be required to cover the sides and bottom of a cistern 10 feet long, 6 feet 6 inches wide, and 7 feet deep? (2) What weight of water will the cistern hold, a cubic foot of water weighing 1000 ounces?

4. Calculate the numerical value of—

$$(1)\ \sqrt[4]{x^4-x^3yz+xy^3z-xyz^3-4y^4+2z^4},$$
when $x=-2,\ y=-1,\ z=+2.$

Also find the value of—

$$(2)\ \frac{ac+bd-ad+bc-1}{ag+2f(b-a)+bg},$$

when $a=4,\ b=5,\ c=6,\ d=7,\ f=3$, and $g=0$.

5. A person travelled by a railway from London at the rate of x miles an hour for 5 hours, and afterwards travelled back towards London on the same railway by a slower train at the rate of y miles an hour for 4 hours. Form an algebraical expression for his distance from London, by that railway, at the end of his second journey.

6. Find the sum of—

$$a^3+ab^2+ac^2+a^2b+a^2c-abc+2def$$
$$-2a^3+3ab^2-3ac^2+5a^2b-7a^2c+3abc+3ade$$
$$11a^3-5ab^2+7ac^2-3a^2b+2a^2c-4abc-7acd$$
$$-a^3+ab^2-2ac^2-7a^2b-2a^2c+2abc-3def.$$

7. From $7ab-12ac+4ad-5fg+6gh$
take $3ab-12ac-5ad+7fg-5gh$.

8. (1) Find the product of x^2+7x-5 and x^2-3x+7.
(2) Also multiply $bxz-cxy+ayz$ by $bcx+acy-abz$.

9. Divide $2a^3-7a^2-46a-21$ by $2a^2+7a+3$.

10. Reduce $\dfrac{1}{2x+1}+\dfrac{1}{2x-1}-\dfrac{7}{4x^2+1}$ to a single fraction.

11. Find the G. C. M. of $21x^3-15x^2-14x+10$
and $21x^3+27x^2-14x-18$.

12. Solve the following equations :—
(1) $7x-15=2x+35$.
(2) $\dfrac{9x+1}{7}-\dfrac{7x-5}{4}=\dfrac{3x+6}{8}-1\frac{1}{28}$.
(3) $4(3x-2)-5(6x+1)=2(9x-7)-12(4x-3)+2x$.
(4) $\begin{cases}8x-9y=20\\7x+3y=61\end{cases}$

13. The breadth of a rectangular sheet of paper (not twice as long as it was broad) was 5 inches less than its length. A square piece has been cut off the end, and the breadth of the remaining piece is $\frac{4}{7}$ths of its length. Determine the original length and breadth.

LIII.

1. Multiply (1) ·002897 by 3020 ; (2) ·47923 by 90·24.
2. Express (1) $\frac{9}{28}$, $\frac{108}{37}$ as decimals
(2) $\cdot4\dot{2}\dot{9}$, $7\cdot4\dot{2}1\dot{9}$, as vulgar fractions.
3. Reduce (1) 14*s.* 9¾*d.* to the decimal of £1 ; and
(2) 10 drms. to the decimal of 1 lb. avoirdupois.
4. What would be the amount paid for wages in 11 months 2 weeks 5 days, at £4 19*s.* 10½*d.* a month ?

5. Find when the hands of a clock will be together between the hours of 5 and 6.

6. Express 77 miles in kilomètres; a kilomètre=1000 mètres and 35 yards=32 mètres.

7. Divide 1 by $1-x$ to five terms.

8. Find the G.C.M. of $a^5+a^4b-3a^2b^3-5ab^4-2b^5$ and $a^4+2a^2b^2-ab^3+2b^4$.

9. Find the L.C.M. of $x^2-3x-70$ and $x^3-39x+70$.

10. Add the fractions—

$$\frac{1+x}{1-x^3}+\frac{1-x}{1+x^3}-\frac{2}{1-x^2}.$$

11. Solve the equations—

(1) $\frac{xy}{x+y}=1;\ \frac{xz}{x+z}=2;\ \frac{yz}{y+z}=3.$

(2) $\left.\begin{aligned} x^4+y^4&=337 \\ x+y&=\ \ \ 7\end{aligned}\right\}$

12. Prove that $\log(3^5\times 4^6)=5\log 3+6\log 4$; and find by the tables $(\cdot 0874)^{\frac{1}{5}}$.

13. Show how to draw the four common tangents to two given circles.

LIV.

1. Find (1) the number of acres in a square mile; and (2) reduce 895485 oz. avoirdupois to pounds, quarters, cwts. &c.

2. What is the L.C.M. of two numbers? Find the L.C.M. of 198, 495 and 2475. By what simple tests can it be ascertained whether 198, 495, 2475 are divisible by 9 and by 11 or not?

3. Why are fractions reduced to a common denominator previously to finding their sum or difference? Find the sum of $\frac{11}{12}$, $\frac{13}{18}$, $\frac{17}{30}$, in its lowest terms.

4. Express the difference between $(2\cdot 8284)^2$ and 8.

5. Find the difference between

$$\frac{2a}{3}-\frac{b}{2}+c \text{ and } \frac{a}{2}+\frac{2b}{3}-3c.$$

6. Find (1) $(1+2x^{\frac{1}{3}}+3x^{\frac{3}{2}})^2$;

(2) $\sqrt{\{x^4-6x^5+17x^6-24x^7+16x^8\}}$.

7. Prove that $\dfrac{2x^2-11x-21}{6x^2-9x-27}=\dfrac{x-7}{3(x-3}$.

8. Solve the equations—

(1) $\dfrac{12x-8}{6}-\dfrac{18-4x}{3}=x+2$.

(2) $\dfrac{x}{2}+\dfrac{y}{3}=13$ and $\dfrac{x}{5}+\dfrac{y}{8}=5$.

9. A garrison was provisioned for 30 days; after 10 days the garrison was reinforced by 3000 men, and the provisions were then exhausted in 5 days; find the number of men in garrison at first.

10. When are magnitudes in harmonic progression? Find the H.M. between $a+b$ and $a-b$.

11. If x, y, z, be in geometrical progression, prove that

$$x^2y^2z^2(x^{-3}+y^{-3}+z^{-3})=x^3+y^3+z^3.$$

12. If two finite straight lines intersect and bisect each other, the straight lines joining their extremities will form a parallelogram.

13. If ABC be any triangle, and AD, BE, CF, the perpendiculars drawn from the angles on the opposite sides, prove that AF : AE :: AC : AB.

LV.

1. (1) Express as vulgar fractions ·00125, ·75.

(2) Express as decimals $\frac{1}{25}$, $\frac{1}{4}$, $\frac{1}{27}$.

(3) Divide ·05344 by 83·5, and 4·32 by ·00036.

2. Reduce 5 poles 4 yards $2\frac{1}{2}$ feet to the decimal of a furlong.

3. If 72 men dig a trench 20 yds. long, 1 ft. 6 in. broad, 4 ft. deep, in 3 days of 10 hrs. each, how many men would be required to dig a trench 30 yds. long, 2 ft. 3 in. broad, and 5 ft. deep, in 15 days of 9 hrs. each?

4. Find the interest on £527 8*s.* 6*d.* in 2 yrs., at $3\frac{1}{2}$ per cent.?

5. How much stock can be bought with £5328 when the funds are at $98\frac{2}{3}$?

6. Find by duodecimals the content of a beam 10 ft. 6 in. long, 4 in. wide, and 2 in. thick.

7. Given the logarithms of two numbers, prove the rule for finding the logarithm of the product of the numbers.

8. Find by the tables the values of (1) $\cdot 03571 \times \cdot 2568$.

$$(2)\ \frac{8352 \times 3\cdot 69}{(30\cdot 57)^3}.$$

9. Solve the equations :—

$$(1)\ 9xy=20(x+y),\ 10xz=24(x+z),\ 11yz=30(y+z).$$

$$(2)\ x^{-1}+x^{-\frac{1}{2}}=6.$$

10. Solve the equations $xy+xy^2=12$; $x+xy^3=18$.

11. What is the number of balls in an incomplete square pile, the number of balls in a side of the bottom course being 44, in a side of the top 22?

12. Show how to find a rectangle equal to a given parallelogram.

13. Show how to describe a circle to touch a given circle and a given straight line, so that the straight line joining the points of contact may be equal to a given straight line. How many such circles can be drawn?

LVI.

1. (1) Reduce $4375\frac{1}{2}$ yards to miles; (2) How many moidores of 27*s.* each are equivalent to 81 guineas; (3) Express 600 fathoms in yards.

2. A truck, with its load of 128 equal packages, weighs 8 cwts. 12 lbs.; supposing the truck to weigh 4 cwts. 1 qr., what is the weight of a package?

3. A field is 300 yds. long, 200 yds. broad; if a belt of trees 30 yds. wide be planted round it, find the area of the interior space. Find also the distance from corner to corner.

4. Define a vulgar fraction. Find the value of the expressions :—

$$(1)\ \frac{1}{3\frac{1}{5}}-\frac{2\frac{1}{4}}{9}+\frac{3\frac{5}{8}}{2}+\frac{\frac{4}{7}}{4\frac{4}{7}};$$

$$(2)\ \tfrac{3}{5} \text{ of } \frac{13}{16}-\frac{1\frac{2}{3}}{6\frac{2}{3}} \text{ of } \frac{19}{20}+\tfrac{3}{7} \text{ of } \frac{6\frac{5}{12}}{3\frac{2}{3}}.$$

5. What fraction expresses 6534 square yards in acres?

6. A bankrupt owes £2000, and has property to the amount of £775. How much will he pay in the £?

7. Show how to determine without the tables the number of which 1·5 is the logarithm to the base 10.

8. Determine by the tables (1) $\sqrt[3]{\cdot 00061475}$;
(2) The number of digits in 13^{18}.

9. Find x and y from the equations $3x=2y$ and $x^3=y^2$.

10. Describe a circle about a given triangle; and find its radius when the sides of the triangle are 60, 80, and 100 ft.

11. Find the side of a regular hexagon which shall be equal in area to an equilateral triangle whose side is 150 ft.

12. A body of infantry is marching in regular column with 10 men more in depth than in front; on the enemy coming in sight the front is increased by 720 men, and then the infantry is drawn up in 9 lines. Find the number of men on the march.

13. If AC be the diameter of a circle, and if any other circle be described with centre C, chords of this latter which, produced if necessary, pass through A are all bisected by the former circle.

LVII.

1. Find which is the greater of the fractions $\frac{21}{23}$ and $\frac{23}{25}$; divide the sum of these fractions by their difference, and multiply the result by $11\frac{1}{2} \div 43\frac{11}{12}$.

2. A person whose income is £1000 a year requires annually two-thirds of his income for his own expenditure; he also spends $\frac{1}{12}$th of his income in purchasing pictures,

and half this amount in charities, and saves the rest; in how many years will he have saved £1250, and what will be the value of his pictures at that time?

3. Subtract ·0057642 from 1·2576, and multiply the result by ·0000045.

4. Find the value of $\frac{3}{4}$ of £2·718, and determine the decimal that £549 2*s.* 6*d.* is of £40.

5. A rectangular field, 400 yards long and 396 yards broad, is let for £54 a year; find the rent of an acre of the same field.

6. A man, walking 7 hours a day for 6 days, completes a journey of 168 miles; in how many days would he complete a journey of 720 miles, walking 6 hours a day at the same rate?

7. Define interest and discount.

Find the amount of £1000 placed out at simple interest, at $3\frac{1}{2}$ per cent. per annum, for 30 years. Also find the present value of £1000 due 30 years hence, at the same rate.

8. Find the highest common divisor of—

(1) $7x^4+14x^3+18x^2-6x-9$ and x^4+2x^2+9; and the least common multiple of—

(2) x^2+x-2, x^2-x-6 and x^2-4x+3.

9. Simplify $\dfrac{1}{4a^3(a+x)}+\dfrac{1}{4a^3(a-x)}-\dfrac{1}{2a^2(a^2+x^2)}$.

10. Reduce to their simplest forms :—

(1) $(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2})(\sqrt[3]{a}-\sqrt[3]{b})$;

(2) $(2\sqrt{32}-3\sqrt{2}+4)\div 8$.

11. Solve the equations :

(1) $\dfrac{4}{x-1}=\dfrac{1}{6-x}$;

(2) $\sqrt{x-3}+\sqrt{x}=\dfrac{3}{\sqrt{x-3}}$.

(3) $\dfrac{3}{1-2x}-\dfrac{3}{1+2x}=4$

(4) $\left\{\begin{aligned} x^3+y^3&=9 \\ xy&=2 \end{aligned}\right\}$

12. (1) Insert 4 arithmetic means between a and b ; sum the series—

(2) 12, $11\frac{1}{4}$, $10\frac{1}{2}$ to 9 terms.

(3) $\frac{5}{7}$, $\frac{15}{14}$, $\frac{45}{28}$, to 6 terms.

(4) $\sqrt{6}$, $\sqrt{2}$, $\sqrt{\frac{2}{3}}$, to infinity.

13. A sets out from a certain place and travels one mile the first day, two miles the second day, three miles the third day, four miles the fourth day, and so on ; B sets out five days after A, and travels twelve miles a day. How far will B travel before overtaking A.

LVIII.

1. Add together $a+3b+4c+5d$, $2a-b-3c-6d$, $7a-5b+9c-11d$, and $a+14b+c+23d$.

2. Find the value of the expression

$$\frac{4a^2+b^2-c^2}{4a^2+c^2-b^2}\times\frac{b^2+c^2-a^2}{b^2+c^2-4a^2}\times\frac{b+5a}{6c-b},$$

when $a=2$, $b=3$, and $c=4$.

3. (1) Multiply $x^5+x^4-11x^3+x+1$ by x^2-x-1, and

(2) Find the cube of x^2+3x+1.

4. (1) If $b>c$ and $a>b-c$, prove that

$$a-(b-c)=a-b+c.$$

(2) Simplify the expression—

$$12a-[7a+b-\{11a-5b-(17a+b)\}].$$

5. (1) Divide x^8+x^4+1 by x^2-x+1 ;

(2) Reduce $(11x+5y+7)^2-(4x-y-3)^2$ into two simple factors.

6. Solve the equations—

(1) $\frac{x-1}{4}+\frac{11x-2}{3}=\frac{1-x}{4}+\frac{59}{3}$.

(2) $\frac{7}{x}+\frac{11x-1}{x}=15-\frac{4-4x}{x}$.

7. Prove that

$$(x^2+y^2)(a^2+b^2)=(ax\pm by)^2+(ay\mp bx)^2,$$

and illustrate this formula by putting

$$x=y=4,\ a=5,\ \text{and}\ b=6.$$

8. Find two numbers such that their sum is 58; and that half of one exceeds one-sixth of the other by 15.

9. Find the greatest common measure of

$$x^3-5x^2+5x-4 \text{ and } x^3-7x^2+16x-16.$$

Will the quantity thus obtained be arithmetically the greatest common measure if a numerical value be given to x?

10. A number has two digits, of which the second is double the first; and, if the digits be reversed, the new number exceeds the original number by 36; find the number.

11. Simplify the expressions—

(1) $$\frac{\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}}{\dfrac{a-b}{a+b}-\dfrac{a+b}{a-b}}\times\frac{ab^3-a^3b}{a^2+b^2} \text{ and}$$

(2) $$\frac{1+4x}{1-4x}\cdot\frac{1-16x^2}{1+16x^2}\cdot\frac{1-256x^4}{1+8x+16x^2}.$$

12. Solve the equations—

(1) $$\left.\begin{aligned} x-15y&=52\\ 11x-\ \ y&=80\end{aligned}\right\}.$$

(2) $$\frac{x+a+b}{x+a-b}=\frac{x-a-b}{x-a+b}.$$

(3) $(11x-4)^2-(7x-3)^2=0.$

13. An express train starts from London for Cambridge, the distance between which is 57 miles, going at the rate of 35 miles an hour, and half an hour after another express, going at the same rate, starts from Cambridge for London, find where these trains pass each other.

LIX.

1. Divide (1) $1+2x$ by $1-x-x^2$ to 5 terms.

(2) Extract the square root of

$$\frac{x^4}{4}+ax^3+\frac{4a^2x^2}{3}+\frac{2a^3x}{3}+\frac{a^4}{9}.$$

2. (1) Find the G. C. M. of $x^6+x^2y-x^4y^2-y^3$, and $x^4-x^3y-x^2y^2+xy^3$.

(2) Reduce $\dfrac{x^2+2x-3}{x^2+5x+6}$ to its lowest terms.

3. Simplify $\frac{4}{7}\sqrt[3]{\frac{3}{16}}$ and $\sqrt{98a^2x}$.

4. Solve the equations:—

(1) $x=14-6x$.

(2) $\dfrac{3x}{4}+\dfrac{7x}{15}+\dfrac{11x}{6}-366=0$.

(3) $x-\dfrac{x-2}{3}=5\tfrac{3}{4}-\dfrac{10+x}{5}+\dfrac{x}{4}$.

(4) $\sqrt{x}-\sqrt{\dfrac{a}{x}}=\sqrt{a+x}$.

(5) $9x+\dfrac{8y}{5}=51$ and $7x-\dfrac{13y}{3}=-44$.

5. A person bought 9 sheep and lambs for 8 guineas : the sheep cost 26*s*. each, the lambs 15*s*.; how many of each did he buy?

6. A privateer, running 10 miles an hour, discovers a ship 18 miles off, making away at the rate of 8 miles an hour; how long will the chase last?

7. A person having to walk 10 miles, finds that by increasing his speed $\frac{1}{2}$ a mile an hour, he might reach his journey's end $16\frac{2}{3}$ minutes sooner than he otherwise would: what time will he take if he quicken his pace halfway?

8. Insert (1), 2 arithmetic means; (2) 3 geometric means between a and b.

9. Sum the series:—

(1) 8, 5, 2, &c. to 12 terms.

(2) $\dfrac{32}{105}$, $\dfrac{16}{35}$, $\dfrac{24}{35}$, &c. to 6 terms.

(3) $\dfrac{\sqrt{3}}{2}$, $\dfrac{4\sqrt{3}-3}{8}$, $\dfrac{19\sqrt{3}-24}{32}$, &c. to infinity.

10. Assuming $1^2+2^2+3^2+\ldots.n^2=\dfrac{n(n+1)(2n+1)}{1\cdot2\cdot3}$; find

the number of shot in an incomplete triangular pile of 5 courses, having 55 shot in the bottom course.

11. Solve the equations $\left\{\begin{matrix} a^{2x}-b^{2y}=c \\ a^x-b^y=d \end{matrix}\right\}$.

12. Prove that a number is divisible by 9, if the sum of its digits is divisible by 9.

13. Find the present value of £1 due 12 years hence, money bearing $4\frac{1}{2}$ per cent. compound interest.

LX.

1. Find the greatest common measure of x^5-4x^3+3x and $2x^4-5x^2-3$.

2. Simplify—

$$\frac{x(x+1)(x+2)}{1\cdot2\cdot3}+\frac{x(x-1)(x-2)}{1\cdot2\cdot3}+\frac{2(x-1)(x+1)}{3},$$

and find its value when $x=2$.

3. Find the L. C. M. of $3x^3-2x^2-x$ and $6x^2-x-1$.

4. Find (1), the sum of $\frac{a}{a+x}$ and $\frac{x(1+a)}{(1-x)(a+x)}$; (2) the product of $\frac{a+x}{x^4}$ and $\frac{a^4x}{(a+x)(a-x)}-x(a^2+x^2)$.

5. Find (1), the cube of $2a^2x^2-5ax^3$;

(2) the 5th root of $\frac{32x^{10}y^{15}}{3^5}$.

6. Determine the values of p and q, when the expression $4y^4-12y^3+py^2+qy+16$ is a complete square.

7. Find the square of $(a+b\sqrt{-1})^{\frac{1}{2}}+(a-b\sqrt{-1})^{\frac{1}{2}}$; and $\sqrt{31-12\sqrt{-5}}$.

8. Solve the equations:—

$$(1)\ 5-\frac{5x}{2}\left(1-\frac{3}{4x}\right)-\frac{10}{3}\left(3-\frac{5x}{2}\right)=31\tfrac{7}{8}.$$

$$(2)\left\{\begin{matrix} x+\frac{1}{2}y=10-\frac{1}{3}z \\ \frac{1}{2}(x+z)=9-y \\ \frac{1}{4}(x-z)=2y-7 \end{matrix}\right\} \qquad (3)\left\{\begin{matrix} \sqrt{x^2+ay-b^2}=c+x \\ \sqrt{y^2+mx}=x+y \end{matrix}\right\}$$

(4) $\sqrt{7x+1}=5+\sqrt{x}$; (5) $\left.\begin{array}{r}x+y+3(x+y)^2=310\\ xy=21\end{array}\right\}$

(6) $\left\{\begin{array}{l}x+y=1\\ x^6+y^6=x^5+y^6\end{array}\right\}$

9. Find all the solutions in positive integers of the equations (1) $11x+13y=1005$; (2) $4x+11y=19$.

10. Sum the series (1) 27, $\frac{51}{2}$, 24 &c. to 21 terms.

(2) $\frac{x}{y}$, $\frac{x}{2y}$, $\frac{x}{4y}$ &c. to infinity.

(3) 20, 10, 5 &c. to 6 terms.

11. Insert 5 harmonic means between -1 and 2^{-1}.

12. Find (1) log 128, (2) log 125, (3) log 2500; having given log 2=·3010300.

LXI.

1. If $x^3(1-b^2)=ab$, and $y^3b^2=a\sqrt{1-b^2}$, prove that $x^2y^2=a\sqrt{x^2+y^2}$.

2. If $\left.\begin{array}{l}xz+yx-yz=2a^2\ (\alpha)\\ xy+yz-xz=2b^2\ (\beta)\\ yz+xz-xy=2c^2\ (\gamma)\end{array}\right\}$ prove that

$$x^2=\frac{(c^2+a^2)\,(b^2+a^2)}{b^2+c^2}.$$

3. Extract the square root of

$$(a-b)^4-4(a^2+b^2)\,(a-b)^2+4(a^4+b^4)+8a^2b^2.$$

4. Solve the equations :—

(1) $4(x-3)-7(x-4)=6-x$.

(2) $\frac{1}{3}(2x-10)-\frac{1}{11}(3x-40)=15-\frac{1}{5}(57-x)$;

(3) $5\{1+\sqrt{1+x^2}\}=5\sqrt{1-x^2}+2\sqrt{1-x^4}$;

(4) $3x-2y=3y-4x=1$;

(5) $(3x-2)\,(x-1)=14$; (6) $x^{10}+3x^5=1120$.

5. Find the number of positive integral solutions of the equation $7x+34y=4030$.

6. Determine by logarithms the value of

$$8^{3\cdot 6} \text{ and } \frac{35^{\frac{1}{2}}\times 12^{17}}{13}.$$

7. Given $\dfrac{3^x}{2^{x+y}}=8$ and $x=3y$, find x and y.

8. If £2653 7*s.* 6*d.* be invested at $3\frac{1}{2}$ per cent. per annu compound interest, payable quarterly, what will it amou to in $4\frac{1}{2}$ years?

9. The shorter edge of the base of a rectangular pile co tains as many shot as the edge of the base of a square pil and the number of shot in the former pile is double of t number in the latter; find the number of shot in each pil supposing there are 22 in the top row of the rectangul pile.

10. How many permutations can be formed of the lette in the word *Examination*, taken altogether?

11. On how many nights may a different patrol of 5 m be draughted from a company of 36 men? and on h many of these would any one man be taken?

12. Find by Horner's method one value of x from t equation $x^3+8x-15=0$, to four places of decimals; fi demonstrating that one root at least must lie between t numbers 1 and 2.

LXII.

1. Simplify (1) $x\left(\dfrac{x-2y}{x+y}\right)^3+y\left(\dfrac{2x-y}{x+y}\right)^3$;

(2) $\dfrac{\dfrac{m^2+n^2}{n}-m}{n^{-1}-m^{-1}}\times\dfrac{m^2-n^2}{m^3+n^3}$.

(3) $\sqrt[3]{x^5y^3}$, $\sqrt[3]{\dfrac{192}{125}}$; $\sqrt[n]{a^{\frac{1}{n+1}}\cdot a^{\frac{1}{n-1}}}$.

2. Find (1) the sum of $7\sqrt{45}$, $2\sqrt{180}$ and $11\sqrt{245}$;

(2) the product of $2\sqrt[3]{4}$ by $3\sqrt[3]{54}$;

(3) divide $x^{\frac{3}{2}}-xy^{\frac{1}{2}}+x^{\frac{1}{2}}y-y^{\frac{3}{2}}$ by $x^{\frac{1}{2}}-y^{\frac{1}{2}}$.

3. Solve the equations :—

(1) $\frac{a-mx}{abx}=\frac{1}{b^2}-\frac{n}{ab}$;

(2) $\frac{x-4}{x-8}=\sqrt{x}-2$.

(3) $\left.\begin{array}{l}\frac{x}{6}+\frac{y}{4}=\frac{1}{6}\\ 4x+3y=3\end{array}\right\}$

(4) $\frac{a}{\sqrt{x}}+\frac{b}{y^2}=m$ and $\frac{b}{\sqrt{x}}+\frac{a}{y^2}=n$.

(5) $\sqrt{\{x^2+\frac{1}{2}\sqrt{x^2+96}\}}=x+1$.

(6) $(x-m)\sqrt{an}-(a-n)\sqrt{mx}=0$.

4. A detachment from an army was marching in regular column, with 7 men more in depth than in front; but the front being increased by 336 men, the detachment was drawn up in 5 lines. Find the number of men.

5. Show that if $x+\sqrt{y}=a+\sqrt{b}$; $x=a$, $y=b$; and apply the principle to determine the square root of $12-4\sqrt{5}$. Express $\frac{12-3\sqrt{3}}{12-4\sqrt{3}}$ as a fraction with a rational denominator.

6. The 7th term of a series in an arithmetical progression is 20, and the 17th, 50; find the nth term and the sum of 100 terms.

7. Sum the series (1) $1, \frac{1}{2}, \frac{1}{4}$ &c;

(2) $1, -\frac{1}{5}, \frac{1}{25}$ &c. ad infinitum.

8. How many shot are there in an incomplete triangular pile of 17 courses, having 5 shot in the side of the top course ?

9. A and B have together 100 sovereigns; if A counts his by 8's, he has 7 over, and if B counts his by 10's, he has 7 over. How many sovereigns has each ?

10. From a company of 50 men, 6 are draughted off every night on guard ; on how many nights can a different guard be posted, and on how many of these will any one man be off duty ?

11. Given $\log_{10}7 = \cdot8450980$, $\log_{10}2 = \cdot3010300$;
(1) $\log_{10}50$; (2) $\log_{10}\cdot0005$; (3) $\log_{10}196$;

$$(4) \log_{10}\left(\frac{28}{5}\right)^{\frac{3}{4}}$$

12. The sides of a rectangle are as 4 : 3, and the ence between the longer sides and the diagonal is 2 the sides.

LXIII.

1. Two persons A and B have each a number of in a gas company. Now, if A receive half of B's sha addition to his own, he would then have 54 shares; B's remaining number of shares were trebled, they amount to six more than three times the difference o original shares. How many shares had each ?

2. The number of men in both fronts of two colun troops, H and K, when each consisted of as many ra it had men in front, was 84; but when the columns ch ground, and H was drawn up with the front K had, with the front H had, the number of ranks in both co was 91; find the number of men in each column.

3. Express—

$$(1) \frac{4}{\sqrt[3]{5}-\sqrt[3]{4}}; \quad (2) \frac{x}{\sqrt{(x^2+x)}-x}; \text{ and}$$

(3) $\sqrt{(61-28\sqrt{3})}$ in simpler forms.

4. Prove the formula for the sum of a geometrica gression, and apply it in finding the value of the rec decimal ·77777 &c.

5. Find the sum of—

(1) $16\frac{1}{3}+14\frac{2}{3}+13+$ &c. to 21 terms.

(2) $\frac{5}{6}+\frac{5}{9}+\frac{10}{27}+$ &c. to n terms.

6. There are 1200 shot in the base of a complete recta *pile, and* four times as many in the longer side of the b

there would be in the top edge if one shot were removed; find the number of shot in the whole pile.

7. There is a number consisting of three digits which are in arithmetical progression; but if the digits be reversed, they will express a number greater by 426 than half the original number; find the number.

8. If it be assumed that, for all values of n,

$$(1+x)^n=1+nx+\text{A}x^2+\text{B}x^3+\text{C}x^4+\&c.,$$

determine the values of A, B, C &c. in terms of n, and then develope $(1-x)^{-\frac{1}{2}}$ in a series as far as five terms.

9. Give the definitions of a logarithm of a number, and the characteristic of a logarithm. Show that—

$$\log \text{P}^{\frac{1}{n}} = \frac{1}{n}\log \text{P}; \text{ and } \log_a \text{P}=\frac{\log_e \text{P}}{\log_e a}.$$

10. Assuming that

$$\log_e(1+x)=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\&c.,$$

show that

$$\log_e(n+1)=\log_e n+\frac{2}{2n+1}+\frac{1}{3}\cdot\frac{2}{(2n+1)^3}+\frac{1}{5}\cdot\frac{2}{(2n+1)^5}+\&c.$$

11. Solve, by means of logarithms, the equations :—

$$(1)\ \left(\frac{6}{11}\right)^{2x^2}=\left(\frac{3}{4}\right)^{4x};$$

$$(2)\ 2^{3x}\times 6^{3x-5}=3^{2(x-1)}\times 2^{5x}.$$

12. If an engine of 167 horses' power can raise from a reservoir $(240)^2$ cubic feet of water in a given time, what horses' power will be required to raise $(8575000)^{\frac{1}{3}}$ cubic feet in half the time.

LXIV.

1. If $a=6$, $b=5$, $c=4$, $d=1$, $n=0$, find the values of—

$$(1)\ \frac{2abc}{ab+c(b-a)}; \qquad (2)\ \frac{2abc}{ab-c(b-a)};$$

(3) $\dfrac{2abc}{c(a+b)-ab}$.

(4) $\{(a-b)^n-(b-c)^{2n}+(c-d)^{3n}\}^{\frac{1}{2}}+$

$\left\{(a-b)^2+2b^2-\dfrac{3c^2}{2}-\dfrac{n}{abcd}\right\}^{\frac{2}{3}}$

2. Resolve into factors

$25x^2-16$; x^3-8; x^3+27y^3; $a^3x^3-64b^3$.

3. Divide

$hkx^4+2(h-k)x^3-(h^2+4-k^2)x^2+2(h+k)x-hk$

by kx^2-h+2x.

4. Find the L. C. M. of

$a^3-4a^2b+9ab^2-10b^3$ and $a^3+2a^2b-3ab^2+20b^3$.

5. Reduce to lowest terms:—

(1) $\dfrac{a(a+2b)+b(b+2c)+c(c+2a)}{a^2-b^2-c^2-2bc}$;

(2) $\dfrac{a^{3m}+a^{2m}-2}{a^{2m}+a^m-2}$.

6. Express

(1) $\dfrac{6}{\sqrt[3]{6}+\sqrt[3]{5}}$; (2) $\dfrac{\sqrt{a+b}+\sqrt{a}}{\sqrt{a+b}-\sqrt{a}}$;

(3) $\sqrt{5-12\sqrt{-1}}$, in simpler forms.

7. Solve the equations:—

(1) $\dfrac{x}{x+1}+\dfrac{x+4}{x+3}=\dfrac{47}{24}$;

(2) $\dfrac{x-1}{\sqrt{x}+1}=4+\dfrac{\sqrt{x}-1}{2}$;

(3) $x^2+xy=a$, $y^2+xy=b$;

(4) $x+y+z=a+2b$, $b(x+y)+az=b(x+z)+ay=2ab+b^2$

(5) $\sqrt{x-1}+\sqrt{x+1}=2$;

(6) $\dfrac{15-4x^2}{\sqrt{15-4x}}=2$.

8. A certain number of sovereigns, shillings and sixpences, together amount to £8 6*s*. 6*d*., and the amount of the shillings is a guinea less than that of the sovereigns, and a

guinea and a half more than that of the sixpences; find the number of each coin.

9. The sides of a triangle being 4, 9, 12, show that the length of the line bisecting the angle between the two shorter sides is $2\frac{4}{13}$.

10. Sum the series:

(1) $\frac{n-1}{n}$, $\frac{n-2}{n}$, $\frac{n-3}{n}$ &c. to n terms;

(2) $2, -4, 8$ &c. to 9 terms;

(3) $2, -\frac{4}{3}, \frac{8}{9}$, &c. to infinity.

11. Assuming $a^x = 1 + \mathrm{A}x + \mathrm{B}x^2 + \mathrm{C}x^3 +$ &c. find B, C, D &c. in terms of A.

12. There are 3 Nos. in A. P., and if the first be increased by 1, or the third increased by 2, they will be in G. P. Find the numbers.

LXV.

1. (1) Divide $x^{16} + a^8x^8 + a^{16}$ by $x^4 - a^2x^2 + a^4$;

(2) Simplify the expression—

$\frac{x^3+y^3}{x-y} \cdot \frac{x^3-y^3}{x+y} \cdot \frac{x^4-x^2y^2+y^4}{x^4+x^2y^2+y^2}$; find its value when $x = 2y$.

2. Prove that, if n be a positive integer, $x^n - y^n$ is divisible by $x-y$ and $x^{2n} - y^{2n}$ by $x^2 - y^2$.

3. State the assumptions on which the equation $a^0 = 1$ depends. What is the value of $4^{\frac{1}{4n}}$, when n a positive quantity is infinite?

4. Simplify $\sqrt[3]{a^2bc} \cdot \sqrt[4]{abc^2} \div a^{\frac{11}{12}} b^{\frac{7}{12}} c^{\frac{2}{3}}$.

5. Solve the equations :—

(1) $\frac{7x-5}{4} - \frac{11-3x}{7} = \frac{4-x}{28} - \frac{2-x}{14} + 10 + \frac{1}{28}$.

(2) $2x^2 - 22x + 56 = 0$;

(3) $x^2 - ax + 2a\sqrt{x^2 - ax + a^2} = 14a^2$;

(4) $x^2 + (3a+4b)x + 2a^2 + 5ab + 3b^2 = 0$;

(5) $\left.\begin{aligned} x^3 + 4xy + y^3 &= 38 \\ x + y &= 2 \end{aligned}\right\}$

6. If $a : b = c : d = e : f$, show that each of these ratios

$$= \sqrt{ac+ae+ce} : \sqrt{bd+bf+df}$$
$$= (a^n+c^n+e^n)^{\frac{1}{n}} : (b^n+d^n+f^n)^{\frac{1}{n}}.$$

7. Having given the first and the last terms and the number of terms in a geometric progression, find the intermediate terms.

Sum the series $\left(\frac{n+1}{n-1}\right)^2$, $\frac{n+1}{n-1}$, 1, $\frac{n-1}{n+1}$ to infinity.

8. A cricket club has 20 members; how many different elevens can be formed, and in how many of these will (1) any one member, (2) any three particular members appear?

9. (1) Expand $(3a-4x)^{\frac{4}{5}}$ to four terms; (2) write down the $r+1$th term of $(1-x)^{-3}$.

10. Explain what is meant by variation, and if $a \propto b$ and $c \propto d$, prove that $ac \propto bd$ and $\frac{a}{c} \propto \frac{b}{d}$.

11. The sum of the semiperimeters of two similar rectangles is 35 inches, the sum of their areas 204 square inches; if the two homologous sides be interchanged, each area becomes 48 square inches. Find the sides of the rectangles.

12. The product of the base and perpendicular of a triangle is 60, the hypothenuse is 13; find the base and perpendicular. Draw a figure representing all the solutions.

LXVI.

1. Multiply $a+mx-nx^2$ by $a-2mx+nx^2$ and by $a+2nx-mx^2$.

2. Extract the square root of $x^{\frac{2}{3}}+2\sqrt{x}+3\sqrt[3]{x}-2x^{-\frac{1}{6}}+x^{-\frac{1}{3}}-1$.

3. If $x=\frac{1}{2}(\sqrt{3}+1)$, find the value of $4(x^3-2x^2)+2x+3$.

4. Reduce to lowest terms $\dfrac{a^3+ab^2-a^2b-b^3}{4a^4-2a^2b^2-4a^3b+2ab^3}$.

5. Solve the equations (1) $\frac{9-2x}{2}+\frac{7x-18}{10}=\frac{3}{2}$

(2) $\frac{x}{2}+\frac{14-x}{3}-\frac{20-x}{2}=0$

(3) $\left.\begin{aligned} 2y-\frac{x+3}{4}&=14\tfrac{1}{4}+\frac{3x-2y}{4} \\ 4x-\frac{8-y}{3}&=25\tfrac{1}{2}-\frac{2x+1}{2} \end{aligned}\right\}$

6. A gamester at one sitting lost $\frac{1}{8}$ of his money, and then won 10*s*.: at a second, he lost $\frac{1}{3}$ of the remainder, and then won 3*s*.: and then had 3 guas. left : how much had he at first?

7. Sum the series :— (1) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$ etc. to 7 terms;

(2) $1-\frac{\sqrt{3}}{2}$, $1+\sqrt{3}$, $1+\frac{5\sqrt{3}}{2}$, etc. to 9 terms;

(3) 5, 4, $3\frac{1}{5}$ etc. to infinity.

8. If 3 numbers be in arithmetical progression, and a fourth proportional to them be found, the last 3 terms of the proportion will be in harmonical progression.

9. The first term of a geometric series is 1, and if the series be continued indefinitely, any term is the limit of all the following terms: find the series.

10. Prove, $\log_{10}N=\log_e N\times\frac{1}{\log_e 10}$. What is the use of this formula?

11. Separate $\frac{3x-1}{x^3-x^2-2x}$ into 3 fractions, having simple denominators.

12. Write down the eighth term of $(3+x)^{10}$.

LXVII.

1. Prove that if r_1, r_2 are the roots of the quadratic equation $x^2+ax+b=0$, then

$$r_1+r_2=-a, \text{ and } r_1r_2=b.$$

2. Find the values of x and y from the equations

$$x^3+y^3=189 \text{ and } x^2y+xy^2=180.$$

3. The product of four consecutive numbers is 3024; find the numbers.

4. If the first term of an arithmetical series is n^2-1, and the common difference -2, show that the sum of n terms is $=n^3-n^2$.

5. Prove that if

$$s_1=a+\frac{a}{r}+\frac{a}{r^2}+\frac{a}{r^3}+\ldots\ldots,$$

$$s_2=b-\frac{b}{r}+\frac{b}{r^2}-\frac{b}{r^3}+\ldots\ldots,$$

$$s_3=c+\frac{c}{r^2}+\frac{c}{r^4}+\frac{c}{r^6}+\ldots\ldots;$$

then will

$$\frac{s_1s_2}{s_3}=\frac{ab}{c}.$$

6. If the number of shot in the top course of a complete rectangular pile be equal to the number of shot in the less side of the base, and the number of shot in the rectangular pile be to the sum of the shot in a triangular and square pile, each having the same number of courses as the rectangular pile, as 14 to 9; what is the number of courses in each pile?

7. Find the fourth term of the expansion of $\sqrt{2ax-x^2}$, by the method of indeterminate coefficients.

8. Extract the cube root of $x^{15}-3x^{12}+3x^{10}+3x^9-6x^7-x^6+3x^5+3x^4-3x^2+1$.

9. Explain the method of finding the characteristic of $\log_{10}\cdot0005$.

10. Find, by logarithms, the value of the expression

$$\left\{\frac{(\cdot21)^{\frac{1}{2}}\times(\cdot21)^{\frac{1}{3}}\times(\cdot21)^{\frac{3}{4}}\times365^2}{(\cdot00416)^2\times(\cdot3125)^3\times\sqrt[4]{365}}\right\}^{\frac{2}{3}}.$$

11. Given $3^{2x}+3^x=6$, and $4^{2xy}-2\times4^{xy}=8$ to find the *values of x and y.*

12. (1) Reduce by 2 the roots of the equation

$$x^3-6x^2+8x+1=0;$$

(2) find all the roots of the equation,

$$x^4-2\,(x^3+x^2-x)+1=0.$$

LXVIII.

1. Solve the equations— (1) $\frac{x}{x+3}+\frac{x+3}{x}=2{\cdot}9$;

(2) $x^2+x^{-2}+x^{-1}+x=4$; (3) $x^2-xy=3$, $x^2+y^2=13$.

2. There is a number composed of two figures, of which ıe figure in the tens' place is four times that in the units'; ıd if 54 be subtracted from the number the difference is ɛpressed by the same digits reversed: what is the number?

3. Sum the series, (1) $2\frac{1}{2}$, $3\frac{3}{4}$, 5, etc. to 12 terms:

(2) $15-3+\frac{3}{5}$ — etc. to infinity;

$\frac{a+b}{2}$, a, $\frac{3a-b}{2}$ etc. to n terms.

4. Show that $\frac{a+b+c+d}{p+q+r+s}$ is greater than the least and ss than the greatest of the fractions $\frac{a}{b}$, $\frac{p}{q}$, $\frac{c}{r}$, $\frac{d}{s}$; each letter ›presenting a positive quantity.

5. Two casks A and B are filled with two kinds of sherry, ıixed in A in the ratio of 2 : 7; and in B in the ratio of : 5. What quantity must be taken from each in order to ave 11 gallons mixed in the ratio of 2 : 9.

6. Three quantities a, b, c are in A.P., G.P., or H.P. according as $\frac{a-b}{b-c}=\frac{a}{a}$, or $=\frac{a}{b}$, or $=\frac{a}{c}$.

7. Prove that the number of combinations of n things taken r together is the same as the number taken $n-r$ together. How many words can be made of the letters in the word *rotation*?

8. In what scale of notation will the common number

5261 be expressed by 40205? What are the greatest an least numbers that can be formed with 5 digits in that scale

9. Express $\sqrt{50}$ in the form of a continued fraction.

10. A person makes 20 lbs. of tea at 4*s.* 9*d.* by mixin three kinds at 3*s.* 6*d.*, 4*s.* 6*d.*, and 5*s.*; how can this b done, using no fractions of a pound?

11. Find the number of balls in a triangular pile of 2 courses.

12. Obtain the rationalising factor of $3^{\frac{1}{2}}-5^{\frac{2}{3}}$.

13. Expand (1) $(a-x)^6$, (2) $(1-2x)^{\frac{5}{2}}$ to 6 terms by th binomial theorem.

LXIX.

1. Express without brackets
 (1) $3(2a-b-c)-5\{a-(2b+c)\}+2\{b-(c-a)\}$.
 (2) $(x+y)(x-y)-x(x+y)+y(x+y)$.

2. Multiply $a+2b-3c$ by $a-2b+3c$.

3. Divide
 $mn(x^5+1)+(n^2+m^2)(x^4+x)+(n^2+2nm)(x^3+x^2$
 by nx^2+mx+n.

4. Find the G.C.M. of $3x^3-3x-18$ and $2x^2+2x-12$.

5. Simplify $\dfrac{x^2-y^2}{xy}\times\dfrac{x^2}{x+y}\div\dfrac{x-y}{y^2}$.

6. Raise $1+2x+x^2$ to the square, and $1-x$ to the 4 power.

7. Extract the square root of $16+8a+a^2-8a^3-2a^4+a$

8. Prove that in an equation any quantity may be tran ferred from one side to the other by changing its sign.

9. Solve the equations:—

 (1) $\dfrac{a-2x}{abx}=\dfrac{1}{b^2}-\dfrac{1}{ab}$;

 (2) $\dfrac{2}{x}=\dfrac{3}{x-1}-\dfrac{1}{x-2}$.

10. (1) Extract the square root of $103-20\sqrt{21}$; and

(2) Simplify $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$.

11. What are the logarithms of 10, 100, ·001 to the base); and of 1000 to the base ·01?

12. Given

g $2=\cdot3010300$, $\log 18=1\cdot2552725$, $\log 21=1\cdot3222193$; ıd (1) $\log\cdot00075$ and (2) $\log 31\cdot5$.

13. $3+\sqrt{-1}$ is a root of the equation

$$x^4-6x^3+13x^2-18x+30=0.$$

nd all the other roots.

LXX.

1. Simplify the expressions :—

(1) $\left\{\frac{a^{-2}}{b^{-2}}\sqrt{\frac{a}{b}}\right\}^{\frac{1}{3}}$

(2) $(\sqrt{5}-\sqrt{2})(\sqrt{2}+1)(\sqrt{5}+\sqrt{2})(\sqrt{2}-1)$.

2. Solve the equations :—

(1) $3x^2-4x=32$;

(2) $\frac{x-1}{x+3}+\frac{x-3}{x+1}=2$.

(3) $x^2+y^2=\frac{5}{2}$ and $x^2-xy+y^2=\frac{7}{4}$.

3. Sum the series :—

(1) 1, $2\frac{1}{2}$, 4 &c. to 10 terms; (2) $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{6}$ &c. to 6 terms;

(3) ·23, ·0023, ·000023 to infinity.

4. Find the harmonic mean between 2 and 7.

5. If $a : b :: c : d :: e : f$ show that

(1) $\frac{a+b}{a-b}=\frac{c+d}{c-d}=\frac{e+f}{e-f}$; (2) $\frac{a+c+e}{b+d+f}=\frac{a}{b}$.

6. A number of two digits : the number formed by inerting the digits :: 7 : 4. Show that the first digit : second igit :: 2 : 1.

7. In any system, what are the logarithms (1) of the base, 2) of 1, (3) of 0?

8. Prove that, in the common system, the log of a number having n digits in its integral part is $n-1$; and that the characteristic of the log of a decimal having n cyphers between the point and the first significant digit is $-(n+1)$.

9. Given

$\log 2 = \cdot 3010300$, $\log 3 = \cdot 4771213$, $\log 7 = \cdot 8450980$; find (1) $\log 60$, (2) $\log \cdot 03$, (3) $\log 1\cdot 05$.

10. Given

$\log 1752 = 3\cdot 2435341$ and $\log 1752\cdot 1 = 3\cdot 2435589$,

construct a table of proportional parts and find $\log 17\cdot 52087$.

11. Divide $\overline{14}\cdot 326847$ by 9.

12. Prove that, when $a > b$ and $c > d$,

$$(a-b)(c-d) = ac - bc - ad + bd.$$

On what grounds is this equation asserted to be true in all cases?

13. Show how to find the number of numbers less than any proposed number and prime to it.

If 5 be subtracted from the sum of the squares of 4 consecutive numbers all prime to 5, the remainder will be a perfect square.

LXXI.

1. Solve the equations :—

(1) $7x^2 - 13x = 2$; (2) $x^6 - 7x^3 = 8$;

(3) $\dfrac{\sqrt{2}}{x} = \dfrac{3}{\sqrt{x+x^2}} - \dfrac{\sqrt{2}}{1+x}$; (4) $\left.\begin{array}{l} x^2 - xy = 10 \\ 2x - 3y = 1 \end{array}\right\}$

2. Simplify the surds :—

$$\sqrt{75a^5b^3};\ \sqrt[3]{135};\ \left(a^{\frac{q}{p-q}} \cdot a^{\frac{p}{q-p}}\right)^{\frac{1}{p^2-q^2}};\ \frac{5}{\sqrt{7}-\sqrt{2}};\ \sqrt{10-4\sqrt{-6}}.$$

3. Insert (1) 5 arithmetic and (2) 3 harmonic means between 1 and 2.

4. Sum the series (1) $1\frac{1}{2}$, 1, $\frac{1}{2}$ &c. to 7 terms;

(2) 27, 18, 12 &c. to infinity;

(3) $1\frac{1}{2}$, $2\frac{1}{4}$, $3\frac{1}{8}$ to n terms.

5. If $b-a$ be the harmonic mean between $c-a$ and $d-a$, ow that $d-c$ is the harmonic mean between $a-c$ and $b-c$.

6. If $\frac{1}{10}$ be the harmonic and $\frac{1}{8}$ the geometric mean be-een two fractions, determine the fractions.

7. If $a : b :: c : d$, show that $\dfrac{ax+b}{cx+d}$ has always the same lue, whatever be the value of x; also show that

$$\sqrt{a+b} : \sqrt{b} :: \sqrt{c+d} : \sqrt{d}.$$

8. If $\text{A} \propto \text{B}+\dfrac{1}{\text{C}}$ and $\text{B} \propto \text{C}$; and if when $\text{C}=1, \text{B}=1, \text{A}=4$; id the value of B when $\text{A}=5$.

9. How many different numbers can be formed with the gits 1, 2, 3, 4; if a decimal point be put before each of em, what sum will be obtained by adding them together.

10. Solve the equations :—

$$\left.\begin{aligned} -\frac{l}{x}+\frac{m}{y}+\frac{n}{z}&=a \\ \frac{l}{x}-\frac{m}{y}+\frac{n}{z}&=b \\ \frac{l}{x}+\frac{m}{y}-\frac{n}{z}&=c \end{aligned}\right\}$$

11. Calculate the sum which must be paid to purchase 1 annuity of £100 to begin immediately and to go on for er, each payment being supposed to be half the preceding ie : reckoning compound interest at 5 per cent.

12. Prove the binomial theorem when n is a positive action.

13. Form the rational equation two of whose roots are $+\sqrt{3}, -1-\sqrt{3}$.

LXXII.

1. Simplify $\dfrac{2\frac{1}{2}}{3\frac{1}{4}}+\dfrac{1\frac{1}{2}-\frac{5}{6}}{1\frac{1}{4}+\frac{5}{6}}+1\frac{2}{39}$.

2. Divide $\frac{5}{28}$ of $1\frac{1}{3}$ of £2 16*s.* 3*d.* by $\frac{8}{49}$.

3. Multiply (1) 76·045 by 1·0305; (2) by ·01305; (3) divide the result by 14·5.

4. At what rate per cent. per annum, simple interest, will £936 13*s.* 4*d.* amount to £1157 7*s.* $4\frac{1}{2}$*d.* in $4\frac{7}{8}$ years?

5. Find the present value of £323 7*s.* $10\frac{1}{2}$*d.* due $2\frac{3}{4}$ years hence at $3\frac{1}{2}$ per cent.

6. What is the value of £10667 10*s.* Bank stock at $188\frac{7}{8}$. brokerage $\frac{1}{8}$ per cent.?

7. Solve the equation $\dfrac{x}{x+1}+\dfrac{x}{x+1}=\dfrac{13}{6}$.

8. The number of combinations of n things taken 5 at a time is to the number taken 3 at a time, as 18 to 5: find n.

9. Find, by the aid of tables, (1) $(34)^{\frac{1}{5}}\times(24{\cdot}68)^{\frac{1}{3}}$; (2) a fourth proportional to 254·5, 4000·6, and ·028.

10. Show that the equilateral triangle described about a circle has double the perimeter of that inscribed in the same circle.

11. ACB is a triangle whose base AB is divided in E and produced to F, so that AE : EB :: AF : FB :: AC : CB; prove that (EF − CF) (EF + CF) = EC^2.

12. The paving of a semicircular courtyard at 5*s.* a foot cost £20, find the radius of the semicircle.

13. Find the volume of a segment of a sphere 7·5 inches high; the radius of the sphere being 12 inches.

LXXIII.

1. Solve the equations:—

(1) $x^2-16x=80$. (2) $\frac{7x+1}{2x-1}+\frac{3x}{5}=7$.

(3) $\begin{cases} x+y=3 \\ x^2+y^2=\frac{13}{2}. \end{cases}$

2. Square $3\sqrt{2}-2\sqrt{7}$, and extract the square root of $57-12\sqrt{15}$.

3. Each one of a company of persons bows once to each of the others. If the company had been a third less, the number of bows would have been less by 18. Find the number of persons present.

4. State the different methods of measuring angles. Show that English seconds may be reduced to French seconds by multiplying by ·324.

5. Define the sine, secant, and tangent of an angle. Show that the versed sine of an angle lies between 0 and 2.

6. Find cos 30° and sin 45°. Express all the trigonometrical ratios of an angle in terms of the versed sine.

7. Trace the changes of cot A in sign and magnitude, as A changes continuously from 0° to 360°.

8. Prove that $\cos(A-B)=\cos A.\cos B+\sin A.\sin B$. Deduce the expression for $\sin(A-B)$.

If $A+B$ is $<90°$, then $\cot A.\cot B$ is >1.

9. Find x in the equations (1), $\sin 2x=\tan x$;

$$(2)\ \cos^{-1}\frac{\sqrt{3}}{2}+\sec^{-1}x=\frac{\pi}{4}.$$

10. In any triangle, prove that $\cos A=\frac{b^2+c^2-a^2}{2bc}$, and deduce the relation $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$.

11. What is the logarithm of $81\sqrt[3]{3}$ to base $3\sqrt{3}$? How would you transform a system of logarithms from base 8 to base 4? What advantages belong to a system with 10 as the base?

12. Show how to solve a triangle, having given two sides and the included angle.

13. From a ship sailing due SE. at the rate of seven miles per hour a lighthouse is observed to bear N. 30° E., and after two hours its bearing is due N.; find the distance of the ship from the lighthouse at each observation.

LXXIV.

1. (1) Multiply $\frac{1}{4}a^{\frac{1}{2}}-\frac{1}{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+\frac{3}{8}b^{-\frac{1}{2}}$ by $\frac{1}{4}a^{\frac{1}{2}}+\frac{1}{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+\frac{3}{8}b^{-\frac{1}{2}}$.

(2) Divide $(b-c)a^3+(c-a)b^3+(a-b)c^3$ by $a^2-ab-ac+bc$.

2. Reduce to their simplest forms—

(1) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$. (2) $\sqrt{18}+\sqrt{200}-\sqrt{338}$.

3. Solve the following equations:—

(1) $\frac{x-2}{2}+\frac{3}{x-1}=3$. (2) $\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=9$.

4. If $a:b::c:d$, show that $a+b:a-b::c+d:c-d$.

5. Show that the arithmetical mean, between a and c, is greater than the geometrical.

6. Two vessels contain each a mixture of water and wine, in the ratios of 2 : 3, and 3 : 7 respectively. What quantity must be taken from each to form a mixture which shall consist of 5 gallons of water and 11 of wine?

7. Convert $24^g\ 15'\ 24''$ into English measure; find the complement of the result, and reconvert it into grades.

8. Prove that the circumference of a circle varies as its radius; and explain the formulæ $C=2\pi r$, and $\theta=\frac{A^\circ}{180^\circ}\pi$. What is the angular unit in circular measure?

9. The radius of a fly wheel in a stationary engine is $3\frac{1}{2}$ feet in length, and goes round uniformly 20 times a minute. At what rate per minute does the extremity move?

10. Express all the trigonometrical ratios in terms of the sine.

11. Prove the following formulæ:—

(1) $\sin(A+B)=\sin A\cos B+\cos A\sin B$, where A is $>90^\circ$ and $A+B<180^\circ$.

(2) $\tan\frac{A}{2}=\frac{1-\cos A}{\sin A}$.

(3) $\sin(A+B)+\sin(A-B)=2\sin A\cos B$.

(4) $\sin(A+B)\times\sin(A-B)=\sin^2 A-\sin^2 B$, or $\cos^2 B-\cos^2 A$.

12. The angle of elevation of the top of a steeple is 60° from a point on the ground. That of the top of the tower on which the steeple rests is 45° from the same point. What proportion does the height of the steeple bear to that of the tower?

13. Two ships at sea are 1000 yds. apart. Each of them makes for the same object, one bearing NW. at an $\angle$ of 45° from the line of 1000 yds., and the other at an $\angle$ of 30°, in a direction also northerly. Find the distance each will have to sail.

LXXV.

1. Simplify (1) $\sqrt{2}+\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{8}}$. (2) $\sqrt{27+7\sqrt{5}}$.

2. Solve the equations—

(1) $\frac{x^2+1}{2x}+\frac{x-1}{4}=3x-2$. (2) $\frac{x+7}{x}-\sqrt{\frac{x+7}{x}}=\frac{15}{4}$.

3. If a, b, c, be continued proportionals, then $a+c$ is $>2b$.

4. Sum the series—

(1) $9+8\frac{1}{3}+7\frac{2}{3}+\ldots\ldots\ldots$ to 50 terms.

(2) $\frac{1}{3}+2+12+\ldots\ldots\ldots\ldots$ to 5 terms.

5. If D, G, be the number of degrees and grades respectively in the same angle, prove that $\frac{D}{90}=\frac{G}{100}$. Express 1° 7′ 30″ in French measure.

6. Define the sine, secant, and tangent of an angle. Show geometrically the absurdity of the equation cosec $A=\frac{1}{2}$.

7. Find sin 60° and tan 45°; and express all the other trigonometrical ratios in terms of the cosine.

8. Trace the changes of sec A in sign and magnitude as A changes continuously from 0° to 360°.

9. Prove the following formulæ:—

(1) $\cos(A+B)=\cos A \cdot \cos B-\sin A \cdot \sin B$.

(2) $\cot^2\frac{A}{2}=\frac{1+\cos A}{1-\cos A}$.

(3) $\tan 3A=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$.

10. In any triangle prove that

$a : b : c :: \sin A : \sin B : \sin C$.

11. In a triangle having given a, b, and A $(a<b)$ prove that there will be two solutions, one, or none, according as a is $>$, $=$, or $<b \sin A$.

12. The angle of elevation of the top of a steeple is 45° from a point in the same horizontal plane as its base, and is 30° from a point 30 feet directly above the former point; find the height and distance of the steeple.

13. An object C being inaccessible from B, a line BA, 408 yards in length, is measured, and the angles CBA, CAB, observed to be 22° 37′ and 58° 7′ respectively; find BC, having given L sin 80° 44′=9·994295, L sin 58° 7′=9·928972, log 4·08=·610660, and log 3·51=·545337.

LXXVI.

1. If $a^3-pa^2+qa-r=0$, then x^3-px^2+qx-r is exactly divisible by $x-a$.

2. Simplify (1) $\frac{1+\sqrt{2}}{1+\sqrt{2}+\sqrt{3}}$. (2) $\sqrt{75-12\sqrt{21}}$.

3. If α, β are the roots of $ax^2+bx+c=0$, find in terms of a, b, and c, values of—

(1) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$. (2) $\alpha^3+\beta^3$.

4. Solve the equations:—

$$(1)\ \frac{x-1}{x}+\frac{x}{x-1}=3\tfrac{1}{3}.\quad (2)\ x+y\sqrt{-1}=a+b\sqrt{-1}.\quad (3)\ \left.\begin{array}{r}\dfrac{xy}{z}=4\\ \dfrac{xz}{y}=9\\ \dfrac{yz}{x}=16\end{array}\right\}$$

5. What are eggs per dozen when if there are two less in a shilling's worth the price is increased one penny per dozen?

6. Sum (1) the arithmetic series $58+54+\ldots$ to 30 terms.

(2) the geometric series $\sqrt{\frac{2}{3}}+\sqrt{\frac{3}{2}}\ldots$ to 5 terms.

(3) the series $1+2x+3x^2+\ldots$ to n terms.

7. Explain the terms complement and supplement, illustrating them by examples. Show, by example, how to convert English degrees into French. Explain the symbol π and the No. 3·1416 ; and exhibit the algebraical signs of the sine, cosine, and tangent, in the third quadrant.

8. Show, geometrically, that $\sin(90°+A)=\cos A$; $\cos(90°+A)=-\sin A$; $\tan(-A)=-\tan A$.

9. Find the numerical values of sin 60°, cos 60°, tan 75°

10. In any triangle, $\dfrac{\sin\left(\frac{A}{2}+B\right)}{\sin\frac{A}{2}}=\dfrac{b+c}{a}$.

11. Prove the following formulæ:—

(1) $\dfrac{\sin A+\sin B}{\cos A+\cos B}=\tan\frac{1}{2}(A+B)$.

(2) $\tan 2A-\tan A=\dfrac{2\sin A}{\cos A+\cos 3A}$.

12. In any triangle, area $=\sqrt{s.(s-a).(s-b).(s-c)}$; and if the triangle be right-angled at C,

$$\tan^{-1}\frac{a}{b+c}+\tan^{-1}\frac{b}{a+c}=\frac{\pi}{4}.$$

13. If in a triangle ABC, BC=70, AC=35, and $\angle$ ACB= 36° 52′ 12″, find the remaining angles, the tables giving—

$$\log 3 = \cdot 4771213.$$

$$\log \cot 18^\circ\, 26'\, 6'' = 10\cdot 4771213.$$

LXXVII.

1. Define a surd; and simplify—

(1) $\sqrt{128}-\sqrt{32}+\sqrt{8}-\sqrt{16}$. (2) $\sqrt{5}\,.\,\sqrt[3]{2}\,.\,\sqrt[4]{4}$.

2. Find the 6th term, and the sum of 6 terms, of the series—

(1) $3+4\frac{1}{2}+6+$&c. (2) $3+4\frac{1}{2}+6\frac{3}{4}+$&c.

3. The sum of three numbers in arithmetical progression =15, and their product=105. Find them.

4. Solve the equations—

$x^2+y^2 : xy :: 5 : 2.$ $x+y : 5 :: 3 : y-4.$

5. Find the ratio of two angles, of which the one contains as many English seconds as the other does French minutes.

6. Define the sine and secant of an angle; and show that the secant can never be a proper fraction.

7. Find cos 45° and tan 30°; and if cot A$=\frac{3}{4}$, find sin A and sec A.

8. Trace the changes of tan A in sign and magnitude, as A changes continuously from 0° to 360°.

9. Prove that—

(1) $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}=\frac{\pi}{2}$.

(2) $\dfrac{\cot A\,.\,\cos A}{\cot A+\cos A}=\dfrac{\cot A-\cos A}{\cot A\,.\,\cos A}$.

10. In a right-angled triangle ABC, prove that $abc= a^3\cos A+b^3\cos B$; C being the right angle.

11. Investigate an expression for the radius of the inscribed circle of a triangle in terms of the sides.

12. If d be the perpendicular dropped from the angle C upon the opposite side of a triangle, show that $\sin C=\dfrac{cd}{ab}$.

13. A pole is fixed on the top of a mound, and the angles of elevation of the bottom and top of the pole are 30° and 60° respectively; prove that the height of the pole=twice the height of the mound.

LXXVIII.

1. What is the "mantissa," and what is the "characteristic," of a logarithm? Prove that the same mantissa serves for all numbers consisting of the same digits in the same order.

2. Prove that $\log \frac{b}{a} = \log b - \log a$; and that $\log b^a = a \log b$.

3. Construct a table of proportional parts corresponding to a difference of 64.

4. Define the cosecant of an angle and trace its variation in sign and magnitude through the four quadrants.

5. Prove (1) $\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$;

(2) $\frac{1}{2} \sin^{-1} \frac{4}{5} + \tan^{-1} \frac{1}{3} = \frac{\pi}{4}$;

(3) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$.

6. In any plane triangle $\cot \frac{1}{2} A = \frac{b+c}{b-c} \tan \frac{1}{2}(B-C)$.

7. Show that $\frac{\tan(45°+A)}{\tan(45°-A)} = \frac{2\cos A + \sin A + \sin 3A}{2 \cos A - \sin A - \sin 3A}$.

8. Two straight roads, which cross one another, meet a canal at angles of 60° and 30° respectively. If it be 3 miles by the longer of the two roads, from the crossing to the canal, how far is it by the shorter? If there be a foot-path which goes the shortest way to the canal, what is the distance by it?

9. Find the solid content of a log of timber 10 yds. 2 ft. 7 in. long, 2 ft. 11 in. broad, and 2 ft. 5 in. thick.

10. If a pressure of 15 lbs. to the square inch be applied

to a circular plate 3 ft. in diameter, what is the total pressure?

11. Find the number of cubic feet in a hexagonal room, each side of which is 20 ft. and its height 30 ft.; and which is finished above with a roof in the form of a hexagonal pyramid 15 ft. high.

12. What is the solid content of a sphere, when its surface is equal to that of a circle 4 feet in diameter?

13. If a quadrilateral ABCD be inscribed in a circle,

$$AB \cdot BC + AD \cdot DC : BC \cdot CD + BA \cdot AD :: BD : AC.$$

LXXIX.

1. Prove the rule for finding the greatest common measure of two quantities.

2. Reduce to its lowest terms

$$\frac{ab\,(x^2-y^2)\;+xy\,(a^2-b^2)}{ab\,(x^2+y^2)\;+xy\,(a^2+b^2)}.$$

3. Solve the equations —

$$(1)\quad (a+x)\,(b+x)\;-a\,(b+c)=\frac{a^2c}{b}+x^2$$

$$(2)\quad \left(\frac{a+x}{a-x}\right)^2=1+\frac{cx}{ba}.$$

4. Find by logarithms the value of

$$(1)\quad \frac{\cdot 6325\times\cdot 001355}{1\cdot 191\times 5\cdot 355}$$

(2) the sum of the series 12, 36, 108 etc. to 7 terms:

(3) the value of x in the equation $\left(\dfrac{7}{3}\right)^x=100.$

5. A hemispherical basin holds 1 gal. of water; find its diameter, if 1 gal. $=277\cdot 27$ cubic inches.

6. How many square yards of sheet iron would be required to make a pipe $1\frac{1}{2}$ ft. in diameter, and 48 ft. long; and what would be its weight if the iron were ·1 in. thick and 1 cubic foot of iron weighed 486 lbs.?

7. Prove that; (1) $\frac{\cos A - \cos 3A}{\sin 3A - \sin A} = \tan 2A$

(2) $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2 \frac{1}{2}(A - B)$.

8. In any plane triangle show that $\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$.

9. What is the angle of depression of an object? From the top of a hill the angles of depression of two consecutive milestones on a straight, level road, were found to be 12° 13′ and 2° 45′: find the height of the hill.

10. If r be the radius of the circle inscribed in a regular pentagon, R the radius of the circumscribed circle; show

that $\frac{r}{R} = \frac{\sqrt{5}+1}{4}$.

11. Find the area of the equilateral triangle circumscribing the circle whose radius is 60 feet.

12. Show how to find the area of a trapezoid.

13. Find the volume of a pyramid.

Ex.: A pentagonal pyramid, side of base 7·25 feet, height 12·6 feet.

LXXX.

1. Resolve the following expressions into factors:—

(1) $x^3 + y^3 + 3xy(x+y)$.

(2) $a^2 - ab + 2(b^2 - ab) + 3(a^2 - b^2) - 4(a-b)^2$.

2. Calculate by logarithms (1) $\sqrt[3]{\cdot 0000083825}$;

$$(2)\ \left(\frac{\cdot 03214}{\cdot 01762}\right)^{\frac{1}{4}}.$$

3. Find the amount of £250 in 10 years, at 4 per cent. compound interest.

4. Prove the formulæ

(1) $4\cos^3 A \sin 3A + 4\sin^3 A \cos 3A = 3\sin 4A$;

(2) $\sin^3 A \sin 3A + \cos^3 A \cos 3A = \cos^3 2A$.

5. Show from the expression for the area of a triangle in terms of its sides, that of all triangles which have a given

base and the sum of their sides given, the isosceles triangle has the greatest area.

6. The angle of elevation of the top of a hill is observed to be 5°; after walking 1 mile directly towards the hill the angle of elevation is 14° 30′. Find the height of the hill.

7. Investigate a formula for determining the sine of half an angle of a triangle in terms of the sides. The sides of a triangle are 32, 40, 66; find the angle intermediate in value between the other two.

8. If a pyramid with a square base be cut by a plane parallel to the base, show that the section is a square.

9. In any plane triangle, if a and b are the sides opposite to the angles A and B, prove that—

$$\frac{a}{b} = \sqrt{\frac{(1-\cos A)(1+\cos A)}{(1-\cos B)(1+\cos B)}}.$$

10. A balloon ascends uniformly in a vertical line; an observer takes up a position in the horizontal plane at a distance of 350 yards from the point where the balloon left the earth; after a time he observes the angle of elevation of the balloon to be 55° 17′, and after an interval of three minutes he observes the angle of elevation to be 60° 18′. Find the rate of the ascent of the balloon.

11. The slant side of a right cone is 10 ft., and its height 8 ft.; find its volume.

12. Bisect a trapezium by a straight line drawn through one of its angular points.

13. In a circle describe a triangle having each of the angles at the base one-third of the vertical angle.

LXXXI.

1. Find the number of combinations of n things taken 4 together.

The number of combinations of n things, taken 4 together, is to the number taken 2 together as 51 to 2. Find n.

2. The sum of 5 numbers in A. P. is 50; the sum of their squares is 540. Find the numbers.

3. Find by the aid of logarithms—

(1) $\frac{(\cdot 0476)^3}{(3\cdot 005)^{\frac{1}{3}}}$;

(2) A mean proportional to $\sqrt{4\cdot 756}$ and $\sqrt[3]{\cdot 0078^2}$.

4. Prove that the second term of the expansion of $(1+x)^n$ is nx; and write the 7th term of $(1+x)^{10}$.

5. The sides of a right-angled triangle are 1·5 and 2 ft. respectively; find, without the use of Trigonometry, the perpendicular drawn from the right angle to the hypothenuse.

6. A quadrilateral has two of its sides parallel; these sides are 10 and 12 ft. respectively; the perpendicular distance between them is 4 feet. Find the area of the figure.

7. If two straight lines, AEB, CED, in a circle intersect in E, the angles subtended by AC and BD at the centre are together double of the angle AEC.

8. Prove the sine of an angle equal to the sine of its supplement. For what angle is the sine of the supplement equal to the sine of the complement? Find sin 15°.

9. Prove—

(1) $\text{Cot}\,(A+B)=\frac{1-\tan A\tan B}{\tan A+\tan B}$, and hence find cot 75°;

(2) $4\cos A\cos(120°-A)\cos(120°+A)=\cos 3A$.

10. Show that the square described round a circle is equal to $\frac{4}{3}$ of the inscribed do-decagon.

11. The sides of a quadrilateral taken in order are 27, 36, 30, 25 feet; the angle between the first two is 90°. Find its area.

12. Find the content of a right cone whose height is 210·6 feet, and radius of base $23\frac{1}{2}$ feet.

13. Find the surface and solid content of a cylinder 80 ft. long, the radius of the base being 8 feet.

LXXXII.

1. Solve the equations—

(1) $\frac{1}{18+x}=\frac{1}{7}+\frac{1}{11}+\frac{1}{x}$.

(2) $\frac{9}{1+x+x^2}=7-x-x^2$;

(3) $x^2+xy=15$ and $xy-y^2=2$.

2. Calculate by logarithms—

(1) $\frac{35\cdot125\times\cdot3397}{10500\times\cdot009126}$; (2) $\frac{\sqrt[8]{\cdot07197}}{\sqrt[5]{27}}$.

3. The number of combinations of n things, taken 3 at a time, is to the number taken $n-2$ at a time as $5:1$; find n.

4. Find by logarithms the sum of the series 1, $\frac{15}{16}$, $(\frac{15}{16})^2$ &c., to 12 terms.

5. Find the number of gallons of water contained in a cylinder, the radius of the base of which is 18 inches, the height 20 ft.; 1 gal.$=277\cdot27$ cub. in.

6. A sphere has a radius of $25\frac{1}{4}$ feet; find its surface, and determine the radius of a sphere whose surface is $\frac{1}{4}$ of that of the former.

7. Define the tangent of an angle, and find tan 90°, tan 135°, tan 60°.

8. Prove that $\tan(\text{A}+\text{B}+\text{C})=\frac{\tan(\text{A}+\text{B})+\tan \text{C}}{1-\tan(\text{A}+\text{B})\tan \text{C}}$.

9. Given three sides of a triangle, investigate a formula, adapted to logarithmic computation, by means of which an angle may be found.

10. Find the least angle of the triangle whose sides are 63·4, 72·2, 81·8.

11. In any triangle, show that

$$c=(a-b)\times\frac{\cos\frac{1}{2}\text{C}}{\sin\frac{1}{2}(\text{A}-\text{B})}=(a+b)\times\frac{\sin\frac{1}{2}\text{C}}{\cos\frac{1}{2}(\text{A}-\text{B})}.$$

12. The sides of a triangle are 65 and 25; the difference *of the opposite* angles is 60°; find all the angles.

13. The distance between two headlands A and B is found to give the extreme width of a harbour. An eminence C is taken lying inland at distances from A and B of 127·3 and 891 yards. The angle between the directions of the two measurements is 120°. Find the width of the harbour.

LXXXIII.

1. Find the cost of 7 cwts. 2 qrs. 14 lbs. at £1 9*s.* 2*d.* per cwt.

2. A person, after paying 7*d.* in the £ income-tax, has £816 9*s.* 5*d.* of his income remaining; what was his gross income?

3. Prove the rule for dividing one decimal by another; and divide ·0063612 by 2·052.

4. Reduce $\dfrac{2x^3-7x^2-46x-21}{2x^4+11x^3-13x^2-99x-45}$ to its lowest terms.

5. Solve the equations—

(1) $\dfrac{11y-5x}{22}=\dfrac{x+3y}{32}$, $8y-5x=1$.

(2) $\dfrac{1}{x-12}=\dfrac{1}{x}-\dfrac{1}{5}-\dfrac{1}{7}$.

(3) $x^2-xy+y^2=7$, $x^4+x^2y^2+y^4=133$.

6. Insert 6 arithmetical means between 5 and 33.

7. Find the present value of an annuity of £90, to continue 17 years, allowing interest at 5 per cent.

8. Decompose (1) $\dfrac{x^2+7x+1}{(x-1)(x-2)(x-3)}$;

(2) $\dfrac{5-10x}{2-x-3x^2}$ into simple fractions.

9. Investigate the expansion for $\log(1+x)$ in powers of x, and show that the series is convergent when x is less than unity.

10. There are two vessels, each containing a mixture of water and wine, the first in the ratio of 2 to 3, the second

in the ratio of 3 to 7; what quantity must be taken from each to make a mixture which shall contain 5 gallons of water and 11 of wine?

11. A lighthouse was observed by a ship at sea to bear SE.; after the ship had sailed NE. for 12 miles, the lighthouse was observed to bear 15° E. of S. Find the distance of the lighthouse from each position of the ship.

12. An object 10 ft. high is placed on the top of a tower, and subtends an angle of 6° at a place which is in the same horizontal plane as the foot of the tower, and is 50 ft. distant from it. Determine the height of the tower.

13. The centres of two circles which intersect are 12 ft. apart; the radius of one circle is 9 ft., that of the other 8 ft. Find the area of the part which is common to both circles.

LXXXIV.

1. Find the cost of carpeting a room measuring 12 ft. 4 in. by 16 ft. 3 in., at 1*s.* 6*d.* a square foot.

2. A, B and C can build a wall in 10 days, A and B in 13 days, B and C in 15 days; in what time would A and C perform the work?

3. Calculate by logarithms—

(1) $\left(\frac{347}{123}\right)^{\frac{1}{5}}$; (2) $\sqrt[5]{\cdot 023476}$; (3) $\left(\frac{5}{7}\right)^{\cdot 1345}$.

4. Sum the series 1, 3, 5, 7, to n terms.

5. Find the sine and cosine of 15°.

6. Prove that $\sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$.

7. If A, B, C be the angles of a plane triangle, show that $\tan \frac{A+B}{2} = \cot \frac{1}{2} C$.

8. In a triangle, given $a=215$, $b=248$, $C=30°$; find its area.

9. In a triangle, given $c=1124$, $b=640$, $A=128° 4'$; find B and C.

10. A well is 100 ft. deep; how many coils of rope will

be required to reach to the bottom, the roller on which they are wrapped being 8 inches in diameter?

11. Find the greatest triangle having a given chord of a circle for its base, and its vertex on the circumference of the circle.

12. AB, AD, the sides of a rectangle, are 8 and 6; parallelograms are described about the diagonal BD, and the diameter of the smaller of them is $3\frac{3}{4}$. Find how the sides AB, AD are divided, and determine the areas of the complements.

13. Compare the areas of two regular pentagons, one inscribed in a given circle, the other described about it.

LXXXV.

1. Solve the equations—

(1) $\frac{x}{7-x}=\frac{7-x}{x}-\frac{5}{6}$.

(2) $\sqrt{\frac{5x}{x+y}}+\sqrt{\frac{x+y}{5x}}=2$ and $xy-x-y=75$.

2. State the rule for supplying the characteristics of the tabular logarithms (1) of whole numbers; (2) of decimals.

3. Find by the aid of tables $\frac{(15\cdot6)^{\frac{1}{3}}\times(\cdot0045)^{\frac{3}{4}}}{(\cdot00065)^{\frac{1}{2}}}$.

4. Find the present value of £1800 due 5 years hence at 4 per cent. compound interest.

5. Find an expression for the number of combinations of n things, taken 3 at a time.

6. Find the cosine of an angle of a plane triangle in terms of the sides of the triangle.

7. In any triangle ABC

$$a+b+c=(a+b)\cos C+(a+c)\cos B+(b+c)\cos A.$$

8. B and C are two stations 3 miles apart; travellers start from B and C towards a third station, A: the traveller from B, walking 4 miles an hour, reaches A in $\frac{3}{4}$ hr.; the traveller from C, at the rate of 5 miles an hour, reaches A in $\frac{1}{2}$ hr. Find either angle of the triangle ABC.

9. Find (1) sin 18° ; (2) cos 36°.

10. Having given $\frac{\tan^2 \alpha}{\tan^2 \beta}=\frac{\cos \beta (\cos x-\cos \alpha)}{\cos \alpha (\cos x-\cos \beta)}$ show that $\tan^2 \frac{x}{2}=\tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}$.

11. From the top of a tower by the sea-side 150 ft. high, it was found that the angle of depression of a ship's hull was 36° 18′. Find the distance of the ship from the foot of the tower.

12. What length of a gun 6 inches bore will be filled with 20 lbs. of powder of which 30 cubic inches weigh 1 lb.?

13. ABC is an isosceles triangle, C being an obtuse angle, AD a perpendicular drawn from A to BC produced; the square on AB is equal to twice the rectangle contained by BC and BD.

LXXXVI.

1. If gold be beaten out so thin that 1 oz. will form a leaf of 20 square yards, how many of these leaves will make ·01 inch thick, if 1 cub. foot of gold weighs 1095 lbs.?

2. Solve the equations—

$$\left\{\begin{array}{l} x+y+z=0 \\ ax+by+cz=0, \\ bcx+cay+abz+(a-b)(b-c)(c-a)=0. \end{array}\right\}$$

3. Two merchants, A and B, start in trade with the same capital; at the end of a year, A has gained £3000, and B has lost £1000. A's capital is then double that of B: what did they start with?

4. Explain why $\log_{10} 3{\cdot}56$ and $\log_{10} {\cdot}0356$ have the same mantissa; $\log_{10} {\cdot}0356$ and $\log_{10} {\cdot}04$ the same characteristic.

5. Find by the aid of tables—

$$(1)\ \frac{1{\cdot}265 \times {\cdot}01628}{2{\cdot}2825 \times 64{\cdot}28};\ (2) \sqrt{2543 \times {\cdot}1726}.$$

6. A person with a capital of £a, for which he receives *interest* at $100r$ per cent., spends every year £b, which is

more than his original income. In how many years will he be ruined? Apply the result to numerical calculation, where $a=1000$, $r=\cdot05$, $b=90$.

7. Given two angles and a side of a plane triangle; show how to solve the triangle.

8. From the top of a tower 150 feet high, the angles of depression of the top and bottom of a vertical column, standing on the same horizontal plane, were observed 22° 15′ and 44° 30′: find the height of the column.

9. A lighthouse was observed from a ship to bear N. 45° E.; and after the ship had sailed 4 miles SE., the bearing of the lighthouse was observed to be N. 15° E. Find the distance of the lighthouse from each position of the ship.

10. Prove that $\frac{\sin 45°-\sin 30°}{\sin 45°+\sin 30°}=\frac{\sec 45°-\tan 45°}{\sec 45°+\tan 45°}$; and $\tan 2A+\tan A=\frac{2 \sin 3A}{\cos A+\cos 3A}$.

11. The sides of a parallelogram, including an angle 60°, are 20 and 30; and the sides of another, including an angle 120°, are 30 and 40. Compare their areas.

12. The slant side of a right cone is 20 feet, and its axis 16 feet; determine its volume.

13. Find the volume of a frustum of a cone, the altitude being 9 ft., and the diameters of the ends 4 ft. and 2 ft. respectively.

LXXXVII.

1. A field is in the form of a rectangle whose conterminous sides are 510 and 680 ft. respectively; find the length of a path from one corner to the opposite one.

2. A brigade consisting of 5 regiments marched into a garrison, the duty of which required 760 men a day; what number of men must each regiment furnish, their strength being 540, 510, 480, 390, 360 men, respectively?

3. Express $3a-(b+2c-d)$ without brackets; and show that the result obtained is the correct one.

4. The product of 5 numbers in A. P. is 945, and their sum is 25; find the numbers.

5. Expand $(a^2-x^2)^{-2}$ to 6 terms by the binomial theorem; and write down the 12th term.

6. In what time, at compound interest at 5 per cent., will £100 become £1000?

7. If θ be the circular measure of an angle, show that as θ diminishes $\frac{\sin\theta}{\theta}$ approaches 1 as its limit.

8. A right pyramid on a square base, having 20 ft. in each side of the base, has an altitude of 35 ft. Find the sum of the plane angles forming either of the solid angles at the base.

9. Prove (1) $\sin 2A = \frac{2 \cot A}{1+\cot^2 A}$.

(2) $\frac{\sin 2A + \sin A}{\cos 2A + \cos A} = \tan \frac{3}{2}A$.

(3) $\tan \frac{1}{2}A = 2-\sqrt{3}$; when $\sin 2A = \cdot 5$.

10. The altitude of a tower, observed at the end of a horizontal base of 100 yards measured from its foot, is 30°; find its height.

11. In a plane triangle $c=732$, $b=846$, $a=945$; find the greatest angle.

12. Find x from the equation $\cos^{-1}\frac{1-x^2}{1+x^2}+\tan^{-1}\frac{2x}{1-x^2}=\frac{4\pi}{3}$,

and prove that $\sin(36°+A)-\sin(36°-A)$
$=\sin A+\sin(72°+A)-\sin(72°-A)$.

13. Investigate the area of a circle, and of any sector of a circle.

LXXXVIII.

1. If 9 tons $7\frac{1}{4}$ cwts. of iron be sold for £245, and the gain upon it be 20 per cent., what was the cost of 1 cwt.?

2. Solve the equations $\begin{cases} xy+\dfrac{x}{y}=\dfrac{5}{3} \\ \dfrac{1}{xy}+\dfrac{y}{x}=\dfrac{20}{3} \end{cases}$

3. A garrison loses 5 men on the first day of a siege, 10 on the second, 15 on the third, and so on. At this rate the garrison would be all destroyed in 50 days. How many men must be introduced at the end of the fifteenth day of the siege to bring the strength up to 8000 men?

4. Find by tables: (1) a fourth proportional to 3^5, 2^7, 15^3. (2) the sum of 7 terms of the series 7, 21, 63, &c.

5. Find the amount of £2900 in 7 years at $4\frac{1}{2}$ per cent. compound interest?

6. If $a : b :: c : d : e : f$, prove

$$(a^2+c^2+e^2)^{\frac{1}{2}} : (b^2+c^2+d^2)^{\frac{1}{2}} :: a : b.$$

7. Show how to calculate sin 7° 30′.

8. If A, B, C be the angles of any plane triangle, prove that

$$\sin 2A+\sin 2B+\sin 2C=4 \sin A \sin B \sin C.$$

9. X and Y are two stations on a coast; X is N. 45° E. of Y, and 3 miles distant from it: a ship at sea observes at the same instant the bearing of X to be N. 20 15′ E., and that of Y to be N. 42° 20′ W.; find the distance of the ship from the nearer station.

10. One side of a triangle is 200 yds., another is 88 yds.; determine the third side, when the area is 5095 square yds.

11. O is the middle point of the hypothenuse AB of a right-angled triangle, ABC; show that OA, OB, OC are all equal to one another.

12. A, B, C, D, E are the angles of a regular pentagon taken in order; join CA, CE; then,

(1) The angle BCD is trisected.

(2) CA is parallel to DE.

13. The base and altitude of a right-angled triangle are respectively 7 and 4 feet; find the altitude of a right-angled triangle 5 times as great as the former, the base of which is 10 feet.

LXXXIX.

1. What fraction of the habitable globe, which is computed to contain 3767300 square miles, is the area of Great Britain, which contains 74688000 acres?

2. In the decimal ·5074 explain the local value of each digit. Divide ·00169 by ·013, and 25·6 by ·000016.

3. Reduce $\frac{5}{7}$ to a decimal. In reducing a fraction to a decimal, show that when the quotient does not terminate it must recur; and limit the number of recurring digits.

4. Multiply $3x^4+6x^3y-9xy^3+12y^4$ by $2x^2-3xy+y^2$.

5. Divide—

$$a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6$$
by $a^3-3a^2b+3ab^2-b^3$.

6. Show that $1-\dfrac{b^2+c^2-a^2}{2bc}=\dfrac{(a+b-c)\,(a+c-b)}{2bc}$.

7. Solve the equation $\dfrac{6x+13}{3}-\dfrac{3x+5}{x-5}=2x$.

8. If a, b, c be in A. P., and b, c, d in H. P., show that $a : b :: c : d$.

9. If e be the base of Napier's system of logarithms; assuming the series for $\log_e\,(1+x)$, prove that

$$\log_e\frac{m}{n}=2\left\{\frac{m-n}{m+n}+\frac{1}{3}\left(\frac{m-n}{m+n}\right)^3+\frac{1}{5}\left(\frac{m-n}{m+n}\right)^5+\&c.\right\}$$

10. The length of a ditch horizontal at top and bottom is 100 yards, its depth 12 ft.; width at top 12 ft., at bottom 8 ft. Find its content.

11. Show that the difference of the squares on the side of a regular pentagon and a regular decagon, inscribed in the same circle, is equal to the square on the radius.

12. The side of an equilateral triangle inscribed in a circle

is equal to a diagonal of a rhombus, each of whose sides is the radius of the circle.

13. If A, B, C are the angles of a plane triangle, prove that

$$\frac{\sin A+\sin B+\sin C}{\sin A+\sin B-\sin C}=\cot \tfrac{1}{2} A \cot \tfrac{1}{2} B.$$

XC.

1. Express $\frac{4}{27}$ of a right angle, (1) in degrees &c. English; (2) in grades; (3) in circular measure.

2. Show that—

$$\cos A=\cos (2n \,.\, 180+A)=-\cos \{(2n+1)\ 180°-A\}$$
$$= -\cos \{(2n+1)\ 180°+A\} =\cos (2n \,.\, 180°-A),$$

n being a positive integer.

3. If $\sin A=\sqrt{2} \sin B$, and $\tan A=\sqrt{3} \tan B$, determine A and B.

4. Prove that—

(1) $\dfrac{\sin A\pm\sin B}{\cos A+\cos B}=\tan \tfrac{1}{2} (A\pm B)$.

(2) $\cos A=\tfrac{1}{2} \{\sqrt{1+\sin 2 A}+\sqrt{1-\sin 2 A}\}$, A being less than 45°.

(3) $\sin n A+\sin (n-2) A=2 \sin (n-1) A \cos A$.

(4) $\dfrac{\sin 2 A}{1+\cos 2 A}\times\dfrac{\cos A}{1+\cos A}=\tan \tfrac{1}{2} A$.

5. Given $b=252$, $c=528$, $C=124° 34'$; determine the angles of the triangle—

log 252 = 2·4014005, log 528 = 2·7226339,
Log sin 55° 26′ = 9·9156460, Log sin 23° 8′ = 9·5942513,
Log sin 23° 9′ = 9·5945469.

6. If R be the radius of the circle inscribed in a triangle, $R=\tfrac{1}{2} (a+b+c) \tan \tfrac{1}{2} A \tan \tfrac{1}{2} B \tan \tfrac{1}{2} C$.

7. In any triangle the length of the perpendicular from A upon the opposite side $=\dfrac{b^2 \sin C+c^2 \sin B}{b+c}$.

8. Adapt the formula $c^2=a^2+b^2-2ab \cos C$ to logarithms.

9. Find the area of a triangle whose sides are 16·1, 23·45, 29·53.

10. Find the area of the sector of a circle the angle of which is 60°, the radius of the circle being 60 ft.

11. Find A from the equation $\sin 2A = 3 \tan A \cos 2A$.

12. Show that in any plane triangle $a = b \cos C + c \cos B$.

13. Solve a right-angled triangle, having given the hypothenuse and one side. If the side be very small compared with the hypothenuse, what difficulty arises in calculating the angles by means of the logarithmic tables? and how is the difficulty obviated?

XCI.

1. Find the value of $(a+b)\sqrt[3]{(x-b)y^2} - a\sqrt{\{y(x-b)\}} + x$, when $a = \frac{4}{5}$, $b = 2$, $x = \frac{10}{3}$, $y = \frac{4}{3}$.

2. Divide the product of—

$$x^2 + xy + y^2 \text{ and } x^3 + y^3 \text{ by } x^4 + x^2y^2 + y^4.$$

3. Find the G.C.M. of—

$$x^4 - x^3 - 2x^2 + 3x - 1 \text{ and } 2x^3 - 2x^2 - x + 1.$$

4. Solve the equations—

$$(1)\ \frac{5x-6}{7} + \frac{9x-14}{11} = \frac{9x-16}{5}.$$

$$(2)\ x = 4y,\ \tfrac{1}{5}(2x+7y) - 1 = \tfrac{2}{3}(2x-6y+1).$$

5. The fore-wheel of a carriage makes 6 revolutions more than the hind-wheel in going 120 yards; if the circumference of the fore-wheel be increased by $\frac{1}{4}$ of its present length, and that of the hind-wheel by $\frac{1}{5}$ of its present length, the fore-wheel will make 4 revolutions more than the hind-wheel in 120 yards. Find the circumferences of the wheels.

6. Find by the tables the values of

$$(1)\ \left(\frac{7432}{\sqrt{29}}\right)^{\frac{3}{7}};\quad (2)\ \left\{\frac{5\sqrt[3]{138}}{\sqrt[5]{\cdot 01}}\right\}^{\frac{1}{2}}$$

7. Prove the binomial theorem when the index is a negative integer.

8. Find the coefficient of x^n in the expansion of

$$\frac{(1+x)^2}{(1-x)^3}$$

in a series of ascending powers of x.

9. Resolve $\dfrac{3x-2}{(x-1)(x-2)(x-3)}$ into partial fractions.

10. Show that the square on the sum of two lines, together with the square on their difference, is double the squares on the two lines.

11. If two circles intersect in A and B, and through P, any point in the circumference of one of them, the chords PA, PB be drawn to cut the other circle in C and D, show that CD is parallel to the tangent at P.

12. The sum of the alternate angles of any hexagon inscribed in a circle is equal to four right angles.

13. Solve the equation $\sin 5x = 16 \sin^5 x$.

XCII.

1. What length of matting $\frac{3}{4}$ yd. wide will be required for a room 15 ft. 8 in. long by 11 ft. 3 in. wide; and what will be the cost of the matting at 9*d.* a yard?

2. Reduce $\frac{17}{625}$ to a decimal; and show that $\cdot 000\dot{9} = \cdot 001$.

3. Find the annual interest on £3654 15*s.* at $3\frac{1}{2}$ per cent. Which is more profitable to the state,—an income-tax of 1*s.* 2*d.* in the pound, or one of 6 per cent.?

4. If 2 cubic inches of iron weigh as much as 15 of water, and a cubic foot of water weighs 1000 oz., find the weight of a beam of iron 5 yards long, 2 feet wide, and 1 inch thick.

5. If m and n are integers, show that $a^m \cdot a^n = a^{m+n}$; $(a^m)^n = a^{mn}$. What is the numerical value of $8^{-\frac{2}{3}}$?

6. Arrange $x^4 + x^{-4} - x^{-2} - x^2$ according to decreasing powers of x, and divide the expression by $x - x^{-1}$.

7. Extract the $\sqrt{9a^2-3ax+\frac{x^2}{4}}$ and $\sqrt{\frac{x^2}{4}-3ax+9a^2}$; explain why the results differ.

8. Solve the equations—

$$(1)\left.\begin{array}{l}\frac{x-2}{5}-\frac{10-x}{3}=\frac{y-10}{4}\\ \frac{2y+4}{3}-\frac{2x+y}{8}=\frac{x+13}{4}\end{array}\right\}$$

$$(2)\ 5x-\frac{3(x-1)}{x-3}=2x+\frac{3x-6}{2}.$$

$$(3)\ \frac{5xy}{6}=x+y,\quad \frac{3xz}{10}=x+z,\quad \frac{2yz}{15}=y+z.$$

9. Find a factor to rationalise $a^{\frac{1}{p}}-b^{\frac{1}{q}}$; and express as fractions with rational denominators—

$$(1)\ \frac{3}{\sqrt{5}+\sqrt{2}},\ (2)\ \frac{3}{\sqrt{3}+\sqrt[3]{5}}.$$

10. Find the present value of an annuity of £A to be continued for n years at r per cent. compound interest.

11. If the annuity become a perpetuity, find the number of years' purchase that must be paid for it at 5 per cent.

12. Show how to find the number of shot in a complete rectangular pile; and apply the result to find the number of shot in an incomplete pile having 200 and 160 shot in the sides of its base, 70 and 30 in the sides of its top course.

13. ACB is an isosceles triangle, APB is an equilateral triangle within ACB; find AC when the perimeter of APB is half that of ACB.

XCIII.

1. If four magnitudes be proportionals, the sum of the greatest and least is less than the sum of the other two.

2. Prove that $\log_{10} \text{PQR}^{\frac{1}{m}} = \log_{10} \text{P} + \log_{10} \text{Q} + \frac{1}{m}\log_{10} \text{R}$.

3. Find by the tables, (1) a third proportional to ·0374 and 34·762; (2) the fifth root of ·00374.

4. If e be the base of Napier's system of logarithms, expand e^x, and show that the series for calculating e converges.

5. One side of a rectangle is 6 in.; find the other side, when the area is equal to that of a trapezoid whose parallel sides are 7 in. and 8 in., the perpendicular distance between them being 12 inches.

6. Regular hexagons are inscribed in circles whose radii are 3 and 5 inches; what is the radius of the circle in which the area of the inscribed hexagon is a mean proportional between the areas of the other inscribed hexagons?

7. Express in degrees, grades, and in circular measure, (1) the angle of a regular pentagon; (2) each angle of an isosceles triangle in which the angle at the vertex is half of each of the angles at the base.

8. From the edge of one bank of a river, a person ascends 100 yards up a slope of 1 in $4\frac{1}{2}$, and observes the angle of depression of an object on the opposite bank close to the edge of the river to be $1\frac{3}{4}°$. Find the breadth of the river.

9. Prove that $\sin^2 30° : \sin^2 45 : \sin^2 60° :: 1 : 2 : 3$.

10. Given $\sin(A-B)=\sin A \cos B-\cos A \sin B$, to prove that $\cos(A+B)=\cos A \cos B-\sin A \sin B$.

11. Prove the expressions—

(1) $\cos A-\cos B=-2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$.

(2) $\dfrac{1-\cos A}{1+\cos A}=\tan^2 \dfrac{A}{2}$.

(3) $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A \tan B}$.

(4) $\tan(45°+A)-\tan(45°-A)=2 \tan 2A$.

12. In any triangle, prove that—

$$\operatorname{Tan} \frac{A}{2} \tan \frac{B}{2}+\tan \frac{A}{2} \tan \frac{C}{2}+\tan \frac{B}{2} \tan \frac{C}{2}=1.$$

13. If in the "ambiguous case" c, c' be the two values of the third side, and the given angle be 45°, show that the

angle between the two positions of the side opposite is, $\cos^{-1}\dfrac{2cc'}{c^2+c'^2}$.

XCIV.

1. If the rent of a farm of 17 ac. 3 ro. 12 po. be £39 4*s.* 7*d.*, what would be the rent of another farm containing 26 ac. 2 ro. 38 po.; 6 acres of the former being worth 7 acres of the latter?

2. In a rectangular court, measuring 96 ft. by 84 ft., there are four rectangular grass plots, each measuring $22\frac{1}{2}$ ft. by 18 ft.; find the cost of paving the remainder at $8\frac{1}{2}d.$ per square yard.

3. Solve the equations—

$$(1)\quad \frac{2x+3}{x+1}=\frac{4x+5}{4x+4}+\frac{3x+3}{3x+1}.$$

$$(2)\quad \frac{ax}{a+x}+\frac{by}{b+y}=\frac{(a+b)\,c}{a+b+c},\quad x+y=c.$$

4. What is the amount of a debt which can be discharged in 36 weeks by paying 1*s.* the first week, 3*s.* the second, 5*s.* the third, and so on? How much was paid the 13th week?

5. Find a point without two concentric circles from which, if tangents be drawn to the circles, one shall be double the other.

6. A and B are two fixed points without a circle, PQR; it is required to find a point P on the circle, such that the sum of the squares described on AP and BP may be the least possible.

7. Show that $\tan 7\frac{1}{2}^\circ=\dfrac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}$; and find sin 15° and cos 15°.

8. In any plane triangle $\dfrac{\tan\frac{1}{2}A-\tan\frac{1}{2}B}{\tan\frac{1}{2}A+\tan\frac{1}{2}B}=\dfrac{a-b}{c}$.

9. Two sides of a triangle being 12 feet and 20 feet, include an angle of 120°; find the third side, without finding *the remaining* angles, by a formula adapted to logarithms.

10. Show how to determine the height and position of an elevated object when a base can be measured which is not horizontal.

11. The sides of a quadrilateral are 10, 7, 8, 9, and the angle between the second and third sides is 120°; find the area of the figure.

12. The perpendicular height of the frustum of a cone is 7 feet, and the radii of the ends are 4 and 5 feet; find its volume.

13. Find the whole surface and solid content of a square pyramid, each side of the base being 12 ft., and the slant height 25 ft.

WOOLWICH PAPERS.

XCV.

1. Supposing the number of sheep in the country to be 13000000; what would be the value of their wool in a year, estimated at £8 15*s.* per cwt., 15 sheep yielding 29 lbs. of wool?

2. How many men would it employ for $5\frac{1}{2}$ days to cultivate a field of $2\frac{5}{8}$ acres, if each man completed 77 square yards in 9 hrs., and they worked 10 hrs. a day?

3. Solve the equations—

$$(1) \left\{\begin{array}{l} x-y \;=3 \\ x^4-y^4=609 \end{array}\right\}$$

$$(2)\ \frac{y}{x}-\frac{x}{y}=\frac{x-y}{x^2+y^2} \quad \text{and} \quad \frac{x^2}{y^2}-\frac{y^2}{x^2}=\frac{x+y}{y^2}.$$

4. Calculate the value of $\sqrt[3]{\dfrac{2+\sqrt{3}}{2-\sqrt{3}}}$.

5. Two parties have a joint capital of £2000: one withdraws his money after 12 months, and receives for capital and profit £1040; the other lets his remain 17 months, and then finds it amount to £1710. What was the original capital of each partner, the profit being uniform throughout?

6. The annual growth of a shrub is 3 in. more every successive year; in the first year it grew 2 in., and its height is now 442 in. How many inches did it grow in the last year?

7. Expand $\dfrac{1+4x+x^2}{(1-x)^4}$ in a series ascending by powers of x, by the method of indeterminate co-efficients.

8. Find the value of x in the equation $\sqrt[x]{100}=7$.

9. What are the respective limits of magnitude within which the values of the sine, cosine, and secant of an angle are always included? Are there any such limits to the value of the tangent? Given the secant of an angle$=3$, find its sine, cosine, and tangent.

10. Prove that—

$$(\sin A+\sec A)^2+(\cos A+\operatorname{cosec} A)^2=(1+\sec A \operatorname{cosec} A)^2.$$

11. The three sides of a triangle are 460, 575, 805; find the greatest angle.

12. In general if a, b, c be the sides of a triangle, how may it be determined whether or not the triangle contains an obtuse angle?

13. If E be a point without a circle, and straight lines E A B, E C D are drawn to cut the circle in A, B and C, D, show that the angle C E A is equal to half the difference of the angles subtended by A C and B D at the centre of the circle.

XCVI.

1. Multiply ·076 by ·0007; divide the product by ·000019, and prove that the numbers of decimals you point off in the product and in the quotient are the right ones.

2. Find the value of—

·175 tons + ·195 cwts. + ·145 qrs. + ·15 lbs.

3. Assuming the area of the British Islands to be 91000 geographical square miles, and 122000 square statute miles, what is the ratio of the length of a geographical mile to that of a statute mile?

4. A salvo of 5 guns is fired from a battery, and the guns are thenceforward fired at intervals of 12, 16, 21, 52 and 70 seconds respectively. What interval will elapse before the guns are again fired all together?

5. Solve the equations—

$$(1)\ \frac{2x-3}{3x-4}=\frac{4x-6}{6x-7}; \quad (2)\ \frac{\sqrt{x}+\sqrt{3}}{\sqrt{x}-\sqrt{3}}=\frac{2}{3}.$$

6. The sum of 29 terms of an A. P. is 145, and the common difference $\frac{1}{4}$. Find the middle term.

7. Find the fifth root of $\frac{1000}{999}$, correct to 12 places of decimals by the binomial theorem.

8. Calculate the first six terms of $(1+x)^{1+x}$.

9. Find the value of $\dfrac{x^3+2x^2-14x-3}{x^2-x-6}$ when $x=3$; and give a proof of the principles upon which your method rests.

10. Find all the values of θ which satisfy the equation—

$$\cos 8\theta-\cos 5\theta+\cos 3\theta=1.$$

11. Express $\sin 3A$ in terms of $\sin A$; and determine its numerical value when $\sin A=\frac{1}{2}$.

12. One side of a right-angled triangle is 29 ft. 8 in., and the adjacent acute angle $27^\circ\ 33'\ 49''$. Find the side opposite to the right angle.

13. A cone whose vertical angle is a right angle, and radius of base 10 feet, is bisected by a plane parallel to the base. Find the radius of the section.

XCVII.

1. Reduce to their simplest forms :—

$$(1)\ \tfrac{2}{3}+\tfrac{1}{6}+\tfrac{5}{9}-\tfrac{11}{12}; \quad (2)\ \tfrac{2}{3}(6\tfrac{2}{3}+2\tfrac{1}{2});$$

$$(3)\ \frac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}-2\frac{1}{2}}; \quad (4)\ \frac{11781}{29799};$$

$$(5)\ \cfrac{3}{1+\cfrac{2}{5+\cfrac{7}{9}}}.$$

2. A contractor engaged to finish a work of 3150 cu. yds. in 50 days, and employed at once 60 men upon it, but at the end of 35 days he finds only 1800 cu. yds. completed. How many extra men must he put on to complete the work in the given time.

3. Simplify $\frac{1}{2x+y}+\frac{1}{2x-y}-\frac{3x}{4x^2-y^2}$.

4. Solve the simultaneous equations—

$$\begin{Bmatrix}(x-y)\ (x^2-y^2)=a\\(x+y)\ (x^2+y^2)=b\end{Bmatrix}$$

5. Explain fully the principles of the process of finding the greatest common measure of two quantities.

6. Find two geometric means between 7 and 96. Solve the question both with the aid of tables and without it.

7. Investigate an expression for the logarithm of a number to a given base, in the form of a converging series.

8. Find by logarithms the 5th root of $\frac{17}{52}$ and the square root of $\frac{\sqrt[3]{\cdot 0125}\times\sqrt{31\cdot 15}}{\cdot 00081}$.

9. From any point O draw straight lines to the vertices of a plane triangle A B C; prove that—

sin O A C sin O C B sin O B A = sin O C A sin O B C sin O A B.

10. Find the remaining parts of the triangle A B C, when the parts given are $c=24992$, $a=7946$, $A=8°\ 19'\ 35''$.

11. Find x from the formula—

$$\sin 3A = 4 \sin A \sin (x+A) \sin (x-A).$$

12. An object 12 ft. high, standing on the top of a tower, subtends an angle of 1° 54′ 10″, at a station which is 250 feet from the base of the tower. Find the height of the tower.

13. Two concentric circles have radii of 10 ft. and 15 ft. respectively. Calculate the area of the figure bounded by these circles and by radii inclined at an angle of 40° to each other.

XCVIII.

1. Multiply £12 15*s.* 4½*d.* by 1800.

2. Find, by the rule of Practice, the value of 8764 things £10 7*s.* 7½*d.* each.

3. If the carriage of 6 cwts. 3 qrs. for 124 miles cost 4*s.* 8*d.*, what weight would be carried 93 miles for 0*s.* 7¼*d.*?

4. Solve the equations:—

(1) $\sqrt{x+13}=2+\sqrt{x-11}$.

(2) $x^{\frac{1}{m}}y^{\frac{1}{n}}=a$, and $x^{\frac{1}{p}}y^{\frac{1}{q}}=b$.

(3) $\sqrt[3]{a+x}+\sqrt[3]{a-x}=\sqrt[3]{b}$.

5. A sum of £6 is to be divided among a certain number men, and £3 among as many women; and it is found that there were 5 more men, and 6 less women, each woman uld receive as much as a man. How many of each were ere?

6. Given P the pth term, and Q the qth term of an arith-tical progression to find N the nth term.

7. Having given the first and last terms, and the sum the terms, of a geometrical progression; to investigate an pression for the number of terms.

8. Find the value of sin 60°; and show that—

$$\sin A=\sin(60°+A)-\sin(60°-A).$$

9. Show how the distance from one another of two ccessible objects, C and D, may be determined by observa-ns from A and B, in the same plane, whose distances are own.

10. A spectator observes the explosion of a meteor due rth of him, and at an altitude of 30°. To another spec-or, at a place 10 miles NNE. of the former, it appears the same moment to have an altitude of 45°; investigate height from the earth at the time.

11. Expand sin mx in terms of sin x.

12. If two circles intersect in A and B, and A C, A D be two diameters, prove that the line C D will pass through B.

13. A slice is cut from a sphere by two parallel planes: the distance between the planes is 4 feet, the radius of one circular section is 9 feet, and of the other 12 feet. Determine the radius of the sphere and the volume of the slice.

XCIX.

1. If 1 lb. of silver cost £3 6*s.*, what is the price of a cup weighing 10 lbs. 6 oz. 10 dwts. subject to a duty of 1*s.* 6*d.* per oz., and also to a charge of 1*s.* 9*d.* per oz. for workmanship?

2. Divide ·014904 by $3\frac{6}{25}$ and 14·904 by ·000324: find the value of ·375 of a guinea + $\cdot\dot{5}\dot{4}$ of 8*s.* 3*d.* + $\cdot0\dot{2}\dot{7}$ of £2 15*s.*

3. Solve the equations :—

$$(1)\ \frac{2x+25}{13}+\frac{30-x}{8}=\frac{10x-327}{19}.$$

$$(2)\ \frac{a^2}{x}+\frac{b^2}{y}=\frac{(a+b)^2}{c} \text{ and } x+y=c.$$

4. Define arithmetical progression and harmonical progression. Show that if a^2, b^2, c^2 are in A. P., $b+c$, $c+a$, $a+b$ are in H. P.

5. Calculate by logarithms—

$$(1)\ \frac{(42{\cdot}666)^{12}}{1{\cdot}147}\times\frac{({\cdot}0765)^{10}}{194{\cdot}32};\quad (2)\ \sqrt{4\tfrac{1}{3}+5\tfrac{1}{3}}.$$

6. In any triangle show that,

$$(1)\ \cot B-\cot A=\frac{a^2-b^2}{ab}\operatorname{cosec} C;$$

$$(2)\ a^2+2ab\sin(C-30^\circ)=c^2+2bc\sin(A-30^\circ).$$

7. Find the area of the triangle whose sides, 84 and 73 feet in length, include an angle of 48° 27′ 53″.

8. A sector of a circle contains an angle of 45°, and the area of the sector is 200 sq. yds.: find the radius.

9. From C, the foot of a tower, A C is drawn in the horizontal plane; the angle of elevation of B, the top of the

tower, as seen from A is α; AD ($=a$ ft.) is measured from A, in the horizontal plane, perpendicular to AC; at D the angle of elevation of the tower is β; prove the height of the tower to be $\dfrac{a \sin \alpha \sin \beta}{\sqrt{\sin(\alpha+\beta)\sin(\alpha-\beta)}}$: apply the formula, when $a=350$ ft., $\alpha=10^\circ\ 45'$, $\beta=8^\circ\ 20'$.

10. Prove that the sum of the three perpendiculars drawn from any point within an equilateral triangle on the three sides is constant.

11. If a line be drawn from the vertex of an isosceles triangle to the base or the base produced, the difference between the square on this line and the square on the side of the triangle, is equal to the rectangle contained by the segments of the base.

12. A pyramid is cut by a plane parallel to the base; show that the section is a figure which is similar to the base.

13 AC is the diameter of a circle, and a diagonal of the inscribed quadrilateral ABCD; given AB=30, BC=40, CD=10, find AD and the area of the figure.

C.

1. Find the value of 6 tons 1 cwt. 14 lbs. at £7 13*s.* 6*d.* per cwt.

2. Divide each of the following to 4 places of decimals: 16·25 by 4·35; ·01 by ·85; ·5 by ·0065; ·0004692 by ·000365.

3. Extract the cube root of 44·6 to 4 places of decimals.

4. Reduce £15 8*s.* 7½*d.* to the decimal of £1; and 3 fur. 66 yds. to the decimal of 1 mile.

5. A does $\frac{2}{5}$ of a piece of work in 4 hrs., B does $\frac{3}{4}$ of what remains in 1 hr., C finishes it in 20 minutes. How long would they have been doing the whole if they had worked together?

6. $\left.\begin{array}{r} x^2+xy+y^2=49 \\ x^4+x^2y^2+y^4=931 \end{array}\right\}$ (2) $\left.\begin{array}{r} 4(x+y)=3xy \\ x+y+x^2+y^2=26 \end{array}\right\}$

Find x and y in each case.

7. On the ground are placed n stones in a line; the distance between the 1st and 2nd is 1 yd.; between the 2nd and 3rd, 2 yds.; between the 3rd and 4th, 3 yds., and so on. How far will a person have to travel in order to bring them one by one to a basket placed at the first stone?

8. Calculate, by logarithms, the 4th proportional to the quantities $\sqrt{1\cdot027}$, $\sqrt[3]{2\cdot546}$, $(31\cdot027)^2$.

9. A, B, C are three points, the distances of which are, $AB = 320$ yds., $AC = 600$ yds., $BC = 435$ yds. From S, a station, it is observed that $ASB = 15°$, $BSC = 30°$. Find the distances of S from A, B, and C; the point B being nearest to S, and the angle ASC being the sum of ASB and BSC.

10. Prove that $\cot 3A = \dfrac{1-3\tan^2 A}{3\tan A - \tan^3 A}$.

11. Show that any straight line which passes through the middle point of a diameter, and is terminated by two opposite sides, bisects a parallelogram.

12. Divide a given circle into two segments, so that the angle contained in one segment shall be double the angle contained in the other.

13. Two sides of a triangular field, the lengths of which are 16 and 40 poles respectively, are inclined to each other at an obtuse angle; find that angle, and the third side of the triangle, when the field contains exactly an acre.

CI.

1. Reduce to a vulgar fraction $\cdot 7 + \frac{3}{11}$ of $\cdot 825 + 4\cdot 1\dot{3}$.

2. At what rate per cent., simple interest, will a sum of money treble itself in 25 years?

3. If the French 3 per cents. are at 60, when the English are at 95, the exchange between the countries being 25 francs to the pound, how much French stock in francs can be bought by selling £6000 stock out of the English funds?

4. Simplify $\dfrac{ab}{(b-c)(c-a)} + \dfrac{bc}{(c-a)(a-b)} + \dfrac{ca}{(a-b)(b-c)}$.

5. Solve the equation $\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-5}{x-6}-\frac{x-6}{x-7}$.

6. A passenger train sets out from L at the same time that a luggage train, travelling 10 miles an hour slower, sets out from S, and they meet at a certain place on the road. On their return they again set out at the same time, and meet at a place 30 miles nearer L. What is the distance of this place from L, and at what rates do the trains travel? The distance from L to S=80 miles.

7. Express cos 3A in terms of cos A.

8. If $A+B+C=\pi$, show that—

$$\sin A-\sin B+\sin C=4\sin\tfrac{1}{2}A\cos\tfrac{1}{2}B\sin\tfrac{1}{2}C.$$

9. Sum the series—

$$\sin A+\sin 2A+\sin 3A+\sin 4A+\dots \text{ to } n \text{ terms.}$$

10. A line AB, 450 yds. long, is measured close by the brink of a river, and a point C, close to the bank of the river on the other side, is observed both from A and B; the angle CAB is 52°, CBA, 70°; find the width of the river.

11. If 30 cubic inches of powder weigh 1 lb., show that it will require nearly 9 lbs. to fill a shell whose internal diameter is 8 inches.

12. If c, c_1 be two chords of a circle that intersect at right angles, d its diameter, x, y the segments of the chord c; prove that $2(x^2+y^2)=d^2+c^2-c_1^2$.

13. Find the area common to two equilateral triangles inscribed in a circle, the position of one differing from that of the other by a given angle (θ): show that when the area becomes a regular hexagon it will be equal $\frac{r^2\sqrt{3}}{2}$.

CII.

1. By payment of 2*s.* 1*d.* in London a banker will give credit at Calcutta for 1 rupee; how many rupees may be received in Calcutta by the payment of £5025 in London?

2. Write down all the numbers that can be composed of the four digits 3, 4, 5, 6, which are divisible by 11.

3. At what rate per cent., simple interest, will £7433 6*s.* 8*d.* amount to £9942 1*s.* 8*d.* in $7\frac{1}{2}$ years?

4. Show that the rule for dividing one fraction by another is consistent with the result of division in whole numbers.

5. Simplify $\dfrac{\frac{2}{3}+\frac{4}{5} \text{ of } \frac{5}{9}-\frac{8}{21}}{1+\frac{2}{3}\times\frac{3}{7}-\frac{5}{9}}$.

6. Simplify (1) $2-x+3x^2-\frac{1}{2}(4-2x+x^2)$;

$$(2)\ \frac{(1-x^2)(1-x^3)}{x(1+x)(1-x)^2}-\frac{x^3+\dfrac{1}{x^3}}{x^2+\dfrac{1}{x^2}-1}.$$

7. Find the factors of $1-3x+2x^2$; and determine the value of $\dfrac{a-b}{b-c}+\dfrac{1}{2}\cdot\dfrac{5(a+c)}{2b}-\dfrac{2b-c}{2}$ when $a=4$, $b=2$, $c=1$.

8. Solve the equations:—

$$(1)\ 1+\frac{x}{2}-\frac{x}{3}=4-\frac{x+1}{7}-\frac{x-1}{5}.$$

$$(2)\ \left\{\begin{array}{c} x\sqrt{\dfrac{x}{y}}+y\sqrt{\dfrac{y}{x}}=34 \\ x-y=12 \end{array}\right\}$$

9. Extract the square roots of 4·04010 and of
$(x^2+1)^2+4x(x^2-1)$.

10. Find what value of x will make $x^2+2ax+b^2$ the square of $x+c$. What does the result become when $a=b=c$?

11. When are four quantities said to be in proportion? What value must be given to x to make $1+x$, $2+x$, $8-x$, $10-x$, proportionals?

12. If $a : b :: c : d$, prove the equality—

$$\frac{a^3+b^3}{c^3+d^3}\times\frac{b}{d}=\left(\frac{a+b}{c+d}\right)^4$$

13. Sum the series (1) $-\frac{1}{3}$, $-\frac{1}{12}$, $\frac{1}{6}$, &c. to 20 terms; (2) 1, $\frac{1}{3}$, $\frac{1}{9}$, &c. to infinity.

14. If 3 dice be thrown, what is the chance that they will all turn up aces?

15. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - \&c.$ show how to determine x.

CIII.

1. A man bought £500 3 per cent. stock when consols were at $93\frac{1}{2}$; after having received one dividend he sold at $96\frac{3}{4}$. What did he gain by the transaction?

2. What must be the rate per cent. that £75 may amount to £100 in 25 years?

3. Find the value of $\frac{3}{8}$ of 13*s.* 4*d.* + $\frac{3}{8}$ of 7*s.* 6*d* + $\cdot 1$ of 9*s.* 9*d.*

4. Simplify $$\frac{\dfrac{1}{1-\dfrac{1}{2-\frac{1}{2}}} - \frac{1}{3} + (1+\frac{1}{2})}{3\left(1+\dfrac{2}{3\frac{1}{2}}\right) - 4}$$

5. Explain the rule for reducing a recurring decimal to a fraction; and reduce $2\cdot4171717$ &c.

6. If unity be taken from the square of any number, prove that the difference is equal to the product of two numbers—one greater by unity and the other less by unity than the original number.

7. Prove the rule that, in multiplication of Algebraical quantities, 'like signs give plus, and unlike, minus.' What assumptions are made as to the nature of the quantities multiplied?

8. Prove that $\{\sqrt{3+4\sqrt{-1}} + \sqrt{3-4\sqrt{-1}}\}^2 = 16$.

9. If the sum of nth and $2n$th terms of a geometrical progression be given, and also the sum of the $2n$th and $3n$th terms; find the first term and the common ratio.

10. Sum the series of which $2r+3+2\times3^r$ is the rth term.

11. Find the logarithm of $a^{\frac{p}{q}}$ to the base $a^{\frac{m}{n}}$.

12. Show that the characteristic of the logarithm of any number can be determined by inspection when the base is 10. Having given $\log_{10} 2 = \cdot 301030$ and $\log_{10} 3 = \cdot 477121$, find $\log_{10} \cdot 0000432$.

13. If the logarithms of numbers in the ordinary tables were all doubled, what would be the base to which they would then be the logs of the same numbers as before?

14. Sum the series—

(1) $\frac{1}{1.2}$, $\frac{1}{2.3}$, $\frac{1}{3.4}$ &c. to infinity.

(2) $\frac{1}{2.3.4}$, $\frac{2}{3.4.5}$, $\frac{3}{4.5.6}$ to n terms.

15. In a bag there are a coins each worth £b, a' worth £b' each, and so on. If a person has a right to draw r coins, prove that the value of his expectation is

$$£r \times \frac{ab + a'b' + \ldots\ldots}{a + a' + \ldots\ldots}$$

CIV.

1. Express as decimals—

(1) $\frac{4}{10} + \frac{7}{1000} + \frac{8}{100000}$;

(2) $\frac{11}{4} - \frac{17}{8}$; (3) $\frac{13}{70}$.

Can you determine by inspection that the last fraction will give a recurring decimal?

2. Reduce $\frac{(2 \cdot 05)^2 \times 2 \cdot 24}{\cdot 0041}$.

3. Extract the square root of $529 \cdot 092004$.

4. Find the value of $x^2 - 5x + 7$ when $x = 3$; and explain why $(x-4)^2$ and $(4-x)^2$ have the same value for any integral value of x.

5. Simplify $3a + 5b - \frac{c}{2} - \left\{a - \frac{2b}{3} + \frac{c}{4}\right\}$

6. Multiply $x^4-4x^3y+6x^2y^2-4xy^3+y^4$ by $x^2+2xy+y^2$.

7. Reduce $\dfrac{6x^3-19x^2y+18xy^2-5y^3}{2x^2-3xy+y^2}$ to its lowest terms.

8. Solve the equations—

(1) $\dfrac{x-2}{5}-\dfrac{2x-7}{10}+15=\dfrac{16x-39}{10}$;

(2) $\dfrac{3x+2y-3}{4x-5y+16}=\dfrac{9}{4}$ and $3x=5y$.

9. A courier undertakes to perform a journey of 60 miles within 12 hours: he travels $6\frac{1}{2}$ miles the first hour, but afterwards in every successive hour he travels $\frac{1}{4}$ mile less than in the preceding hour. Will he perform his undertaking?

10. Find a third proportional to $\dfrac{2a^2b^2}{a^2+b^2}$ and ab.

11. Prove that the coefficient of x^{2n} in the expansion of

$$\frac{1}{(1-x)(1+x)^4} \text{ is } \frac{(n+1)(2n^2+n+3)}{3}.$$

12. Prove that $\log_a b\times\log_b a=1$; and that when $x=1$, $(\log x)^{\log x}$ is unity. Can the same number have more than one logarithm real or imaginary?

13. Obtain the equations connecting the numbers of degrees and grades in an angle, and also its circular measure. Find the circular measure of 179°.

14. Prove that $\sin 2A \sin A=\cos A-\cos A\cos 2A$; and express $\cot 2A$ in terms of $\cot A$.

15. A balloon was observed due S. and at an elevation of 36° at noon, and its shadow was 1 mile from the place of observation, the sun's altitude being 45°; at 11 A.M. and 1 P.M. the bearings of the balloon were SE. by S. and SW. by W. respectively. Find the velocity and direction of motion of the balloon, supposing it to move uniformly at a constant height. And show that the nearest distance of the balloon was $\dfrac{1+\sqrt{3}}{4}\sqrt{10+3\sqrt{2}}$ miles.

CV.

1. If a new unit of weight were instituted containing 1 lb. 1 oz. 3·326 drs. avoirdupois, how many of such units of weight would make 537 lbs. 11 oz. 15 drs.?

2. If a company, when its capital was £2000000, paid 7 per cent. to the shareholders, what is its capital now when it can pay no more than 3 per cent., although the profit to be divided is six times as much as it was in the former case?

3. Prove that

$$\frac{a(b+c)}{(a-b)(c-a)}+\frac{b(a+c)}{(a-b)(b-c)}+\frac{c(a+b)}{(c-a)(b-c)}=1.$$

4. Find the limits of value between which x must not lie to secure $4x^2+4x-1$ being positive.

5. Solve the equations—

(1) $\dfrac{31-6x}{15}+\dfrac{2x}{5}=\dfrac{3x-7}{5x+25}$;

(2) $\left.\begin{array}{l} ay^2+bxy-b=0 \\ bx^2+axy-a=0 \end{array}\right\}$

6. Six papers are to be set in an examination, two of them in mathematics; in how many different orders may the papers be given, provided only that the two mathematical papers are not successive?

7. If $\dfrac{1}{x^2+x-6}$ be expanded in ascending powers of x, find the coefficient of x^8.

8. Trace the changes in the sign of $\sin A \cos (A-15°)$ as the angle A increases from 0° to 360°.

9. Prove the formulæ—

(1) $\cos (A-B)=\cos A \cos B+\sin A \sin B$;

(2) $\dfrac{\sin A+\sin B}{\sin A-\sin B}=\tan \frac{1}{2}(A+B) \cot \frac{1}{2}(A-B)$.

10. Express $\sin A$ in terms of $\sin 2A$, and apply the result *to* find $\sin 105°$ from $\sin 210°=-\frac{1}{2}$.

11. Find the greatest angle of the triangle whose sides are 299, 325, and 442 yards.

12. If p_1, p_2, p_3 be the perpendiculars from the angles of a triangle upon the opposite sides, r the radius of the inscribed circle, r_1, r_2, r_3 the radii of the escribed circles, prove $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}=\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$.

13. Two towers stand on a horizontal plane 144 ft. apart; a person standing at the foot of each tower observes the elevation of one tower to be double that of the other, but when he is half way between them, the angle of elevation of one tower is the complement of that of the other. Show that the heights of the towers are 108 feet and 48 feet respectively.

14. Solve the equation $16\sin^4\theta-16\sin^2\theta+1=0$.

15. Construct a square three times as large as a given square.

CVI.

1. If a tradesman gains 2*s*. $3\frac{1}{2}$*d*. upon an article which costs him 7*s*. 6*d*., how much does he gain per cent.?

2. What difference is there between the annual income arising from £15000, invested in the $3\frac{1}{2}$ per cents. at $92\frac{7}{8}$, and the same sum invested in the 5 per cents. at $110\frac{1}{2}$?

3. Find the sum of the series 1^2, 2^2, 3^2, &c. to n terms.

4. Find the fifth root of 31 to 5 places of decimals by Hutton's method; and explain the principle of the rule.

5. In the expansion of the binomial $(a+x)^n$, where n is a positive integer, prove that the coefficients of the second term, and of the last but one, are each n.

6. Show that the sum of the squares of the coefficients in the expansion of $(a+b)^n$ by the binomial theorem $=\frac{1.2.3\ldots\ldots(2n)}{(1.2.3\ldots n)^2}$.

7. A person who receives an annuity of £500 puts it out

always at simple interest at 5 per cent. on the day that he receives it. In how many years will it amount to £6875?

8. Express the numbers 434, and 2170 in the quinary scale.

9. If a number be expressed in the ternary scale, how can you ascertain, by mere inspection, whether it is even or odd?

10. Prove that in any plane triangle:—

(1) $\sin \frac{1}{2}(A-B) = \cos \frac{1}{2} C \cdot \frac{a-b}{c}$.

(2) $\cos \frac{1}{2}(A-B) = \sin \frac{1}{2} C \cdot \frac{a+b}{c}$.

(3) $\frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)} = \frac{a+b}{a-b}$.

11. Prove the formula:—

$$\tan \frac{A+B}{2} \cdot \tan \frac{A-B}{2} = \frac{\cos B - \cos A}{\cos B + \cos A}.$$

12. In a right-angled triangle, given the side $a = 5618{\cdot}97$, and the opposite angle $A = 61^\circ\ 40'$; find the hypotenuse.

13. The sides of a right-angled triangle are 13 and 7543; obtain the formulæ for determining the angles with accuracy.

14. If R, r be the radii of the circles described about, and inscribed in a given triangle, O, O′ their centres; show, that

$$OO'^2 = R^2 - 2Rr.$$

15. The sides A B, A C of a triangle are bisected in E, F. A D is drawn perpendicular to the base B C. If E D, D F be joined, the angle F D E is equal to the angle B A C, and the area of the triangle A B C is double that of the quadrilateral A F D E.

CVII.

1. If £1 = 10 florins = 100 cents = 1000 mils, find the greatest error that could be incurred by applying the following rule for the conversion of shillings, pence, and farthings *into florins* cents and mils.

For every sixpence write 25 mils, and for every farthing in the remainder add 1 mil.

2. Extract the square root of ·025 and the cube root of 13312053.

3. Reduce the fraction $\frac{x^4-4x+3}{3x^5-20x^2+15x+2}$ to its lowest terms.

4. Solve the equations:—

$$(1)\ \frac{21-3x}{3}-\frac{4x+6}{9}=6-\frac{5x+1}{4}.$$

$$(2)\ 13y\sqrt{\frac{x^2}{y}+3}=6x^2+20y \text{ and } 24x^2+y^2=2x(5y+4x).$$

5. A man's age is 40 yrs., and that of his son 9 yrs.; what will be the age of the father when he is twice as old as the son?

6. Express the number 109, as also its double, treble, and quadruple in the binary scale of notation.

7. Expand $\sqrt{29}$ in the form of a continued fraction.

8. Find the value of $\left(\frac{1}{100002}\right)^{\frac{1}{5}}$ to 20 places of decimals by binomial theorem.

9. The sides of an isosceles triangle are 183, 183, and 197; find its angles.

10. The sides of a triangle are 5 and 7, its base 4 feet: find the length of the perpendicular drawn to the base from the opposite angle, by means of Euc. II. 13.

11. Prove (1) $\cos A=1-2\sin^2\frac{1}{2}A$.

$$(2)\ \frac{a}{b}=\frac{\sin A}{\sin B}.$$

$$(3)\ \cos nA+\cos(n+2A)=2\cos(n+1)A\cos A.$$

12. Find the side c and the angles A and B of the plane triangle in which $a=6383{\cdot}53$, $b=3157{\cdot}76$ and $C=37^\circ 26'$.

13. Show how to draw a straight line from a given point to make a given angle with a given plane. Is the problem determinate?

14. Describe a circle touching one side of a triangle, and the other two sides produced.

15. Given the centres of the three escribed circles of a triangle, construct the triangle.

CVIII.

Equations to be solved:—

1. $\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-1\frac{4}{5}}=0.$

2. $\frac{3x}{10}-\frac{y}{15}-\frac{4}{9}=\frac{x}{12}-\frac{y}{18}$ and $2x-\frac{8}{3}=\frac{x}{12}-\frac{y}{15}+\frac{11}{10}.$

3. $\begin{cases} x^{\frac{3}{5}}+y^{\frac{1}{2}}=35 \\ x^{\frac{1}{5}}+y^{\frac{1}{6}}=\ \ 5 \end{cases}$

4. A merchant made a mixture of wine, at 28*s.* a gallon, with brandy at 42*s.* a gallon; and he found that by selling the mixture at 35*s.* a gallon, he gained 15 per cent. on the price of the wine, and 20 per cent. on the price of the brandy. In what ratio were the wine and brandy mixed together?

5. Prove that $\log_{10} \text{N}=\log_{10} 5 \times \log_5 \text{N}$; and find, without tables, $\log_{25} 8$, having given $\log_{10} 2=\cdot 301030$.

6. Find, with the aid of tables, a third proportional to 1·6 and $\frac{\sqrt[3]{44}}{794\cdot 81}$.

7. Find the value of x from the equation $(1\cdot 05)^x=(8\cdot 25)^{\frac{1}{x}}$.

8. Assuming the form of the binomial theorem, show how to find the greatest term of $(1+x)^n$; and find the greatest term of $(1+\frac{1}{5})^{-7}$.

9. Prove that $\sin \frac{1}{2}\text{A}=\frac{1}{2}\{\pm\sqrt{1+\sin \text{A}}\pm\sqrt{1-\sin \text{A}}\}$

$\cos \frac{1}{2}\text{A}=\frac{1}{2}\{\pm\sqrt{1+\sin \text{A}}\mp\sqrt{1-\sin \text{A}}\}$

10. Express the signs correctly in *Quest.* 9 when A lies between 315° and 360°; and find the sine of 9°.

11. If any number of triangles on the same base B C and on the same side of it have their vertical angles equal, and

perpendiculars meeting in D be drawn from B, C, on the opposite sides, find the locus of D; and show that all the lines which bisect the angle B D C, pass through the same point.

12. The perimeters of two similar trapeziums are as 3 : 7, a side of one is 3 in., its area 774 square inches; find the area of the other.

13. Through any point D in the base of a triangle A B C, straight lines D E, D F are drawn parallel to the sides A B, A C, and meeting the sides in E, F, and E F is joined: prove that the triangle A E F is a mean proportional between the triangles F B D, E D C.

14. Given the segments of the base of a triangle and the line drawn from the vertex to the point of section, to describe the triangle.

15. Define the locus of an equation; and from the definition determine the locus of the equation $ax+by=c$.

CIX.

1. Add together $\frac{1}{9}$ guin., £$\frac{3}{8}$, $\frac{5}{24}s.$, and $\frac{1}{4}d.$; reduce the result to the decimal of £1.

2. Reduce $\cdot 7\dot{8}5714\dot{2}$ to a vulgar fraction; and divide 3·1 by ·0025, ·00062 by ·64.

3. Required the sum and difference of $4\sqrt{7}$ and $3\sqrt[3]{5}$ to 4 places of decimals.

4. Solve the equations—

$$(1)\ \frac{6x+1}{15}-\frac{2x-4}{7x-13}=\frac{2x-1}{5}.$$

$$(2)\ x(x+1)+3\sqrt{2x^2+6x+5}=25-2x.$$

5. Find by logarithms the fourth proportional to

$$\sqrt[3]{8\cdot 37},\ (\cdot 84)^2 \text{ and } \sqrt[5]{\cdot 054321}.$$

6. Given the sum of an arithmetical progression, the first term, and the common difference; find the number of terms.

Explain the double answer ; taking for example the case in which the first term is 4, the common difference 2, and the sum 18.

7. If $\pi a^2 \left(b - \frac{a}{3}\right)$ be given as the expression for finding the content of the segment of a sphere, what do a and b represent ?

8. An inverted conical vessel, whose height is equal to the diameter of its rim (4 inches), is supported when filled with water; on the top of it is placed a heavy sphere with a diameter of 5 inches. How much water will be left in the vessel ?

9. Given two sides of a triangle 7 and $9\frac{4}{5}$ miles, and the included angle 30° 20′; find the third side.

10. P is a station on the coast, A and B, two headlands at sea, P A 7 miles, P B, $9\frac{4}{5}$ miles, and the angle which the headlands subtend at P 30° 20′ ; in the line P A, 3 miles from P, a steamer is seen to start at the rate of 9 miles an hour. In what direction relatively to A P must it sail to be seen from P in the line P B after 40 minutes ?

11. A person at a horizontal distance of 400 yards from an elevated vertical straight line observes the angles of elevation of its ends to be 49° 27′ and 38° 51′. Find its length.

12. The angles of a plane triangle form a geometrical progression of which the ratio is $\frac{1}{2}$. Show that the greatest side is to the perimeter as 2 sin $12\frac{6}{7}$° is to unity.

13. In any triangle $\cot \frac{1}{2} A \cot \frac{1}{2} B = \frac{a+b+c}{a+b-c}$.

14. Prove that in general the change of the sine of an angle, and also the change of the L sin, are proportional to the change of the angle. State the exceptional cases.

15. Inscribe a sphere in a triangular pyramid.

CX.

1. If 800 soldiers cost £20000 in 9 months, what will be the cost of 1800 soldiers for a year?

2. A is a cask containing 136 gals. of wine; B, a cask containing 200 gals. of water: 68 gals. are drawn from each, mixed together, and the casks are filled up with the mixture. The operation is repeated. Find the quantity of wine then contained in each cask.

3. If $\left(\frac{y}{x}+\frac{b}{a}\right)\cdot\left(\frac{z}{x}+\frac{c}{a}\right)=\frac{bc}{a^2}$ show that $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=0$.

4. Express the square root of

$$(x+y)^2-\frac{2(x+y)^2}{xy}+\left(\frac{1}{x}+\frac{1}{y}\right)^2 \text{ in its simplest form.}$$

5. Solve the equations—

$$(1)\ \frac{x+4}{x-4}-\frac{x-4}{x+4}=\frac{8}{3}.$$

$$(2)\ \left.\begin{aligned} x^2+y^2+z^2&=50 \\ x&=y+z-6 \\ x(y+z)&=27 \end{aligned}\right\}$$

6. Given $\log_{10} 2=\cdot3010300$, find without tables $\log_{10} 125$.

7. Find by the aid of tables the sixth root of ·000000004096, and the fifth term of the series 5, 15, 45, 135 &c.

8. Find n, a whole number, such that $5^{\frac{1}{n}}$ shall have its nearest value to 1·307.

9. If n be a whole number, show that the fourth coefficient of $(x+a)^n$ is the same as the number of combinations of n things taken 3 at a time.

10. Find the eighth term of $\left(x+\frac{1}{x}\right)^{\frac{7}{2}}$.

11. Prove that

$$\left.\begin{aligned}\left(\frac{1+x}{1-x}\right)^n=1+n\cdot\frac{2x}{1+x}+\frac{n(n+1)}{1\cdot2}\left(\frac{2x}{1+x}\right)^2+ \\ \frac{n(n+1)(n+2)}{1\cdot2\cdot3}\cdot\left(\frac{2x}{1+x}\right)^3+\&c.\end{aligned}\right\}$$

12. Two circles touch each other; a common tangent is drawn to them at their point of contact; from any point in this tangent a tangent is drawn to each circle. Show that these latter tangents are equal to each other.

13. Write down in one formula all angles having $+\frac{1}{2}$ for their sine.

14. In a right-angled isosceles triangle the radius of the inscribed circle is 1 foot. Find the sides.

15. In any triangle

$$\left.\begin{array}{l}\dfrac{a^2-b^2}{c^2}\sin C+\dfrac{b^2-c^2}{a^2}\sin A+\dfrac{c^2-a^2}{b^2}\sin B+ \\ 4\sin\dfrac{A-B}{2}\sin\dfrac{B-C}{2}\sin\dfrac{C-A}{2}=0.\end{array}\right\}$$

CXI.

1. A sovereign, standard gold, weighs 5·136 dwts.; a shilling, standard silver, weighs $\frac{1}{86}$ lb. troy. What weight of standard silver is equal in value to 4 oz. of standard gold?

2. If a merchant with a capital of £20000 gain £500 in 3 months, what sum will he gain with a capital of £30000 in 7 months?

3. Find without tables the number of which $\cdot\dot{3}$ is the logarithm to the base 10.

4. Find by the aid of the tables,

$$(1)\ \frac{(7\cdot25)^{\frac{1}{3}}\times 1\cdot0046}{(\cdot0874)^2};$$

(2) The number of digits in $\dfrac{3^{20}\times 5^{15}}{2^{11}}$.

5. Prove that

$$\frac{1}{x(x-a)(x-b)}=\frac{1}{abx}+\frac{1}{a(a-b)(x-a)}+\frac{1}{b(b-a)(x-b)}.$$

6. Find in its simplest form the value of—

$$\sqrt{7+4\sqrt{3}}+\sqrt{7-4\sqrt{3}}.$$

7. Given $x^3-12x^2+44x-48=0$ and $y=x-2$; eliminate x and solve the resulting equation.

8. Show how to find the present value of an annuity of £A for a given number of years at a given rate compound interest. From the expression obtained, determine the number of years' purchase that should be given for a freehold estate, calculating the interest at 5 per cent.

9. Account for the formula

$$\frac{n(n+1)(2n+1)}{1\cdot 2\cdot 3}$$

always being integral when n is a positive integer.

10. Find the number of balls in an incomplete square pile of 20 courses having 44 balls in each side of the base.

11. Prove that the sine of an angle, or of an arc, is equal to the cosine of its complement.

12. In a right-angled triangle, C being the right angle,

$$\sec 2\text{A}=\frac{c^2}{b^2-a^2}.$$

13. Express vers θ in terms of cosec θ, and hence deter mine the numerical values of vers θ, when

$$\theta=0,\ \theta=\frac{\pi}{2},\ \theta=\pi.$$

14. The plane of the side of a hill running E. and W. is inclined to the horizon at an angle α. It is required to construct a straight road upon it inclined at an angle β to the horizon. Determine the point of the compass to which it must be directed.

15. The diagonals of a rhombus are 8·36 and 7·2 inches. Find its area.

CXII.

1. Divide ·011214 by 5·34 and ·00329875 by ·0754.

2. Add together $\frac{5}{18}$ guins., £$\frac{3}{32}$, $\frac{7}{20}$ crs., and $\frac{5}{16}$*s.*; and reduce the result to the decimal of £1.

3. If a, b, c be positive and unequal numbers, show that

$$(1)\ \frac{a}{b}+\frac{b}{a} > 2;$$

$$(2)\ (a+b+c)\,(ab+ac+bc) > 9\,abc.$$

4. Solve the equation $19x + 23y = 2000$, in positive integers.

5. Find by aid of tables—

$$\frac{(1{\cdot}086)^{11}-(1{\cdot}024)^{11}}{(365{\cdot}73)^{\frac{1}{7}}}$$

6. At what rate per cent. per annum will £500 amount to £740 2*s.* 4$\frac{4}{5}$*d.* in 10 years at compound interest?

7. Prove $\cos 3\,\text{A}=4\cos^3\text{A}-3\cos\text{A}$. Find $\sin 18°$ and thence deduce $\cos 36°$.

8. Find the radius of the circle touching one side of a plane triangle and the other sides produced.

9. Two railways intersect at an angle of 35° 20′; from the point of intersection two trains start together, one at the rate of 30 miles an hour; find the rate of the other train so that after 2½ hrs. the trains may be 50 miles apart.

10. Show that there are two velocities that will satisfy the condition in *Quest.* 9.

11. If the four sides of a quadrilateral figure are all equal, show that the diagonals bisect each other at right angles.

12. In any triangle show that the sum of the squares on the two sides is equal to twice the square on half the base, together with twice the square on the line drawn from the vertex to the middle point of the base.

13. What is the equation of a right line passing through the two given points (x', y'), (x'', y'')?

14. What is the length of the perpendicular drawn from the point (x', y') upon the line $x\cos a+y\sin a-p=0$, the axes being rectangular?

15. What is the area of the triangle formed by joining the three points (x', y'), (x'', y''), (x''', y''')?

CXIII.

1. Find the cost of 7 cwts. 2 qrs. 14 lbs., at £1 9*s.* 2*d.* per cwt.

2. Reduce £3 11*s.* $9\frac{3}{4}$*d.* to the decimal of £1; and to the decimal of £2 10*s.*

3. The length of a room is 7 yards 1 foot 3 inches, the breadth is 5 yards 2 feet 9 inches, and the height 4 yards 6 inches. Find the expense of papering the walls with paper 1 yard wide, at 9*d.* per yard.

4. Reduce $\dfrac{x^3+3ax^2+3a^2x+a^3}{x^6-3a^2x^4+3a^4x^2-a^6}$ to its lowest terms.

5. If $x+x^{-1}=2a$; $x-x^{-1}=2b\sqrt{-1}$; $y+y^{-1}=2c$; $y-y^{-1}=2d\sqrt{-1}$; find $xy+(xy)^{-1}$, and show that $(ac-bd)^2+(bc+ad)^2=1$.

6. Solve the equations—

(1) $\dfrac{x^2}{y^2}+\dfrac{y^2}{x^2}+\dfrac{x}{y}-\dfrac{y}{x}=\dfrac{106}{9}$ and $xy=3$.

(2) $x^4-2x^3-6x^2+8x+8=0$, one root being $1+\sqrt{3}$.

7. Calculate by logarithms—

$$\frac{\sqrt[3]{172500}}{\sqrt[5]{\cdot 01}}.$$

8. Find the time in which a sum of money will be doubled at $3\frac{1}{2}$ per cent. compound interest.

9. Find the greatest term in the expansion of $(1+x)^n$.

Ex. $n=-12$, $x=\frac{1}{5}$.

10. A circle is described about a triangle ABC, which has the side AB=side AC; from A a line is drawn meeting the base of the triangle in D and the circumference of the circle in E. Prove that the circle which passes through B, D, E touches AB.

11. ABCD is a parallelogram, and F a point in the diagonal AC; through F a line is drawn parallel to AB, meeting AD in G, and BC in H; and through F a line is drawn parallel to AD, meeting AB in E, and DC in K. Show that GK and EH produced meet in AC produced.

12. AB is the diameter of a circle, P a point in the circumference; from any point, C, in the diameter AB draw CD perpendicular to AB, meeting the circumference in F, and meeting AP and BP, produced if necessary, in D and E. Prove CD a third proportional to CE and CF.

13. Find the equation to the circle, with radius r having its centre on the axis of x, and passing through the origin.

14. When does the equation $Ay^2+Bxy+Cx^2+Dy+Ex+F=0$ represent a circle, the axes being oblique?

15. What is the equation to the circle circumscribing the triangle whose sides have $\alpha=0$, $\beta=0$, $\gamma=0$, for their equations?

CXIV.

1. If the cost of a cavalry soldier be to that of an infantry soldier as 5 : 2, find the annual cost of 15,000 cavalry, when 750 infantry cost annually £36000.

2. Find by the aid of tables a mean proportional between $(2{\cdot}0736)^{\frac{1}{2}}$ and $({\cdot}000000262144)^{\frac{1}{3}}$.

3. Required the amount of £3650 10*s.* at $5\frac{1}{2}$ per cent. compound interest, in 3 years, interest payable quarterly.

4. Construct an arithmetical progression in which the fifth term from the beginning is 2, and the third from the end -2, the number of terms being 9.

5. Show that—

$$\frac{(1-xy)(1+xy)-(x-y)(x+y)}{(1+xy)^2+(x-y)^2}=\frac{1-x^2}{1+x^2}.$$

6. If $a=\dfrac{-1+\sqrt{-3}}{2}$, prove that

$$\frac{1}{a}=\frac{-1-\sqrt{-3}}{2}, \text{ and } a^3+\frac{1}{a^3}=2.$$

7. Solve the equations—

$$(1)\ \frac{5x}{x-2}-\frac{8}{x+1}=5+\frac{3(2x+1)}{x^2-4}.$$

$$(2)\ x+2\sqrt{xy}+y=143+2\sqrt{x}+2\sqrt{y} \text{ and } 3\sqrt[4]{xy}-\sqrt{y}=9.$$

8. If $x=p+1$ show that $x^6 7 p(x^5+x^4+x^3+x^2+x+1)$.

9. In a garrison of 990 men there was a certain quantity of gunpowder, which was calculated to last a certain number of weeks, at the rate of $\frac{1}{2}$ lb. a week for each man, but after the first weekly distribution 10 men died weekly, and it was found that the powder lasted twice as long as was at first calculated. Find the quantity of powder at first. How long did it last ?

10. What are Napierian logarithms ? Given the Napierian logarithm of a number, show how to find its common logarithm.

11. Obtain the series—

$$\log_e m = 2\left\{\frac{m-1}{m+1} + \frac{1}{3}\left(\frac{m-1}{m+1}\right)^3 + \frac{1}{5}\left(\frac{m-1}{m+1}\right)^5 + \&c. \right\}$$

12. If the middle points of the adjacent sides of two similar trapeziums be joined, show that the figures thus formed are parallelograms, whose areas are in the same ratio as those of the trapeziums.

13. In a parabola, show that the abscissa varies as the square of the semi-ordinate to the axis.

14. Find the area of the portion of a parabola included between two ordinates to the axis, one passing through the focus of the length of 12 ft., the other ordinate being 24 ft. in length.

15. Prove the *latus rectum* of an ellipse a third proportional to the major and minor axes ; and find the *latus rectum* and area of the ellipse whose equation is—

$$25y^2+16x^2=160x.$$

CXV.

1. A legacy of £897 1*s*. is to be divided among 3 persons, A, B, C ; A is to have $\frac{1}{7}$, B $\frac{2}{3}$, and C the remainder. Find the sum received by each, and the fraction of the whole paid to C.

2. In what number of years will £1876 10*s*. amount to £2439 9*s*., at 4 per cent. simple interest?

3. A wine merchant mixes two kinds of wine, and sells the mixture so as to gain 8 per cent.: had he sold each kind of wine at the same price per gallon as he sells the mixture, he would have gained 10 per cent. and 6 per cent. respectively on the cost price of each. In what ratio were the two kinds mixed?

4. Explain what is meant by systems of logarithms calculated to different bases: what is the base of each of the systems most commonly used in mathematical or arithmetical calculations? State briefly any advantage belonging to these systems: show that $\log_2 N = 3 \log_8 N$.

5. Given $\log_{10} 2 = \cdot 3010300$; express without reference to tables the logarithm of the mean proportional between $\dfrac{(1\cdot 25)^{\frac{1}{3}}}{(\cdot 00016)^6}$ and $\dfrac{(1\cdot 25)^{\frac{2}{3}}}{1000}$.

6. Find by the aid of the tables the sum of 10 terms of the series $1 + \dfrac{17}{16} + \dfrac{17^2}{16^2} + \&c.$

7. Find the two simple factors which will divide without remainder $a^2 - b^2 + c^2 - d^2 - 2(ac - bd)$.

8. Extract the square root of—

$$\frac{x^2}{y^2} + \frac{y^2}{x^2} - \frac{1}{\sqrt{-1}}\left(\frac{x}{y} - \frac{y}{x}\right) - \frac{9}{4}.$$

9. Solve the equations—

(1) $3x + 5y - 70 = \dfrac{x}{5} + \dfrac{8y}{3} = x + y + 8.$

(2) $\dfrac{(x+y)^2}{a^2} + \dfrac{(x-y)^2}{b^2} = 8$ and $x^2 + y^2 = 2(a^2 + b^2)$.

10. Find $\sin 15°$, and thence prove that $\tan 15° = 2 - \sqrt{3}$.

11. Prove that—

$$\frac{\cos A + \cos B}{\cos B - \cos A} = \cot \tfrac{1}{2}(A+B) \cot \tfrac{1}{2}(A-B).$$

12. If in a triangle the angle A and the sides a and b are given to solve it, show that the third side c may be obtained from the expression $c=b\cos A\pm\sqrt{a^2-b^2\sin^2 A}$, and thence explain the ambiguity of the expression when b is greater than a.

13. The equation to a circle referred to rectangular co-ordinates being $2x^2+2y^2+8x-12y=6$, find its radius and the position of its centre.

14. Find the length of the perpendicular let fall from the point $(3,-5)$ upon the line $7x-3y=9$.

15. Give Napier's rules for the solution of right-angled spherical triangles.

CXVI.

1. The average year of the Gregorian calendar is greater than the true year by $24{\cdot}3648''$; find in days and decimals of a day the length of the true year.

2. The income derived by a legatee from money invested in his behalf in the 3 per cents., at $94\frac{1}{2}$, is £68 3*s.* 6*d.* What is the amount of the legacy?

3. Rationalise the denominator of the fraction—

$$\frac{1}{a-\sqrt{b}+\sqrt{c}}.$$

4. Prove that the difference of the fractions

$$\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}},\quad \frac{2\sqrt{2}+\sqrt{3}-1}{\sqrt{3}+1}$$

exceeds their product by 1.

5. If $x:a::y:b::z:c$ &c., prove that—

$$mx+ny+pz+\&c. : ma+nb+pc+\&c. :: x:a.$$

6. Solve the equation—

$$\frac{\sqrt{3x+1}+\sqrt{3x}}{\sqrt{3x+1}-\sqrt{3x}}=4.$$

7. Write down as a series in geometric progression the recurring decimal ·PPQQQ . . . . . where P,Q contain p,q digits

respectively: hence find its value in the form of a fraction. Deduce the ordinary rule for obtaining such value.

8. Define the sine of an angle, and prove $\sin A = \sin(n \cdot 180^\circ + (-1)^n A)$, where n is any positive or negative integer. Write down all angles between -800° and 800° which have $\cdot 5$ for the value of their sine.

9. Obtain the equation for determining $\sin A$ in terms of $\sin 3A$, and show why three values are obtained.

10. Prove from equation in *Quest.* 9 the formulæ—

(1) $\left.\begin{array}{r} 4\sin A \sin(60^\circ - A) + 3 = 4 \sin A \sin(60^\circ + A) + \\ 4 \sin(60^\circ - A)\sin(60^\circ + A) \end{array}\right\}$

(2) $4 \sin A \sin(60^\circ - A)\sin(60^\circ + A) = \sin 3A$.

11. The angle between the lines bisecting two adjacent exterior angles of any quadrilateral is equal to the angle between the lines bisecting the two interior opposite angles.

12. If through the extremities of a chord common to any number of circles two lines be drawn cutting the circles, the lines joining their other points of intersection with each circle are parallel.

13. If a and b are the sides, c the hypothenuse of a right-angled spherical triangle, prove that $\cos c = \cos a \cdot \cos b$.

14. Find the length of the perpendicular drawn from the point (3, 4) on the right line $\frac{x}{3} - \frac{y}{4} - 1 = 0$, the axes being rectangular.

15. Find by algebraic geometry the locus of a point the square of the distance of which from a given point is proportional to its distance from a given right line.

CXVII.

1. Add together the fractions $\frac{5}{8}$, $\frac{13}{16}$, $\frac{119}{125}$, $\frac{598}{625}$; and express the result as a decimal.

2. The cost of digging a trench varies as the product of the depth to which it is sunk, and the quantity of earth thrown out; find the cost of digging a trench 180 yds. long,

6 ft. broad, and 12 ft. deep, if the cost of digging one 4 ft. broad and 9 ft. deep be 1*s.* 3*d.* for each yard of length.

3. Of what number is -5 the logarithm to the base 10? What is the logarithm of 256 to the base 2?

4. Given $\log_{10} 2 = \cdot 3010300$, $\log_{10} 7 = \cdot 845098$, find, without using tables, $\log_{10} \cdot 0035$.

5. Prove that—

$$\frac{a^2-bc}{(a+b)(a+c)}+\frac{b^2-ca}{(b+a)(b+c)}+\frac{c^2-ab}{(c+a)(c+b)}=0.$$

6. Solve the equations:—

$$(1)\ \frac{x^2-(a+b)x-bc}{x-b}=\frac{x^2-(a+c)x-bc}{x-c}.$$

$$(2)\ x^4-\frac{5x^3}{6}-\frac{19x^2}{3}-\frac{5x}{6}+1=0.$$

7. (1) Find the product of the first 9 terms of the series

$$\frac{a^4}{b^4},\ \frac{a^3}{b^3},\ \frac{a^2}{b^2};$$

(2) Find the geometrical mean between $a+b$ and $a^3-a^2b-ab^2+b^3$.

8. Write down the 4 least values of θ which satisfy the equation $\tan\theta=1$.

9. Prove that $\tan\left(30^\circ+\frac{\theta}{2}\right)\tan\left(30^\circ-\frac{\theta}{2}\right)=\frac{2-\sec\theta}{2+\sec\theta}$.

10. One side of a quadrilateral inscribed in a circle is 40 ft., the angles which it makes with its adjacent sides are 90° and 75° respectively; if the radius of the circle be 25 ft., find the area of the figure.

11. Given two straight lines, construct the rhombus of which they are the diagonals; and show that it is greater than any other parallelogram having the same straight lines for diagonals.

12. Q is a point without a circle; Q A B, Q C D, are two lines at right angles, cutting the circumference in A and B, C and D; take the arc D E equal to A C; prove the whole B A C D E a semicircle.

13. Find the coordinates of the points in which the line $3x+7y=1$ meets the two rectangular axes.

14. Ascertain whether the circles represented by the equations

$$x^2+y^2-\ \ 6x-6y+13=0$$
$$x^2+y^2-14x-4y+43=0 \text{ intersect or not.}$$

15. If the circles in *Quest.* 14 intersect, find the equation to their common chord, and prove that it is perpendicular to the straight line joining the centres of the circles.

CXVIII.

1. Find the value of 28 cwts. 3 qrs. 21 lbs. at £5 9*s*. 6*d*. per cwt.

2. Find the value of ·334375 of 20*s*.; of ·3275 of 1 day; and of ·4765625 of 1 mile.

3. Find the sum and difference of $\cdot\dot{6}$ and $\cdot\dot{2}9\dot{6}$ both as fractions and recurring decimals.

4. Divide $x+y+z-3\sqrt[3]{xyz}$ by $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}$.

5. Reduce to its simplest form—

$$\frac{x^2+\left(\frac{a}{b}+\frac{b}{a}\right)xy+y^2}{x^2+\left(\frac{a}{b}-\frac{b}{a}\right)xy-y^2}.$$

6. If $\left.\begin{array}{l} x^2-yz=a^2 \\ y^2-xz=b^2 \\ z^2-xy=c^2 \end{array}\right\}$ show that $x+y+z=\dfrac{a^2x+b^2y+c^2z}{a^2+b^2+c^2}$.

7. Calculate by logarithms—

$$(1)\ \frac{70{\cdot}25\times{\cdot}67957}{21000\times{\cdot}018252}. \qquad (2)\ \frac{42\times({\cdot}0016)^{\frac{7}{4}}}{\sqrt[3]{108}}.$$

8. Assuming the binomial theorem, examine the condition that $(1+x)^n$ may converge.

9. In any triangle right-angled at C

$$\sin^2 \tfrac{1}{2}\text{A}-\sin^2 \tfrac{1}{2}\text{B}=\frac{a-b}{2c}.$$

10. In any triangle the radius of the circumscribing circle

$$=\frac{a+b+c}{2(\sin A+\sin B+\sin C)}.$$

11. In an oblique plane triangle ABC, given the angle ACB$=139^\circ\ 58'$, ABC$=22^\circ\ 10'$, BC$=840{\cdot}5$ yds.; determine by how much AB differs from a mile.

12. A is the summit of a hill of which B is the foot; the lower part, BC, of the hill ($840{\cdot}5$ yds.) is a plane inclined to the horizon at $12^\circ\ 10'$; the elevation of the top of the hill, seen from B, is $34^\circ\ 20'$, and seen from C is $52^\circ\ 12'$. Find the perpendicular height of A above the horizontal plane through B.

13. Prove that great circles in a sphere are bisected at their intersections.

14. Find the area of a spherical triangle in terms of the spherical excess; and obtain an expression for the latter, in a form adapted to logarithmic calculation, in terms of the sides.

15. Find the equation to the ellipse referred to polar coordinates, the origin being the focus.

CXIX.

1. (1) Reduce $18\frac{3}{4}$ lbs. to the fraction of 1 ton;
   (2) 11 yds. to the decimal of 1 mile; and
   (3) $\frac{2864400}{6745200}$ to its lowest terms.

2. If £1000 of 3 per cent. stock, at 72, be transferred to the 4 per cents. at 90, find the alteration of income.

3. Find the value of $\sqrt{\cdot 02}-\sqrt{\cdot 002}$.

4. Divide (1) $x^6-6x^4+9x^2-4$ by x^2-1;
   (2) $1+2x$ by $1-3x$ to 4 terms.

5. Multiply $a^{\frac{1}{2}}+b^{\frac{1}{2}}+a^{-\frac{1}{2}}b$ by $ab^{-\frac{1}{2}}-a^{\frac{1}{2}}+b^{\frac{1}{2}}$.

6. Solve the equations:—
   (1) $(x+a)^5-(x-a)^5=242a^5$.
   (2) $\left.\begin{aligned} xy &= 3(x+y) \\ xz &= 8(x+z) \\ 7yz &= 24(y+z) \end{aligned}\right\}$

7. Develope $(1+\sqrt{-3})^6$; and exhibit the result in its simplest form.

8. An officer desires to draw up a regiment of 1,000 strong in a hollow square with the men standing 5 deep on each side of it; how many men must be in the outer side of the square?

9. The number of shot at the base of a rectangular pile being 220, and along the ridge 10, find the number in each side of the base, and in the whole pile.

10. Prove the formulæ:—

$$(1)\ c^2=a^2-2ab\cos \mathrm{C}+b^2;$$
$$(2)\ 2\sin^2 \mathrm{A}=1-\cos 2\mathrm{A};$$
$$(3)\ \cot \tfrac{1}{2}\mathrm{A}=\operatorname{cosec} \mathrm{A}+\cot \mathrm{A}.$$

11. The parallel sides of a trapezoid are 8 ft. and 3 ft.; its two other sides are 7 ft. and 5 ft.; find its area.

12. Given the radii of the two ends of a frustum of a right cone, made by a section perpendicular to the axis, and the distance between them; find its volume, and the area of its curved surface.

13. Calculate the sides of a spherical triangle in which the angles are 90°, 60°, and 36°.

14. Find the equation to the normal at any point of an ellipse.

15. PM is an ordinate at any point P of an ellipse; MP is produced to meet in Q the circle described on the axis major of the ellipse as diameter: find the distance from the centre of the ellipse, of the point of intersection of the normal to the ellipse at P, and the normal to the circle at Q.

CXX.

1. The French mètre is equivalent to 39·37009 inches English, and corresponds to the ten millionth part of the distance from the pole of the earth to the equator: find the *circumference* of the earth in miles, &c.

2. If 30 cubic inches of powder weigh 1 lb., what must be the internal diameter of a shell to contain 10 lbs. of powder?

3. Two workmen contract to do a piece of work for £25, which they reckon will take them 75 days to execute. After working together 45 days, one of them falls ill, and his mate completes the job in 50 days more. How much is each entitled to receive?

4. Find the periphery of a regular heptagon containing an area of 50 square feet.

5. Given $\sin 60^\circ\ 27'\ 30'' = \cdot 87$, to find $\cos 14^\circ\ 46'\ 15''$ to 4 places of decimals.

6. The three sides of a triangle being 357, 476, 595 respectively, find each of the angles independently, and verify the accuracy of the solution.

7. Given the base, a, the vertical angle, A, and the difference, d, of the sides of a plane triangle, to find the sides.

8. Prove the formulæ:—

(1) $\tan(45^\circ + \frac{1}{2}A) + \cot(45^\circ + \frac{1}{2}A) = 2 \sec A.$

(2) $\dfrac{\sin 60^\circ + \sin 30^\circ}{\sin 60^\circ - \sin 30^\circ} = \dfrac{\tan 60^\circ + \tan 45^\circ}{\tan 60^\circ - \tan 45^\circ}.$

9. If $\sin \theta = m \sin \phi$ and $\tan \theta = n \tan \phi$, find the sines and cosines of θ and ϕ.

10. On the bank of a river stands a tower 200 ft. high, on which is a flagstaff 30 ft. high, and at the bottom of the tower stands a man 6 ft. high. To an observer on the opposite bank the man and the flagstaff subtend equal angles. Find the width of the river.

11. Two circles, whose radii are r and r', intersect; the distance between their centres is c: find the length of their common chord.

12. If circles be described about each of the two pairs of triangles into which any quadrilateral is divided by its diagonals, their centres will be the angular points of a parallelogram.

13. Find the equation to a straight line passing through a given point, and at right angles to a given straight line.

14. All the parallelograms circumscribing a given ellipse are equal to one another.

15. Find the polar equation to a parabola, a focus being the pole.

CXXI.

1. An American dollar is 4*s.* 3·6*d.*, or 5·42 francs; what is the smallest sum which can be paid in either shillings, dollars, or francs?

2. Two men set out at the same time from two places 60 miles apart, and travel, without stopping, continually backwards and forwards between those places at the rates of 4 and 5 miles an hour. Where will they meet for the second time, and where for the third time?

3. Prove that—

$$\frac{x-y}{x^{\frac{1}{3}}-y^{\frac{1}{3}}}-\frac{x+y}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}=2x^{\frac{1}{3}}y^{\frac{1}{3}}.$$

4. Solve the equations—

(1) $\dfrac{(x-2)(x-3)}{x-4}=12-\dfrac{x+1}{x-4}$.

(2) $\left.\begin{array}{r}4x^2+4y^2=17xy\\ \sqrt{x}+\sqrt{y}=3\end{array}\right\}$

(3) $x^2-4x+5=\sqrt{61-52x+11x^2}$.

5. Solve the equation—

$x^3-12x^2+57x-100=0$, by Cardan's method.

6. From 20 privates of a regiment, 6 corporals, and 4 sergeants, how many different guards may be formed, each consisting of 8 privates, 2 corporals, and 1 sergeant?

7. Given $\log_{10} 125 = 2{\cdot}0969100$, find without tables $\log_{10} 2$.

8. Find by the aid of tables the value of x, if

$$x=\frac{(7{\cdot}014)^3-1}{(7{\cdot}014)^3+1}.$$

9. Find without tables, to 3 places of decimals, the number of which $1\cdot\dot{3}$ is the logarithm.

10. Prove $\tan 2A = \dfrac{2 \tan A}{1-\tan^2 A}$.

11. Two sides of a triangle are 850, 960 feet respectively, and the included angle is 56° 13′; find the remaining angles.

12. An observer stands on a wall, so that his eye is 10 ft. above the horizontal plane on which a tower, whose height is 250 ft., stands; the tower subtends at the eye of the observer an angle of 56° 40′; find its horizontal distance from the wall.

13. Investigate the equation to the tangent to an ellipse, the ellipse being referred to its axes as axes of co-ordinates.

14. Find the equation to the tangent at any point of a parabola.

15. Determine the co-ordinates of a point on a parabola such that the perpendicular, from the focus, on the tangent at that point shall be equal to the *latus rectum.*

CXXII.

1. A grocer buys 106 lbs. of tea at 3*s.* 4*d.* per lb., 75 lbs. at 5*s.* 2*d.*, and 94 lbs. at 5*s.* 5*d.*; and, after mixing them, sells the mixture at 5*s.* per lb. How much does he gain in all, and how much per cent.?

2. A florin being the tenth part of £1, a cent the hundredth, and a mil the thousandth; find the value of 4875 times £24 6 fl. 3 c. 4 m.

3. Reduce $\frac{1}{2}+\frac{3}{4}-\frac{1}{8}+\frac{5}{16}-\frac{1}{32}$ to its simplest form, and convert the result into a decimal.

4. Divide 590·4825 by ·03275, and find the product of $\cdot 2\dot{3}$ and $\cdot\dot{1}\dot{5}$.

5. Find the amount of £382 10*s.* in $4\frac{1}{2}$ years, at 5 per cent. simple interest.

6. Extract the square root of 4782·02676484, and of $41\frac{184}{225}$.

7. Solve the equations—

$$(1)\ \frac{1}{24+x}=\frac{1}{13}+\frac{1}{11}+\frac{1}{x}.$$

$$(2)\ x^2-4y^2=36,\ xy+2y^2=12.$$

8. Three numbers, of which the sum is 12, are in A. P., and if 2, 5, 20 are added to them respectively, they are in G. P. Find the numbers.

9. From a company of soldiers mustering 100, a picket of 12 men is to be selected: determine in how many ways this can be done, (1) so as always to include 2 particular men, (2) so as always to exclude the same 2 men.

10. Show that one of the triangles in the figure Euc. IV. 10 is a mean proportional between the other two.

11. Describe a triangle similar to a given triangle and twice as large.

12. The radius of a circle is 10 feet, express in degrees the angle subtended at the centre of this circle by an arc 3 feet in length.

13. Prove the formulæ—

$$(1)\ \tan A+\cot A=2 \operatorname{cosec} 2A.$$

$$(2)\ \cos 10A+\cos 8A+3\cos 4A+3\cos 2A=8\cos A\cos^3 3A.$$

14. In any plane triangle ABC, show that

$$\tan^2\frac{A}{2}=\frac{(a+b-c)(a+c-b)}{(a+b+c)(b+c-a)} \text{ and if } C=90^\circ,\ \tan\frac{A}{2}=\frac{a}{b+c}.$$

15. Given $\sin 50^\circ=·766$; find $\sin 25^\circ$ without using the tables.

16. A tower standing on a horizontal plane is surrounded by a moat, which is as wide as the tower is high; a person on the top of another tower whose height is a, and whose distance from the moat is c, observes that the first tower subtends an angle of 45°. Find the height of the first tower.

17. Two hills rise at the same point with inclinations of

60° and 40° to the horizon, at a distance of 64 feet from the base of the lower hill ; the angles of elevation of the bottom and top of a vertical object on the other hill are 40° and 70°. Find the height of the object.

18. Define a pyramid. Find the content of a hexagonal pyramid ; side of base 10 feet, perpendicular height 15 feet.

CXXIII.

1. Find the cost of 256 lbs. of tea at 3*s.* $3\frac{3}{4}$*d.* per lb. If 16 lbs. of it be spoiled, how much will be gained or lost by selling the remainder at 4*s.* per lb. ?

2. Find the cost of an embankment 1 mile long 4 feet high and $13\frac{1}{2}$ feet wide, at 6*d.* per cubic yard for the first foot in height, and 1*d.* per yard extra for each successive foot in height.

3. One company guarantees to pay 5 per cent. on shares of £100 each ; another guarantees at the rate of $4\frac{5}{6}$ per cent. on shares of £7 10*s.* each ; the price of the former is £124 10*s.*, and of the latter £8 10*s.* Compare the rates of interest which they return to the purchaser.

4. Extract the $\sqrt{12}$ to 7 places of decimals, and show that after $n+1$ figures have been obtained by the ordinary method, n more may be obtained by division only ; $2n+1$ being a whole number.

5. If $a=\frac{5}{7}$, $b=\frac{1}{2}$, $x=5$, $y=\frac{9}{2}$, find the value of

$$10(a+2b)\sqrt{\{(x-b)y\}}-3a\sqrt[3]{y^2(x-b)}+5b.$$

6. Prove—

$$\left.\begin{array}{l}\{(a-b)^2+(b-c)^2+(c-a)^2\}^2= \\ 2\{(a-b)^4+(b-c)^4+(c-a)^4\}\end{array}\right\}$$

7. Prove the rule for finding the L. C. M. of two algebraical expressions, and find the L. C. M. of

$$x^3+2x^2y-xy^2-2y^3 \text{ and } x^3-2x^2y-xy^2+2y^3.$$

8. Simplify—

$$(1)\ \frac{a+b}{(b-c)(c-a)}+\frac{b+c}{(c-a)(a-b)}+\frac{c+a}{(a-b)(b-c)}.$$

(2) $\frac{a-b}{a+b}+\frac{b-c}{b+c}+\frac{c-a}{c+a}+\frac{(a-b)(b-c)(c-a)}{(a+b)(b+c)(c+a)}$.

9. Solve the equations—

(1) $\frac{6x+7}{15}-\frac{2x-2}{7x-6}=\frac{2x+1}{5}$;

(2) $\left.\begin{array}{l} x+y+z=a+b-c\,; \\ bx-cy+az=ay+bz-cx=ab-(a+b)c \end{array}\right\}$

(3) $(7+4\sqrt{3})\,x^2+(2+\sqrt{3})\,x=2$.

10. A railway train, after travelling 1 hr., meets with an accident which delays it 1 hr., after which it proceeds at $\frac{3}{5}$ of its former rate, and arrives 5 hrs. late; if the accident had occurred 50 miles further on the train would have been 3 hrs. 20 min. late. Find the length of the line.

11. Divide the number n into 4 parts in A. P. such that the sum of their cubes shall be $\frac{n^3}{10}$.

12. The sides of any quadrilateral figure are together greater than the sum of its diagonals.

13. The sides A B, A C of a triangle are bisected in D and E, and B E, C D are produced until E F=E B and G D=D C: show that the line G F passes through A.

14. Perpendiculars are let fall, from two opposite angles of a rectangle, upon a diagonal: show that they will divide the diagonal into equal parts, if the square on one side of the rectangle be double that on the other.

15. Define the tangent of an angle; trace the magnitude and sign of the tangent of an angle through the four quadrants.

16. Prove the formula $\cos(180°-A)=-\cos A$.

17. Solve the equation $\sin 7x-\sin x=\sin 3x$.

18. Given $a=175{\cdot}08$, $b=118{\cdot}14$, $B=38°\ 40'$; find the remaining parts of the triangle.

CXXIV.

1. A invests £9450 in the purchase of 3 per cent. stock at $87\frac{1}{2}$, and B invests the same sum in the 4 per cents. at $94\frac{1}{2}$: what will be the difference of their incomes after deducting 16*d.* in the £ income-tax?

2. Solve the equations, $\sqrt{x+y}+\sqrt{x-y}=a$, and

$$\sqrt{x^2+y^2}+\sqrt{x^2-y^2}=b^2.$$

3. If a, b, c, d be the four sides of any quadrilateral figure, δ_1, δ_2 the diagonals joining opposite angles, e the length of the line joining the middle points of the diagonals; prove that,

$$a^2+b^2+c^2+d^2=\delta_1{}^2+\delta_2{}^2+4e^2.$$

4. In the triangle A B C given A$=70°\ 20'$, B C$=956$, A C$=852$; and in the triangle A′ B′ C′ given A′$=75°\ 20'$, B′ C′$=852$, A′ C′$=956$: of these two triangles solve that which is not ambiguous.

5. At what distance may a light, which is 60 ft. above the level of the sea, be just seen from the deck of a ship 12 ft. above the level of the sea, the earth being considered a sphere of 4000 miles radius?

6. At the foot of a hill a visible object has an elevation of 29° 12′ 40″; when the observer has walked 300 yds. up the hill away from the object, he finds himself on a level with it. The slope of the hill being 16°, and the places of observation in a vertical plane with the object, find the distance of the object from the first place of observation.

7. An observer at A wishes to determine the distance between two inaccessible objects B, C. He measures a base A D, 1763 yds., and observes the angles, B A C$=45°\ 1'$, C A D$=30°$, B D C$=36°\ 15'$, B D A$=33°\ 8'$. Find B C.

8. A ditch is 5000 ft. long, 9 ft. deep, 14 ft. broad at top, and 11 ft. broad at the bottom; how many cubic feet of water will fill it? If half that quantity of water is supplied, how high will it rise?

9. Find the equation to the straight line passing through the origin and making an angle of 60° with the line $x+y\sqrt{3}-1=0$.

10. Find the polar equation to a circle, the origin being an external point and the initial line a diameter.

11. Find the locus of the intersection of the tangent to a parabola, with the perpendicular drawn to it from a given point on the axis.

12. Find a point in an ellipse such that the normal may bisect the angle included between the ordinate of that point and the radius drawn to the centre.

13. The line $lx+my=c$ intersects the hyperbola $a^2y^2-b^2x^2=-a^2b^2$: form the equation to the lines through the origin and points of intersection. Deduce the relation that must hold in order that the given line may touch the hyperbola.

14. Trace the curve given by the equation

$$4x^2+7xy+4y^2-8x-8y+4=0.$$

15. Define the differential coefficient of a function, and investigate the differential coefficients with respect to x of $\sqrt{ax}$ and of $\tan ax$.

16. Expand $e^x \cos x$, by Maclaurin's theorem, to 3 terms, writing down the general term.

17. Integrate the following functions of x

$$(1)\ \sqrt{\frac{a+x}{x}};\quad (2)\ \frac{x^2-1}{x^2-4};\quad (3)\ e^{ax}\sin cx.$$

18. Find the area of the loop of the curve,

$$y^2(a^2+x^2)=x^2(a^2-x^2).$$

CXXV.

1. Explain the distinction between interest and discount. What sum will produce for interest £113 8*s.* in $2\frac{1}{4}$ yrs. at $4\frac{1}{2}$ per cent. simple interest?

2. If 5 cylindrical pumps, each with a piston of 6 in. radius, and with a length of stroke of 3 ft., working 15 hrs. a day for 5 days, empty the water out of a mine; how many pumps, each with a piston of 9 in. radius and $2\frac{1}{2}$ ft. stroke, working 10 hrs. a day for 12 days, will be required

to empty the same mine, the strokes of all the pumps being performed in the same time?

3. A contractor has stores worth £300; he sells $\frac{1}{3}$ of them so as to lose 10 per cent.; what must he sell the remainder for so as to gain 20 per cent. on the whole?

4. Prove $\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-a)(b-c)}+\frac{c^2}{(c-a)(c-b)}=1.$

5. Solve the equations:—

(1) $\frac{x+8}{x-2}=3.\frac{x+2}{x-1}.$

(2) $xy+\frac{x}{y}=20$ and $\frac{1}{xy}+\frac{y}{x}=\frac{5}{16}.$

6. There is a number of 3 digits of which the first is to the second as the second to the third: the number itself divided by the sum of its digits$=10\frac{9}{13}$; and if 792 be added to the number the digits will be inverted. Find the number.

7. If two circles cut one another, show that there is a certain straight line, from any point of which exterior to the circles, the tangents drawn to both circles will be equal.

8. When two diagonals of a trapezium and the angle at which they intersect are given, show how to find its area.

Ex. ABCD is a trapezium, AC$=312{\cdot}4$, BD$=612{\cdot}5$, the angle at which AC, BD intersect is $105^\circ\ 20'$: find the area.

9. A regular triangular pyramid is contained by 4 equilateral triangles, the side of each triangle being 20 ft.; find the content of the pyramid.

10. If $\frac{x}{a}+\frac{y}{b}=1$ be the equation to a straight line, what do a and b represent? What is the condition that two straight lines shall be at right angles to each other?

11. Show that $2y^2-3xy-2x^2-7y-x+3=0$ represents two straight lines at right angles to each other.

12. Find the general rectangular equation to a given circle: construct the curve $x^2+y^2-6x-10y-15=0$; and find the equation to the diameter passing through the origin.

13. Find the distance of any point on the ellipse from the focus.

If P S p be any focal chord of an ellipse, S the focus, L the *latus rectum* of the ellipse; prove S P, S p= $\frac{L}{4}$(S P+S p).

14. Define the hyperbola. When are straight lines said to be asymptotes to curves? How may such asymptotes be detected algebraically? Find the equation to the hyperbola referred to its asymptotes.

15. Are the three angles of a spherical triangle sufficient data to enable us to determine the angular measure of the arcs which are the sides of the triangle? Are they sufficient to enable us to determine the lineal measure?

16. The area of an equilateral spherical triangle is $\frac{1}{4}$ of the surface of the sphere: find the sides and angles of the triangle.

17. Differentiate the functions—

$$(1)\ u=(a^2-x^2)^{\frac{3}{2}};\ (2)\ u=\log_e \frac{x^2+\sqrt{x^2-1}}{x^2-\sqrt{x^2-1}};$$

$$(3)\ u=\frac{\sin mx}{(\cos x)^m};\ (4)\ u=\sin^{-1}\frac{x}{\sqrt{1+x^2}}.$$

18. Find the least isosceles triangle which can circumscribe a given circle.

19. Integrate the functions—

$$(1)\int\frac{dx}{x^3(a+bx)};\ (2)\int\frac{x^3dx}{\sqrt{1-x^2}};\ (3)\int dx\,(\sin x)^2.$$

20. Find the differential of the surface of a solid of revolution, and hence find the surface of a given right cone.

CXXVI.

1. The stuff out of a lead mine contains at first 15·9 per cent. of lead; after washing, by which process the amount of lead ore is not diminished, the stuff contains 87·45 per cent. of lead; how much rock was washed away out of 216 tons 5 cwt. of the original stuff?

2. If $y+z : 3b-c :: z+x : 3c-a :: x+y : 3a-b$, then
$$x+y+z : ax+by+cz :: a+b+c : a^2+b^2+c^2.$$

3. Given $x^3-24x-72=0$ to find x by Cardan's method.

4. State and prove the relations between the coefficients and roots of the equation—
$$P_n x^n + P_{n-1} x^{n-1} + P_{n-2} x^{n-2} + \ldots\ldots P_1 x + P_0 = 0.$$

5. Prove that the roots of the equation $x^3+prx-r^2=0$ are the products of every pair of the roots of the equation $x^3+px^2+r=0$.

6. Show that the sum of the cubes of the roots of the equation,
$$x^n+p_1x^{n-1}+p_2x^{n-2}+p_3x^{n-3}+\ldots\ldots=0 \text{ is } 3p_1p_2-3p_3-p_1^3.$$

7. Find the value of $\dfrac{(4{\cdot}8755)^7-1}{108}$ by means of logarithmic tables.

8. Obtain formulæ, adapted to logarithmic computation, for determining the angles of a triangle when its sides are known. If the sides be 2 ft., 8 ft., and 9·99992 ft., show that the cosine of $\frac{1}{2}$ the greatest angle is approximately $\frac{1}{200}$; and that the angle contains approximately $\dfrac{180\pi-1{\cdot}8}{\pi}$ degrees.

9. Explain the observations necessary to be made for determining the height of a visible but inaccessible object; and compare the error, if an observed altitude of the object be erroneous by 1″, with the true height.

10. A captain, marching on the left of his company, observes another company, of equal strength with his own, advancing from the right in a direction cutting his line of

march at right angles; he observes the angle subtended by this company at his eye to be 7° 30′, and the angle between his own line of march and the line joining himself with the other captain, who is also on the left of his company, to be 60°. Find the distance of each captain from the line of march of the other, the breadth of each company being 52 ft.

11. Show how to obtain the content of a rectangular parallelopiped in terms of the three edges that meet in an angle, explaining the relation between the lineal and the solid units.

12. Find the content of a pyramid formed by cutting off a solid angle of the cube, whose side is 20 feet, by a plane bisecting its three conterminous edges.

13. In any spherical triangle show that the sines of the angles are proportional to the sines of the opposite sides.

14. Prove the formulæ—

$$\tan \tfrac{1}{2}(A+B)=\frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \tfrac{1}{2}C.$$

$$\tan \tfrac{1}{2}(A-B)=\frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \tfrac{1}{2}C.$$

15. A and B are two fixed points, and P, a point such that AP always bears to BP a constant ratio: find the locus of P.

16. Find the differential coefficient of $\sqrt{a^2-x^2}$ with respect to x by first principles.

17. Find the values of x which make $x(a-x)^2(2a-x)^3$ a maximum or a minimum.

18. Find the value of the radius of curvature at any point of a curve; for example, a parabola at its vertex.

CXXVII.

1. Find the value of 869 cwts. 2 qrs., at £2 17*s.* 7½*d.* per cwt.

2. What fractions are reducible to terminating decimals? *Is* $\frac{91}{560}$ *so* reducible?

3. (1) Divide ·213419596 by ·0100103; (2) reduce $5\cdot\dot{7}\dot{8}$ to a vulgar fraction.

4. Extract the square root, (1) of ·0531118116;

$$(2) \text{ of } \frac{\cdot 00125}{\cdot 18}.$$

5. Solve the equations—

$$(1)\ \frac{x+a}{x+b}=\left(\frac{2x+a+c}{2x+b+c}\right)^2.$$

$$(2)\ x^4+x^3-4x^2+x+1=0.$$

6. Show how to find the sum of the cubes of the natural numbers to n terms.

7. The interior angles of a right-lined figure are in A. P.; the largest angle is 160°, and the common difference is 5°; find the number of sides.

8. Calculate by logarithms—

$$\frac{1\cdot 265\times \cdot 00271}{2\cdot 382\times 10\cdot 71}.$$

9. Find the amount of £500 in 10 years, at 4 per cent. compound interest.

10. Given the cotangent of an angle, find its sine and secant.

Cot $5A=\frac{1}{\sqrt{3}}$; find sin $5A$, and express generally the values of the angle A.

11. P is a point without any number of concentric circles; O is their common centre; from P any number of tangents are drawn to the circles: show that the circle described in PO as a diameter will pass through all the points of contact.

12. If any two consecutive sides of a hexagon inscribed in a circle are respectively parallel to their opposite sides, the remaining sides are parallel to each other.

13. Draw the figure representing the position of the straight line whose equation, referred to rectangular coordinates, is $3y-5x+7=0$.

14. Ascertain whether the right line $\frac{x}{5}+\frac{y}{4}-1=0$ cuts the circle $x^2+y^2-2x-4y+\frac{17}{4}=0$.

15. $x^2-ax=by$ being the equation of a parabola, find the equation of its direction, and the coordinates of its focus.

16. Prove that $\frac{d(\tan x)}{dx}=\sec^2 x$.

17. Find the differentials of the following expressions :—

(1) $\cos\sqrt{x^2+a^2}$; (2) $\sin\sqrt{x^2+a^2}$; (3) $\log\sqrt{x^2+a^2}$.

18. A given sum £a is to be laid out in the purchase of a rectangular piece of ground and in building a wall round it. The wall on one side will cost £m per lineal foot, and on the other three sides £n per foot ; and the land will cost £p per square foot. How may the greatest piece of land be enclosed subject to these conditions ?

CXXVIII.

1. Solve the equation $x^4-8x^3+14x^2+8x-15=0$, whose roots are in arithmetical progression.

2. Show that the reciprocals of quantities in harmonical progression are in arithmetical progression.

3. Find the harmonic mean between a and $\frac{ab}{2a-b}$.

4. If $\frac{a+b}{c}$, $\frac{a+c}{b}$, $\frac{b+c}{a}$ are in A. P., prove that a, b, c are in H. P.

5. Assuming the binomial theorem, express the ratio of the $(r+1)$th term to the rth term ; and find the greatest term in the expansion of $(5+\frac{1}{2})^8$.

6. Prove that $\tan A=\frac{3\tan\frac{1}{3}A-\tan^3\frac{1}{3}A}{1-3\tan^2\frac{1}{3}A}$.

7. In a plane triangle ABC given AB = 1114·5 feet, $\angle$CAB=38° 41′, $\angle$CBA=45° 39′ ; find BC.

8. A tower 300 feet high, of which C is the top, stands on

a horizontal plane ; a horizontal base A B $=1114\frac{1}{2}$ feet is measured, but not in the same vertical plane as the tower ; the $\angle s$ C A B and C B A are observed and found to be respectively $38^\circ\ 41'$, and $45^\circ\ 39'$. Find the nearest distance of the foot of the tower from A B.

9. Show that unity is the limiting value of $\frac{\sin\theta}{\theta}$ as θ is indefinitely diminished. Investigate a method of obtaining an expression for the area of a circle of given radius.

10. A B C is a triangle, C an obtuse angle, B F the perpendicular on A C produced ; if in the base A C a point E be taken, such that A C$=2$ E F; prove $AB^2+BC^2=2\,(AE^2+BE^2)$.

11. Prove that the rectangle contained by a side of the isosceles triangle Euc. IV. 10, and the base, together with the square on the base, is equal to the square on a side of the triangle.

12. In an isosceles triangle show that the base is a mean proportional between the diameter of the circle inscribed in the triangle and the diameter of the circle that is escribed touching the base, and the equal sides produced.

13. In any spherical triangle a, b, c being the sides, A, B, C the angles, prove that—

$$\tan\tfrac{1}{2}(a-b)=\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\,.\,\tan\tfrac{1}{2}c.$$

14. Find the equation to a straight line (1) drawn from the origin and making an angle of 45° with the straight line $y=ax+b$, (2) to a straight line bisecting the angle between two given straight lines passing through the origin, and explain why the problem admits two solutions.

15. Show that the tangent to a parabola will meet the directrix and *latus rectum* produced in two points equally distant from the focus.

16. If $u=\frac{f(x)}{\phi(x)}$ deduce the rule for finding the differential coefficient of u with respect to x.

Find $\frac{du}{dx}$ in the following examples :—

(1) $u=\log_e(x+\sqrt{a^2+x^2})$;

(2) $u=\tan^{-1}\frac{2x}{1-x^2}$.

17. Show that $u=\phi(x)$ will have a maximum value, when a root of $\frac{du}{dx}=0$ substituted in $\frac{d^2u}{dx^2}$ gives a negative result.

Can there be a maximum value of—

u if $\frac{d^2u}{dx^2}=0$, for the same value of x that makes $\frac{du}{dx}=0$?

18. Given the volume of a right cone, find its altitude when the surface is a maximum.

CXXIX.

1. In the 3 per cent. stock what fraction of a given amount of stock is paid for annual interest, (1) without deduction, (2) after 9*d.* in the pound has been deducted for income-tax?

2. What is the amount of stock for which £115 10*s.* is paid as annual interest, after 9*d.* in the £ has been deducted?

3. If an ounce of standard gold, of which the weight of the alloy is 2 parts out of 12, be worth £3 17*s.* 6*d.*, what is the value of 10 lbs of gold in which the weight of the alloy is represented by the decimal $\cdot 41\dot{6}$, the value of the alloy being neglected.

4. What is the base of the system in which log 10=2? What is the value of log 0?—does it vary with the base?

5. Find by the aid of the tables, $\frac{(3\cdot 1416)^{12}}{(2\cdot 1782)^{20}}$.

6. Find the amount of £680 10*s.* in 20 years, at $3\frac{1}{2}$ per cent. per annum compound interest, payable half-yearly.

7. Solve the equations—

$$\left.\begin{array}{l}(1)\ x^3-7x-6=0\\(2)\ 3x^3+4x^2-35x-12=0\end{array}\right\}\text{ which have a common root.}$$

8. If n be a positive whole number, find the general term in the expansion of $(a+b+c+d)^n$.

9. Any polygons whatsoever described about a circle are to one another as their perimeters.

10. For what angles, less than 360°, is the tangent= sin 90° ?

Investigate a formula from which tan A may be found in terms of tan 2 A, and find tan 22° 30′.

11. Prove—

$$\frac{\tan A+\tan B}{\cot A+\cot B}=\frac{\cos(A-B)-\cos(A+B)}{\cos(A-B)+\cos(A+B)}.$$

12. In any triangle, $(a+c)\sin\frac{1}{2}B=b\,.\cos\frac{1}{2}(A-C)$.

13. Find the angle between the lines, whose equations referred to rectangular coordinates are $y-2x+3=0$ and $y+x-2=0$.

14. Find the equation of the right line which joins the points of contact of tangents drawn to the circle $x^2+y^2=r^2$, from the point (x', y').

15. Find the locus of the intersection of tangents to an ellipse, which cut each other at right angles.

16. Show how to obtain the equation of the tangent to a curve represented by an equation between rectangular coordinates.

17. What values of x render $\frac{x}{m^2+x^2}$ a maximum or a minimum?

18. Assuming that $\frac{d\cos x}{dx}=-\sin x$, and $\frac{d\sin x}{dx}=\cos x$, develop $\sin x$ and $\cos x$ in ascending powers of x, by the method of indeterminate coefficients.

CXXX.

1. Solve the equations:—

$$(1)\quad \frac{5x^2+x-3}{5x-4}=\frac{7x^2-3x-9}{7x-10}.$$

$$(2)\quad x+y=a+b;\ \frac{a}{x}+\frac{b}{y}=2.$$

2. Find, by the aid of tables—

$$(1)\ (\cdot0004582)^{\frac{1}{6}};\quad (2)\ \frac{(1\cdot04)^{17}-1}{(1\cdot04)^{17}+1}.$$

(3) A mean proportional between $\left(\frac{2}{5}\right)^6$ and $\left(\frac{5}{2}\right)^7$.

3. Show how the 6th root of a number may be extracted by the application of the ordinary rules of arithmetic.

4. In the expansion of $(1+x+2x^2)^8$; find the coefficient of x^8, and how many terms precede and follow the term involving x^8.

5. In any plane triangle—

$$\sin^2 A+\sin^2 B+\sin^2 C-2\cos A\cos B\cos C=2.$$

6. The sum of the squares of the cosines of the angles of a triangle is equal to unity; show that the triangle is right-angled.

7. The three sides of a triangle are 508, 401, 299 feet; find the greatest angle.

8. The elevations of two mountains in the same line with the observer are 9° 30′ and 18° 20′; on approaching 4 miles nearer, they have both an elevation of 37°. Find the heights of the mountains in yards.

9. The radius of a sphere is 10 ft.; find the volumes of the two segments into which it is divided by a plane, the perpendicular on which from the centre is 5 ft.

10. A pyramid has a square base, the area of which is 20·25 square feet; each of the edges of the pyramid passing through the vertex is $30\frac{3}{4}$ ft.; find the inclination of either of the triangular faces, to the base: and determine by how much the volume of the pyramid differs from 3800 cubic feet.

11. A B is the side of an equilateral triangle inscribed in a circle whose diameter is A O C and centre O.

Prove (1) the triangle B O C is equilateral.

(2) $AB^2 = 3AO^2$.

12. If two chords A B, C D in a circle cut each other at right-angles, the sum of the opposite arcs A C, B D will be a semicircle.

13. Obtain the equation of a circle of radius a referred to axes of coordinates which touch the circle at distances $a\sqrt{3}$ from the origin.

14. Prove that the equation to the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; at the point (h, k) is $\frac{a^2x}{h}-\frac{b^2y}{k}=a^2-b^2$.

15. If the eccentricity of the ellipse be $\frac{1}{\sqrt{2}}$, find, in terms of a, the coordinates of the point where the normal at $\left(\frac{a}{3}, \frac{2b\sqrt{2}}{3}\right)$ again meets the ellipse.

16. If from any point T on a tangent to a conic section T N, T M be drawn perpendicular to the focal distance, and the directrix respectively, prove that $SN = e.\ TM$.

Hence, show that the polar equation of the tangent at the point where the focal radius vector makes an angle a with the axis major is $\frac{l}{r}=e\cos\theta+\cos(a-\theta)$ where $2l$ is the latus rectum.

17. Expand to 4 terms in powers of x, by Maclaurin's theorem, $e^{\cos x}$.

18. Obtain the equation of the tangent to the curve $\phi(x, y)=0$, and show that it is one dimension lower, at least, than the equation to the curve.

CXXXI.

1. Find the vulgar fraction, in its lowest terms, equivalent to $1\cdot\dot{2}\dot{7}$.

2. Which is the greater $\frac{355}{113}$ or $3\cdot14159$? Show that they differ in value by a quantity less than $\cdot000003$.

3. If 90 cwt. be carried 840 English miles for £121, how many Irish miles ought 270 cwt. be carried for £550? 1 I. M.$=1\cdot\dot{2}\dot{7}$ E. M.

4. How many lbs. of tea at 3*s.* 9*d.* per lb. must a merchant mix with 500 lbs. at 4*s.* 6*d.* per lb.; so that by selling the mixture at 4*s.* 6*d.* per lb., he shall gain 10 per cent.?

5. Given $\log_{10} 2=\cdot3010300$, find $\log_{10} \cdot0000025$.

6. The logarithm of a number to the base 10 is 5, what is the logarithm of the number to the base 2? Find the number.

7. Simplify

$$\left\{\frac{x^2+\sqrt{x^4-a^4}}{x^2-\sqrt{x^4-a^4}}-\frac{x^2-\sqrt{x^4-a^4}}{x^2+\sqrt{x^4-a^4}}\right\}\div 4\sqrt{\frac{x^2-a^2}{x^2+a^2}}.$$

8. If a, b, c, d be in geometrical progression; prove that,

$$(a-b+c-d)^2=(a-b)^2+(c-d)^2+2(b-c)^2.$$

9. Solve the equations $xz=y^2$; $x+y+z=13$; $x^2+y^2+z^2=91$; and state of what problem they are the algebraical statement.

10. Between the two terminal stations, A and B, of a railroad there is an odd number of intermediate stations. A slow train, which stops 5 min. at each intermediate station, performs the whole journey in 5 h. 25 m. A fast train, setting out from A $2\frac{1}{2}$ h. after the slow train, goes 5 miles, whilst the slow train goes 3, and stopping at each alternate station $2\frac{1}{2}$ m., reaches B 10 min. later than the slow train. Find the number of intermediate stations.

11. Prove that $\dfrac{\cos A+\cos 3A+\cos 5A}{\sin A+\sin 3A+\sin 5A}=\cot 3A$.

12. A and B are two stations a mile apart, A due north of

B. At the same instant a balloon is seen from A to bear $60^\circ\ 15'$ west of south, and as seen from B to bear $54^\circ\ 30'$ west of north; also, the angle of elevation of the balloon as seen from A at the same time was $35^\circ\ 25'\ 25''$; find the perpendicular height of the balloon above the horizontal plane passing through A and B.

13. A great circle passing through a fixed point O on the surface of a sphere, meets the circumference of a given small circle in P and P′; prove as a consequence of the formula $\tan \frac{1}{2}(x+y) \tan \frac{1}{2}(x-y) = \frac{\cos y - \cos x}{\cos y + \cos x}$, that $\tan \frac{1}{2} \text{OP} \times \tan \frac{1}{2} \text{OP}'$ is constant.

14. Prove De Moivre's theorem when the index of the power is a positive integer.

15. The equation $3y^2-8xy-3x^2-9y+7x+6=0$ represents two straight lines at right angles to each other.

16. In the parabola prove that if (x', y'), (x'', y'') be points on the curve, the coordinates of the point of intersection of the tangents at those points will be respectively the arithmetic mean between y' and y'', and the geometric mean between x' and x''.

17. Inscribe the greatest ellipse in a given semicircle.

18. Expand (1) $\tan^{-1} x$; (2) $\log_e (1+x)$ by Maclaurin's theorem.

CXXXII.

1. What must be the price of the 3 per cent. stock, so that by investing the sum of £32850 my income may be £1080 a year?

2. If a napoleon be worth $15s.\ 10\frac{1}{2}d.$, find the least exact number of napoleons that must be given for an exact number of English sovereigns, stating the number of each.

3. Divide $(x^2-y^2)^3-z^6$ by $x^2-y^2-z^2$.

4. When is an infinite series said to be convergent or divergent? Show that the series—

$$1+\frac{1}{2^x}+\frac{1}{3^x}+\frac{1}{4^x}+\&c.$$

is convergent if x be greater than unity, and divergent if $x=1$.

5. Solve the equation $x^3-12x=16$, one root being 4.

6. Find an angle of which the sine is to the cosine as $\sqrt{3}$ to 1.

7. (1) Prove $\sin 2A=\frac{2\tan A}{1+\tan^2 A}$, and verify it when $A=30°$;

(2) Prove that $\frac{\cos 3A+\sin 3A}{\cos A-\sin A}=1+2\sin 2A$.

8. Show that the sum of the radii of the circles inscribed in and described about a triangle $=\frac{1}{2}(a\cot A+b\cot B+c\cot C)$.

9. The sides of a four-sided figure taken in order are 135, 180, 150, and 125 feet, and the angle contained by the first two is a right angle. Determine the area of the figure.

10. Show how to determine the content of a right cone with a circular base.

11. $22\frac{1}{2}$ cubic feet of matter are to be formed into such a cone as that in *Quest.* 10; find its height when the circumference of the base is 9 feet.

12. Show geometrically that the area of a hexagon inscribed in a circle is double the area of an equilateral triangle inscribed in the same circle.

13. State and prove the properties that connect the polar triangle with its primitive, in spherical trigonometry.

14. Construct the circle whose equation is—

$$4x^2+4y^2-16y-4x-19=0,$$

and find the equation to the diameter passing through the origin.

15. In an ellipse the normal at any point bisects the angle between the focal distances.

16. Expand $e^x \log_e (1+x)$ by Maclaurin's theorem.

17. Write down the general expression for the equation to the normal at any point of a curve, explaining what each symbol indicates.

18. Find the equation to the tangent at any point of the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$, and show that the portion of the tangent intercepted between the axes of coordinates is constant.

CXXXIII.

1. The bounty money for 1408 recruits is £10 per man, but from this is deducted £6143 17*s.* 4*d.* for clothing; how much does each man receive?

2. Find the value of—

$\frac{4}{5}$ of 21*s.* + $\frac{3}{8}$ of $\frac{1}{2}$ of £1 − $\frac{1}{12}$ of $\frac{3}{4}$ of 5*s.* − $\frac{1}{6}$ of $\frac{13}{8}$ of 1*s.*; and reduce the result to the decimal of £1.

3. Find the compound interest on £313 in 4 years, at 5 per cent.

4. Solve the equations—

$$(1)\ \frac{2x-11}{27}+\frac{x-1}{18}-\frac{x-4}{4}=9\tfrac{1}{3}-x.$$

$$(2)\ \frac{1}{x+16}=\frac{1}{x}+\frac{1}{9}+\frac{1}{7}.$$

5. Find the sum of n terms of a geometrical progression. A person who saved every year half as much again as he saved the previous year, had in 7 years saved £205 18*s.*; how much did he save the first year?

6. Calculate by logarithms the value of—

$$(1)\ \left(\frac{3147}{27{\cdot}64}\right)^{\frac{1}{4}};\quad (2)\ \left(\frac{{\cdot}0275}{\sqrt{{\cdot}0176}}\right)^{\frac{1}{5}}.$$

7. If c be the hypothenuse of a right-angled triangle, whose sides are a and b, prove that—

$$\log_e c=\tfrac{1}{2}\log_e 2+\tfrac{1}{2}\log_e a+\tfrac{1}{2}\log_e b+$$
$$\left(\frac{a-b}{a+b}\right)^2+\tfrac{1}{3}\left(\frac{a-b}{a+b}\right)^6+\tfrac{1}{5}\left(\frac{a-b}{a+b}\right)^{10}+\&c.$$

8. If the sum of the squares of the sines of the angles of a triangle$=2$, show that the triangle is right-angled.

9. Two poles are of equal height; a person standing between them in the line joining their bases observes the elevation of the nearer one to be $60°$. After walking 80 ft. in a direction at right angles to the line joining their bases he observes the elevations of the two to be $45°$ and $30°$. Find their height and the distance between them.

10. A, B, C are three points at known distances from each other; $AB=476$, $AC=513$, $BC=495$. An observer stations himself so as to see B and C in the same straight line, and the side AB then subtends an angle of $23° 10'$ at his eye; find his distance from A.

11. If two sides AD, BC of a quadrilateral inscribed in a circle ABCD be produced to meet in E, the circle described about ECD will have the tangent at E parallel to AB.

12. If from the angles at the base of any triangle perpendiculars be drawn to the opposite sides, produced if necessary, the line joining the points of intersection will be bisected by the perpendicular drawn to it from the middle of the base.

13. In a spherical triangle having given $a=70°$, $b=40°$, $c=38° 30'$, calculate numerically the angle A; assuming a suitable formula.

14. In a right-angled spherical triangle, if c denote the side opposite to the right angle, show that

$$\cos^2 \tfrac{1}{2} c=\cos^2 \tfrac{1}{2} a \cos^2 \tfrac{1}{2} b+\sin^2 \tfrac{1}{2} a \sin^2 \tfrac{1}{2} b.$$

Also that $\dfrac{\sin (c-b)}{\sin (c+b)}=\tan^2 \tfrac{1}{2} A.$

15. Show that the locus of the intersection of the perpendicular from the focus of an ellipse on a tangent is a circle. Show that the locus is also a circle, if the line from the focus, instead of being perpendicular to the tangent, makes a constant angle with it.

16. Determine the numerical value of the radius of curvature at the origin of the curve $y=x^4-4x^3-18x^2$.

17. Investigate the differential expression for determining the area of a plane curve referred to polar coordinates, and apply it to determine the whole area of the looped curve $r^2=a^2 \cos 2\theta$.

18. Expand $\sin^{-1}(x+h)$ to three terms by Taylor's theorem.

CXXXIV.

1. A gentleman in Australia receives 12 per cent. per annum on his capital in the colony; he brings his capital home, invests it in the 3 per cents. at $94\frac{3}{8}$, and his income in England is £2400 a year. What was his income in Australia?

2. In a regiment consisting of English, Irish, and Scotch, ·4 of the regiment were Irish and ·3 Scotch; but after 200 Irish and 200 Scotch were added to the regiment, ·225 were English. What was the original strength of the regiment, and the number of men of each nation?

3. Solve the equation $(1+x+x^2)(5-x-x^2)=9$.

4. Solve the equation $x+y-\sqrt{xy}=7$; $x^2+y^2+xy=133$.

5. Show that, whatever be the value of x, the expression ax^2+bx+c has always the same sign as a, unless the roots of the equation $ax^2+bx+c=0$ are possible and unequal, and x lies between them.

6. There are two towns, A and B; a traveller sets out from A towards B, another from B towards A on the same day, and they meet half way between A and B. One travels 5 miles the first day, 10 the second, 15 the third, and so on; the other travels 20 miles a day for 2 days, and 35 miles a day afterwards, until they meet. Find the number of days they travelled, and the distance from A to B.

7. The edge of a cube is 12 inches; one of the angles of the cube is cut off, so that the part cut off forms a pyramid with each of its edges (terminating in the angle of the cube 5 in. long). Find the volume of the remaining solid.

8. The base of a pyramid is a regular hexagon, each side $=40$ feet; what must be its height that its cubical content may be the same as that of a sphere whose radius is 21·5 feet?

9. Without assuming the formula for $\cos(A+B)$, prove that $\cos 2A=\cos^2 A-\sin^2 A$.

10. Show that $\tan A=\dfrac{\cos A-\cos 3A}{\sin A+\sin 3A}$.

11. If A, B, C be the angles, a, b, c the sides of a plane triangle, $a\sin(B-C)+b\sin(C-A)+c\sin(A-B)=0$.

12. If AB, CD be the opposite sides of a quadrilateral circumscribing a circle whose centre is E; prove that the angles AEB and CED are together equal to two right angles.

13. Find the point of intersection of the two lines—

$$3x-2y+5=0,\ 4x+3y=0;$$

find also the distance of this point from the line $2y=x-1$.

14. Prove that the tangents to a parabola from the same point on its directrix are at right angles to one another.

15. The distance between one vertex and the adjacent focus of an ellipse being given, prove that if the axis major increases without limit the curve will ultimately become a parabola.

16. If $u=(x+1)(x^2+1)$, find the value of $\dfrac{du}{dx}$.

17. Find the values of x which will make

$$\sin(x-a)\cos x \text{ a maximum or minimum.}$$

18. Supposing $x=r\cos\theta$, $y=r\sin\theta$, find the value of $x\,dy-y\,dx$ in terms of r and θ.

CXXXV.

1. Extract the cube root of 233·744896, and the square root of $27\frac{27}{128}$ to 4 places of decimals.

2. A person paid £18 15*s.* for 1 year's income-tax, but

after the tax was increased by 9*d.* in the £, he paid £52 10*s.*; what was his income, and at what rate in the £ was the tax levied at first?

3. Prove that—

$$\frac{a^2d^2-b^2c^2}{(ac-bd)^2+(ad+bc)^2}=\frac{a^2}{a^2+b^2}-\frac{c^2}{c^2+d^2}.$$

4. Show that—

$$\frac{a+b\sqrt{-1}}{(a-b\sqrt{-1})^2}+\frac{a-b\sqrt{-1}}{(a+b\sqrt{-1})^2}$$ is a possible quantity.

5. Show that $x^3-px^2+qx-r=0$ is the equation that results from the elimination of y and z from the equations $x+y+z=p$, $xy+xz+yz=q$ and $xyz=r$.

6. Solve the equation, $x^4+4x^3-18x^2+20x-7=0$, which has equal roots.

7. Find the middle term of $(1+x+x^2)^6$.

8. Prove that the co-efficient of x^{2n} in $\frac{1-x}{(1+x)^2}$ is $4n+1$.

9. Show that $\sin 2\text{A}=2\sin\text{A}\cos\text{A}$, without assuming the formula for $\sin(\text{A}+\text{B})$.

10. Prove that $\sec\text{A}+\tan\text{A}=$

$$\sqrt{\frac{1+\sin\text{A}}{1-\sin\text{A}}}=\frac{\sin(45^\circ+\frac{1}{2}\text{A})}{\sin(45^\circ-\frac{1}{2}\text{A})}.$$

11. Show that two unequal triangles may always be constructed having two sides and an angle opposite to one of those sides, in each triangle equal each to each; and prove that the circles circumscribing these triangles have the same radius.

12. If the data in the preceding question be $a=605$, $b=564$, and $\text{B}=56^\circ\ 15'$, find the radius of the circumscribing circle.

13. Prove the following formula of spherical trigonometry and adapt it to logarithmic computation:—

$$\cos c=\cos a\cos b+\sin a\sin b\cos\text{C}.$$

14. In any right-angled spherical triangle—

$$(1)\ \tan^2 \tfrac{1}{2}\, a=\frac{\tan \frac{1}{2}\,(c+b)}{\cot \frac{1}{2}\,(c-b)}.$$

$$(2)\ \tan^2 (45^\circ+\tfrac{1}{2}\, c)=\frac{\tan \frac{1}{2}\,(\text{B}+b)}{\tan \frac{1}{2}\,(\text{B}-b)}.$$

15. Find the equation to the right line perpendicular to the line whose equation is $y=mx$, the axes being rectangular.

16. Determine the angle between the lines $y=mx+n$ and $y=m'x+n'$.

17. Find the equation to the tangent, at the point $(x'\ y')$, to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

18. Find the integrals of,

$$(1)\ \frac{dx}{a^2+x^2},\quad (2)\ \frac{dx}{\cos x},\quad (3)\ \frac{dx}{\sin x}.$$

CXXXVI.

1. If the circumference of the fore-wheel of a carriage be 54 inches, and that of the hind-wheel 66 inches, how many more turns will be made by the fore-wheel than by the hind-wheel in a distance of 3 miles ?

2. If 5 horses and 12 mules can draw a load of 25896 lbs. for a given distance, how many mules will be required to draw 337 cwts. 3 qrs. 20 lbs. the same distance, with the help of 9 horses—2 horse loads being equal to 3 mule loads ?

3. A regiment has to divide a certain quantity of prize money : the colonel is to receive as much as 2 majors, 1 major as much as 5 captains, 7 captains as much as 15 lieutenants, 3 lieutenants as much as 7 ensigns, and 1 ensign as much as 4 sergeants. The share of a lieutenant being £88 13*s.* 4*d.*, find those of the colonel and a sergeant.

4. Solve the equations—

$$\left.\begin{aligned}(1)\quad &\frac{x}{2}+\frac{y}{3}-\frac{z}{4}=1\\ &4x+y-\frac{3z}{2}=1\\ &\frac{3z}{4}-2x-\frac{y}{3}=1\end{aligned}\right\}\qquad \begin{aligned}&(2)\ x\,(x-2)(x^2-4)=2.\\ &(3)\ x-\frac{4}{\sqrt{x-2}}=4.\end{aligned}$$

5. A and B are two workmen : A alone takes 4 hours more to perform a certain work than B alone takes ; A and B can do it together in $3\frac{3}{4}$ hours. How long would each man take to do the whole ?

6. The first term of an arithmetic progression is 2, and the fifth is 18 ; how many terms must be taken to make the sum 800 ?

7. Find the coefficient of x^6 in $(1+2x+3x^2)^8$.

8. Transform the equation $x^3+px^2+qx+r=0$, whose roots are a, b, c, into one whose roots are ab, ac, bc.

9. Find all the roots of the equation

$$x^4-6x^2-16x+21=0.$$

10. Find the number of degrees in the least positive angle that satisfies the equation $\tan \text{A}+\tan(45°+\text{A})=2$.

11. Two sides of a triangle are 175 feet and 105 feet, the included angle is 37° ; calculate the remaining parts of the triangle and its area.

12. A man ascends a hill by a path which is the shortest distance between the base and the summit ; the inclination of the path to the horizon is at first 20°, but afterwards suddenly increases to 40° ; on arriving at the summit he observes the angle of depression of the point from which he started to be 60°: supposing that he walked 800 yards before the change of inclination of his path; determine the height of the hill.

13. Show that the two tangents which can be drawn from an external point to an ellipse subtend equal angles at each focus.

14. Two ellipses have a common focus, and their major

axes are of equal length and coincide in direction, their eccentricities being different; determine the polar coordinates of the points in which the ellipses intersect.

15. If a right cone with a circular base be cut by a plane parallel to a slant side of the cone, show that the section is a parabola.

16. Prove that a paraboloid is one-half its circumscribing cylinder; and find the content of a paraboloid, the *latus rectum* of which is 4 feet, and the greatest radius 10 feet.

17. Two ships are sailing uniformly with velocities u, v along lines inclined at an angle θ; given that at a certain time the ships are distant respectively a and b from the point of intersection of their courses. Find the least distance between the ships.

18. Investigate Maclaurin's theorem, and expand by means of it $e^{\tan^{-1}x}$ as far as the term involving x^3.

CXXXVII.

1. Solve the equations:—

$$(1)\ \frac{6x+1}{15}-\frac{2x-4}{7x-16}=\frac{2x-1}{5}.$$

$$(2)\ \left(\frac{x-a}{x+b}\right)^3=\frac{x-2a-b}{x+a+2b}.$$

2. Prove the binomial theorem when n is a negative fraction.

3. Find the coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^3$ in powers of x.

4. Find, by logarithms—

$$(1)\ \sqrt[3]{\cdot00006706};\quad (2)\ \left(\frac{\cdot01607}{\cdot00881}\right)^{\frac{1}{4}}$$

5. Investigate an expression for the cosine of an angle of a triangle in terms of its sides. The sides of a triangle are 2, $\sqrt{6}$ and $1+\sqrt{3}$; determine the angles.

6. In an oblique-angled triangle A B C; given A B=2700 ft. $\angle A=50° 20'$, $\angle B=110° 12'$ Find B C.

7. To determine the height of the top, C, of a mountain, a base, A B, of 2700 ft. was measured in the horizontal plane, the angle subtended by C B at A was 50° 20′, the angle subtended by A C at B was 110° 12′, and the angle of elevation of C from B was 10° 7′. Find the height of the mountain.

8. How long will it take to fill a hemispherical tank of 6 ft. radius from a cistern which supplies, by a pipe, 6 gallons of water per minute, a gallon of water containing 277·27 cubic inches?

9. The frustum of a right cone is 6 ft. high, the radius of the smaller end is 2 ft., the radius of the larger end is 3 ft.; find its volume. Find the position of the plane parallel to the ends which will bisect the frustum.

10. A B C is a triangle inscribed in a circle, B D is drawn parallel to the tangent to the circle at the point A, and meets A C in D; prove A C : A B :: C B : D B.

11. From any point in the base of a triangle, lines are drawn parallel to the sides; show that the intersection of the diagonals of every parallelogram thus formed lies in a certain straight line.

12. In a triangle the sides about the vertical angle are 25 and 16, the line bisecting the vertical angle is 12; find the base.

13. State the property from which certain curves are called conic sections: distinguish them.

14. If S be the focus of a parabola, P the point of contact of a tangent to the curve, T its intersection with the axis of the parabola, G the intersection of the normal at P with the axis; prove SP=ST=SG.

15. If tangents be drawn at any two points of a parabola, the angle between the tangents is half the angle contained by the focal distances drawn to the point of contact.

16. Explain the meaning of Integration; and find the

value of $\int_a^b x dx$ regarding it as the limit of a certain summation.

17. Show how to integrate the fraction $\frac{F(x)}{f(x)}$, when the roots of the equation $f(x)=0$ are real and unequal.

18. Integrate the following functions of x:—

(1) $\sin ax \sin bx$; (2) $x^2 \cos x$; (3) $\frac{x^2}{(x-a)(x-b)}$.

CXXXVIII.

1. Extract the cube root of 517 to 4 places of decimals.

2. I bought a horse for 25 gs. and sold him the same day for 30 gs., allowing 6 months' credit; what interest did I gain per cent. for my money?

3. If 40 lbs. of standard gold be coined into 1869 sovereigns, what is the value of 1 oz. of standard gold?

4. Reduce—

$$\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-a)(b-c)}+\frac{1}{c(c-a)(c-b)}$$

to its simplest form.

5. Solve the equations:—

$$(1)\ \begin{cases} 3x+2y+\ \ z=23 \\ 5x+2y+4z=46 \\ 10x+5y+4z=75 \end{cases} \qquad (2)\ \begin{cases} 2x-y=2 \\ 8x^3-y^3=98 \end{cases}$$

6. A railway train running from L to X encounters an accident on the way, in consequence of which its rate is reduced to $\frac{2}{3}$ of what it was at first, and it is 2 hrs. 5 min. late; if the accident had happened 18 miles nearer X, it would have been only 1 hr. 50 min. late. Find the distance from X to the place of the accident.

7. There is a looking-glass whose plate is 2 ft. greater in height than in width, the frame cost 5*s.* per linear foot, inside measure, and the plate 10*s.* per square foot; the whole cost was £11 10*s.*; what is the width?

8. What length of a gun of 9 inch bore will a charge of 15 lbs. of powder fill, if 30 cub. in. of powder weigh 1 lb.?

9. A hemispherical bell of 10 ft. diameter is partially buried with its mouth downwards and in a horizontal position, so that only $\frac{1}{3}$ of the vertical radius of the bell appears above ground. What quantity of earth must be dug out in order to leave the bell entirely uncovered and surrounded by a cylindrical wall of earth?

10. From the top of a tower, whose height is 108 ft., the angles of depression of the top and bottom of a vertical column standing on the horizontal plane are 30° and 60° respectively. Find the height of the column.

11. From each of two ships a mile apart the angle which is subtended by the other ship and a beacon on shore is observed: these angles are 55° and 62° 30′. Determine the distances of the ships from the beacon.

12. If the straight line A B be divided in C so that the rectangle contained by A B and B C is equal to the square on A C: show that if in A C, C D be taken equal to C B, the rectangle A C, A D$=$C D^2.

13. C Q is the perpendicular from the centre C on the tangent at the point P of an ellipse; find the maximum value of P Q.

14. Find the differential equation to the normal drawn at any point of a plane curve.

15. If a normal be drawn at any point of an ellipse, find the distances from the centre at which the normal cuts the major and minor axis of the ellipse respectively, and determine the point at which the normal must be drawn when the rectangle contained by these distances is a maximum.

16. Find the differential coefficient of sin ax with respect to x.

17. Find the minimum value of $\frac{x^2-x-1}{x^2-x+1}$.

18. Find the subtangent of a spiral curve from its polar

equation, and show how to determine whether the curve has an asymptote or not.

Ex. Find the subtangent and asymptote of the curve

$$r=\frac{a}{\theta}.$$

CXXXIX.

1. Find the value of 12 qrs. 3 bu. 3 pks. of wheat at £2 2*s.* 8*d.* a quarter, by practice.

2. Find the side of a square court-yard, the expense of paving which, at 3*s.* 9*d.* per square yard, is £38 10*s.* 5*d.*

3. If 21 horses and 217 sheep can be kept 10 days for £56 8*s.* 4*d.*, what sum will keep 9 horses and 60 sheep for 27 days, supposing that 3 horses eat as much as 50 sheep?

4. Solve the equations—

$$(1)\ \frac{x-3}{x+2}=\frac{1}{2}+\frac{x-3}{2x-1};$$

$$(2)\ \left.\begin{array}{l} x+\frac{1}{2}y+\frac{1}{3}z=32 \\ \frac{1}{3}x+\frac{1}{4}y+\frac{1}{5}z=15 \\ \frac{1}{4}x+\frac{1}{5}y+\frac{1}{6}z=12 \end{array}\right\}$$

$$(3)\ \left.\begin{array}{l} x^4+y^2=49+2x^2y \\ x^4+y^4-x^2=20+(2x^2-1)y^2 \end{array}\right\}$$

5. If $\frac{a}{b}$ and $\frac{a'}{b'}$ be two unequal fractions, prove that $\frac{a+a'}{b+b'}$ is intermediate in value between them.

6. If the same transformation banishes both the second and third terms in the cubic equation $x^3-px^2+qx-r=0$, what relation must subsist between the coefficients?

7. Extract the cube root of 20; and find a root of the equation $x^3+2x=30$ by Horner's method.

8. The difference between the numbers of shot in the two sides of the base of an incomplete rectangular pile is 7, and the number in the longer side of the top course is 15. How many shot are required to complete the pile?

9. The interior angles of an irregular polygon are in

arithmetical progression ; the least angle is 120°, and the common difference 5°. Find the number of sides.

10. An estate which has been surveyed is 100000000 times as large as the map which has been made of it. Express the linear scale of the map in terms of decimals of an inch to 1 mile.

11. Describe a square in a given triangle, and show that the side of the inscribed square standing on the base is half the harmonic mean between the base and the altitude.

12. Four trees, A, B, C, D, are in the same straight line. The distance A B=40 yards, B C=20 yards, C D=60 yards. P is a point, out of the line, such that the distances between the trees appear equal to each other. Find A P, B P, C P, D P.

13. In plane coordinate geometry, given the rectangular equations to two straight lines, find the angle at which they intersect, and hence determine the condition that the two lines shall be perpendicular to each other.

14. Explain and illustrate the meaning of the limit of a variable ratio, and show how such a limit is applied in the fundamental principles of the differential calculus.

15. Investigate the differential coefficient of $\sin^{-1}ax$ with respect to x, and differentiate—

$$(1)\ \log\frac{x-a}{x+a};\quad (2)\ \frac{\cos 3x+\cos x}{\sin 3x-\sin x};$$

$$(3)\ (\tan x)^{\cot^{-1}x};\quad (4)\ \cos\{\cos(\cos x)\}.$$

16. If $y=(a+x)^3(a-x)^2$, find the values of x which make y a maximum or a minimum.

17. Determine the position and magnitude of the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$, at the point for which x and y are each equal to $\frac{a}{4}$.

18. Integrate, with respect to x—

$$(1)\ \frac{x^2-5x+7}{x^2-5x+6};\quad (2)\ \frac{x^2+7}{x^4+5x^2+4};$$

$$(3)\ \frac{x^2+2x+4}{x^3+2x^2+4x+8};\quad (4)\ x^3(\log x)^2.$$

CXL.

1. Convert the decimal ·1415926 into a continued fraction.

2. Find the sum of the fourth powers of the roots of the equation $x^4+x^3-7x^2-x+6=0$.

3. Prove that one real root of the equation $x^3-2x-5=0$, lies between 2 and 2·1; also approximate to the value of this root to 4 places of decimals.

4. Give Cardan's method for the solution of cubic equations.

5. Investigate in how many different ways n elements admit of being formed into distinct groups of m elements each.

6. Having given the perimeter $2s$, and the angles A, B, C of a plane triangle, find its sides.

7. From the lower window of a house the angle of elevation of a church tower is observed to be 45°, and from a window 20 feet above the former, 40°; how far is the house from the church?

8. The angle between a fort and a light-house observed from a ship at sea is 56° 30′, but after the ship has sailed $\frac{1}{2}$ a mile in the direction of the fort, the angle between the same two objects is 70° 45′. What was the ship's distance from the light-house midway between the two places of observation?

9. The distance from A to B is 1·75 miles, from B to C 2·5 miles, and these three stations are in the same straight line. I travel, not in that straight line, from C to a station D, and then observe that the stations B and C subtend at my eye an angle of 23° 19′, and A and B an angle of 17° 49′. Find the distances of D from A and B.

10. A cylindrical column stands 50 feet high, and the diameter of its base is 4·25 feet. Find the entire superficies and solid content of the column.

11. Describe a circle passing through two given points and touching a given straight line or a given circle.

12. Prove that the product of the radii of the four circles, each of which touches the same three intersecting right lines, is equal to the square of the area of the triangle included between these right lines.

13. A right circular cylinder is inscribed in a given right cone; find the radius of the cylinder when the whole surface of the solid is a maximum.

14. Find the radius of curvature at any point of an ellipse, and express it in terms of the focal distances of the point.

15. Integrate—

$$(1) \int \frac{x^3\, dx}{(x-a)(x-b)(x-c)}\ ; (2) \int \frac{x\, dx}{(x-3)^2(x+2)}.$$

16. Find the volume of an oblate spheroid, and show that the volume of the sphere described on the minor axis of an ellipse as diameter is a third proportional to the oblate and prolate spheroids generated by the same ellipse.

17. Prove that—

$$\int \frac{dx}{(x^3+a^3)^n} = \frac{1}{3(n-1)a^3} \cdot \frac{x}{(a^3+x^3)^{n-1}} + \frac{3n-4}{3(n-1)a^3} \int \frac{dx}{(x^3+a^3)^{n-1}}$$

18. Evaluate the integrals—

$$(1) \int_0^{\frac{\pi}{2}} (\sin x)^n\, dx\ ; \quad (2) \int_0^{\infty} e^{-x^2}\, dx.$$

CXLI.

1. (1) Find what decimal 3*s*. 9*d*. is of £1. (2) Extract the square root of ·9 and the cube root of ·8, each to 3 places of decimals.

2. How many lbs. of powder, weighing 932 lbs. a cubic foot, will fill a box whose height is 2 ft. 5 in., breadth 1 ft. 7 in. and length 5 ft. 9 in.?

3. A manufacturer having a capital of £5000, on which he can realise by hand labour 10 per cent. profit, buys a

machine for £1000, by which his profit on the remainder of his capital is raised to 20 per cent. This machine lasts 5 years: by how much will he then have increased his capital, supposing him to have drawn £300 a year for the support of his family?

4. How much would it cost to have a cellar dug 18 ft. 4 in. long, 12 ft. broad, and 13 ft. 6 in. deep, at 6*d.* per cubic yard?

5. A cistern is fed by a spout which can fill it in 3 hours; how long would it take to fill it if the cistern had a leak which would empty it in 17 hours?

6. One side of a room is 5 ft. longer than the other, and 1000 square feet of paper are required to cover its walls. Now, if it were 3 ft. higher, the same quantity of paper would be required for 3 of its walls, that remaining unpapered being one of the longer sides. What are the dimensions of the room?

7. A number of persons undertake to erect a public building, contributing equally to the cost, which is £5002. Whilst it is in progress, 10 of the number became bankrupts, and there are 10 others who refuse to contribute to the deficiency thus occasioned. This deficiency, being equally borne by the remainder, increased the contribution of each by £40. How many subscribers were there in the first instance?

8. A dealer having laid in a stock of a certain article, began to sell it by retail. The first day he made a profit of 3*d.*, the second of 4·2*d.*, and so on, the profit increasing by 1·2*d.* a day, until the stock was disposed of; he then found that he had realised a profit of 14*s.* 3*d.* How many days did he continue to sell the article?

9. Prove that the difference of the squares of any two odd numbers is divisible by 8.

10. Prove that $x^n - a^n$ is divisible by $x - a$; and show that if a be a root of the equation $\mathrm{x}=0$, x is divisible by $x - a$.

11. Prove that if a quadrilateral be bisected by both of its diagonals, it must be a parallelogram.

12. Prove that the area of the isosceles triangle is greater than that of any other triangle on the same base, and having an equal perimeter.

13. Given two sides of a spherical triangle and the included angle; investigate a formula for determining the other sides.

14. Given the coordinates of the extremities of a right line; find the coordinates of its middle point.

15. What are conjugate diameters of an ellipse or hyperbola?

16. Prove that, in an ellipse, the sum of the squares of two semi-conjugate diameters is equal to the sum of the squares of the semi-axes. State the corresponding property of the hyperbola.

17. Find the areas of the curves—

$$(1)\ x^2-2xy+10y^2=a^2; \qquad (2)\ \left(y-\frac{x^2}{a}\right)^2=a^2-x^2.$$

18. Find the equation of the tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$, and show that at the point (a, b) it is the same, whatever be the value of n.

CXLII.

1. By selling an article for 6*s.* 8*d.* I lost $\frac{2}{7}$ of its cost price. What did it cost?

2. At what rate per cent., simple interest, will £250 be increased to £330 12*s.* 6*d.* in $4\frac{1}{2}$ years?

3. Which is the greatest of the fractions, $\frac{4}{5}$, $\frac{7}{9}$, $\frac{9}{11}$?

4. (1) Find a fourth proportional to ·45, ·8, and ·367.

(2) Reduce $\cdot 0000566\dot{6}$ to its equivalent fraction.

5. It is required to cut a piece equal to 1 solid foot from a plank $2\frac{1}{2}$ in. thick and 8 in. wide.

6. Solve the equations:—

(1) $\left.\begin{array}{r}3x+2y=118\\ x+5y=191\end{array}\right\}$ (2) $6\sqrt{2x-1}=15+3\sqrt{2x-9}$.

(3) $\left.\begin{array}{r}xy=\ 8\\ (3-y)z=12\\ (2-x)(4-z)=\ 4\end{array}\right\}$

7. I make a journey of 3240 miles on horseback, on foot, and by water; $3\frac{1}{2}$ times as much is performed on land as by water; and $2\frac{1}{2}$ as much on horseback as on foot. How far did I travel on foot?

8. A messenger sets out at the rate of 30 miles a day, but falls off in his speed 4 miles daily. Four days afterwards another sets out from the same place, on the same route, travelling 46 miles the first day, but falling off, like the first, 4 miles daily, after what time will one overtake the other?

9. Prove the formula $\cos 2A=2\cos^2 A-1$; and show, by means of it, that $2\cos\frac{45^\circ}{2^n}=\sqrt{2+\sqrt{2+\sqrt{2+\&c.}}}$

10. Show that if three angles are together equal to two right-angles, the sum of their tangents is equal to their product.

11. The height of a tower above the level of a river being known, how would you ascertain the river's breadth by observations made from the top of the tower?

12. How would you determine the distance of an object by means of observations made from the top and bottom of a tower?

13. Differentiate $\tan^{-1}\frac{2x}{1-x^2}$; $\log\sqrt[4]{\frac{1+x}{1-x}}+\frac{1}{2}\tan^{-1}x$.

14. Expand $\tan^{-1}x$ in a series of ascending powers of x by Maclaurin's theorem.

15. Find what the area of the base of the roomiest tent of a conical form will be, which can be covered by 200 feet of canvas.

16. Explain how the form of a plane curve is determined

by means of an equation, and illustrate the explanation by reference to some particular curve.

17. Find the radius of curvature, in polar coordinates, at the extremity of the *latus rectum* of a parabola.

18. Trace the curve $a^2y^4=x^4(a^2-x^2)$, and find the area of one loop of it.

CXLIII.

1. Prove that every fraction may be represented either by a decimal of a finite number of places, or by a recurring decimal.

2. A, B, C enter into partnership, and put £1800, £1200, and £700 into the joint stock for 12, 15, and 21 months respectively: the profit realised is £1086; find the share of each.

3. Find the square root of 41605800625, and extract the square root of $\frac{5}{8}$ to 5 places of decimals.

4. Solve the equations:—

$$(1)\quad \frac{x}{2}+\frac{3x-8}{14}-\frac{4x+16}{11}=8.$$

$$(2)\quad \left.\begin{aligned}3x+5y&=171\\7x+2z&=209\\7y+2z&=\ \,90\end{aligned}\right\}$$

5. Two gunners keep up a fire on a battery from two different guns; the first one, who had spent 24 shots before the second opened fire, discharges 8 shots to 7 of those of his comrade, but uses in 4 rounds only the same quantity of powder as the other in 3. How many shots must the second fire before he has consumed as much powder as the first?

6. Expand $\sqrt{11}$ in the form of a continued fraction.

7. If n be a very large number, prove that $\log(n+1)-\log n$ is very nearly equal to $\frac{M}{n}$, M being the modulus of the system of logarithms.

8. Why is it expedient to calculate angles nearly equal to 90° by means of their sines rather than their cosines?

9. Prove the formula $d \cos A = -\sin A \, dA$, and explain the negative sign occurring in it.

10. If a cubic foot of metal weighs 4 cwt. 1 qr., and is worth 10 guineas a ton, what will be the cost of 1 mile of piping made out of it, with a 9-inch bore, and $\frac{3}{8}$ in. thick?

11. A solid sphere, whose radius is a, has a cylindrical hole bored through it, whose axis passes through the centre, and whose diameter is half that of the sphere; determine the volume of the remaining portion.

12. The total length of the edges of a rectangular parallelopiped is 72 lineal feet, its entire surface measures 174 square feet, and its volume is 110 cubic feet. Find its length, breadth, and height.

13. Two sides of a triangle being of given lengths, show that its area will be a maximum when they contain a right-angle.

14. Hence, prove that if all the sides except one of a plane rectilineal figure be given, its area will be a maximum when the remaining side is the diameter of a semicircle circumscribing the figure.

15. And consequently if all the sides of a rectilineal polygon be given in length, its area will be a maximum when it admits of being inscribed in a circle.

16. Assuming the differential equation to the tangent to a plane curve referred to rectangular coordinates, show how the asymptotes to the curve may be drawn, if it has any.

$$\text{Ex. } y^2 = \frac{b^2}{a^2}(2ax + x^2).$$

17. Integrate the functions:—

$$(1) \int x^{\frac{p}{q}} dx; \quad (2) \int \frac{x\,dx}{a^2 - x^2}; \quad (3) \int \frac{dx}{x\sqrt{x^2 - a^2}}.$$

18. Find the differential coefficient of the surface of a solid of revolution, and apply it to find the surface of a given segment of a sphere.

CXLIV.

1. Two paintings are to be framed and glazed, one being 6 in. longer, and 4 in. wider than the other; the framing and glazing of the larger costs 28*s.* 8*d.*, and that of the other 20*s.*, the glass being charged at the rate of 4*s.* per square foot, and the frames at 2*s.* per lineal foot, inside measure. What are the dimensions of the paintings; the part of each concealed by the frame being neglected?

2. Solve the equations :—

(1) $ab(x^2+1)=x(a^2+b^2)$.

(2) $(x+y)^3+z^3=1125$; $x+y+z=15$; $xy=24$.

3. State and prove Descartes' rule relative to the signs of the coefficients of an equation.

4. Give the expressions for the sine and cosine of the sum of three angles in terms of the sines and cosines of the angles themselves.

5. Investigate an expression for the area of a plane triangle in terms of the sides.

6. Show how by observing the angular distances from one another of three distant objects, in the same horizontal plane, whose positions are known, the position of the observer may be determined.

7. What will be the length of the radius of the circle circumscribing the triangle whose sides are 4, 5, 7?

8. In determining the length of a perpendicular let fall on the base c from the opposite angle of the triangle whose remaining sides are a and b, if a small error be committed in measuring the side a; what proportion will it bear to that in the calculated perpendicular?

9. Each of the edges, meeting in the vertex, of a square pyramid is double the side of the base: find the inclination of two contiguous triangular faces to one another.

10. Investigate a converging series representing the circumference of a circle in terms of its diameter.

11. The sides of a triangle are 14, 28, 35 feet respectively: find the angles independently of one another, and verify the result.

12. Find the periphery of a regular decagon which shall contain an area of 1000 square feet.

13. Solve the spherical triangle ABC from these given parts—

$$A=51^\circ\ 30', \quad b=80^\circ\ 19', \quad a=63^\circ\ 50'.$$

14. Find the equations of the lines bisecting the angles between the lines whose equations are $12x+5y=8$ and $3x-4y=3$.

15. Define a tangent to a curve, and, according to the definition, find the equation to the tangent at any proposed point of the parabola $y^2=4ax$. Show that tangents equal in length cannot be drawn to a parabola from an external point, unless that point lies in the axis produced.

16. Find the maximum or minimum value of

$$3x^4+4x^3-6x^2-12x+13.$$

17. Find the nth differential coefficients of (1) $x^2 \log x$, and (2) x^3e^x.

18. Integrate:—

$$(1)\ \frac{xdx}{\sqrt{a^4-x^4}}; \qquad (2)\ \frac{xdx}{(x+1)(x+3)};$$

$$(3)\ \frac{d\theta}{\cos^2\theta-\sin^2\theta}; \qquad (4)\ \frac{1+\cos\theta}{\theta+\sin\theta}d\theta.$$

CXLV.

1. An artisan going to his work every week-day is obliged to pay $\frac{1}{2}d.$ toll, the toll being converted into one of 2 mils. How much will he gain in a year by the change of currency?

2. What amount of 3 per cent. stock will produce 1*d.* a day interest?

3. Reduce 2 hrs. 9 min. 10 sec. ·7 sec. to the decimal of a *day*.

4. A besieged place garrisoned by 10000 men was victualled for 27 days, but after 9 days, 2500 men cut their way out. How long would the provisions last the survivors, the rations remaining unchanged?

5. Solve the equation $3x^2-14x+15=0$.

6. What relations must subsist amongst the coefficients A, B, C, D, in order that Ax^3+Bx^2+Cx+D may be a complete cube?

7. If a, b, B are given to solve a triangle, $b<a$; and if c_1, c_2 be the two values found for determining c, the third side, prove that $b^2+c_1 c_2=a^2$.

8. Given $\tan(a+x)\tan(a-x)=\dfrac{1-2\cos 2a}{1+2\cos 2a}$, find $\sin x$.

9. What are the values of sin 30°, cos 30°, sin 45°, cos 45°? Deduce from them the value of sin 15° to 6 places of decimals.

10. In the triangle ABC given $a=3492{\cdot}76$, $b=2471{\cdot}34$, $B=20°\ 21'$ to find c.

11. The boundary of a breakwater seen from either A or B, two stations 1250 yards apart (of which A is due south of one extremity, B due east of the other), subtends an angle of 15°. Find the rectilineal distance between the ends of the breakwater.

12. How many cubic feet of water are contained in a ditch shaped like the frustum of a wedge, 120 yds. long, 6 ft. deep, 10 yds. broad at the top and 4 at the bottom?

13. What is the function of an independent variable? Explain what is meant by the differential coefficient of such a function.

14. Exhibit, without proving it, the ordinary form of Taylor's theorem, and deduce from it the theorem known as Maclaurin's.

15. Expand $\sin x$ in terms of x to three terms by Maclaurin's theorem; and express the general term of the series for $\sin x$.

16. If a tangent be drawn at any point (x, y) of a plane

curve, examine the trigonometrical meaning of $\frac{dy}{dx}$ with reference to that tangent.

17. If C be the centre of an ellipse, CP its semi-axis minor, PT, PG, PN a tangent, a normal, and an ordinate, at any point P, cutting the axis major, produced if necessary, in T, G, N, respectively: prove that $CT \cdot NG = BC^2$.

18. Integrate the functions—

$$(1)\ \frac{a\,dx}{a^2-x^2}; \qquad (2)\ \frac{x\,dx}{(a^2-x^2)^{\frac{7}{2}}};$$

$$(3)\ \frac{\tan^2 x}{4+\tan^2 x}; \qquad (4)\ \frac{1}{x\sqrt{x^2+2x+2}}.$$

CXLVI.

1. In a competitive examination ·07 of the candidates fail to qualify, $\frac{12}{31}$ of the remainder are unsuccessful, 114 are elected; how many candidates presented themselves?

2. When a vulgar fraction is given in its lowest terms, show under what circumstances it can be converted into a terminating decimal. Predict the number of places in the decimal equivalent to $\frac{17}{3200}$.

3. Show whether 14·7 or 14·673 is nearer to $\sqrt{216}$.

4. Solve the equations—

(1) $4\sqrt{\sqrt{x}+6}+3(\sqrt{x}+3)=x$;

(2) $x^3=2x+2$, by Cardan's method.

5. Sum the series—

(1) $80+74+68+\ .\ .\ .$ to 14 terms;

(2) $1-\frac{2}{3}+\frac{3}{3^2}-\frac{4}{3^3}+\ .\ .\ .$ to n terms.

6. In the expansion of $(1+x)^{-8}$, $x > 1$, prove that the terms continually increase in numerical magnitude.

7. Compute by logarithms the value of

$$\frac{7\times 8{\cdot}73}{{\cdot}54963};$$

show that in a table of logarithms we may expect to find the 'differences' largest at the early part of the table and gradually decreasing.

8. Prove the formula—

$$\frac{\sin 5\,A-\sin 3\,A}{\cos 5\,A-\cos 3\,A}=-\cot 4\,A.$$

9. An observer, whose eye is 5 ft. above the ground, observes a vertical object 100 ft. high standing on the same horizontal plane with him, and finds the angular distance between its highest and lowest points to be 30°. What is the distance of the object from him?

10. Find the length of the side of an equilateral triangle inscribed in a circle whose radius is 1 yard.

11. Show that the distance of any point in the circumference of the circle, from the remotest angle of the triangle, is equal to the sum of its distances from the other angles.

12. ABC is a triangle: bisect AB, AC in G, F; join CG, BF, cutting one another in O. Prove that the triangle GOF is equal to $\frac{1}{12}$ of the whole triangle ABC.

13. Two places on the earth's surface are in latitude 45°N., and their difference in longitude is 90°. Find the distance between them (1) measured on the parallel of latitude, (2) measured on a great circle.

14. If through any point within or without a given ellipse two lines be drawn parallel to two given straight lines, to meet the ellipse, the rectangles of the segments will be to each other in an invariable ratio.

15. If an ellipse and a circle intersect in 4 points, show that the common chords make equal angles with the major axis of the ellipse.

16. Find the equation to the right line joining the centres of the two circles $x^2+y^2+6x=16$, $x^2+y^2+10y=144$, and determine whether the circles intersect or not.

17. Prove that the axis of the greatest parabola that can be obtained by the section of a given finite right cone $=\frac{3}{4}$ the length of the slant side.

18. Show how the integral calculus is applied to determining the lengths of curves, the areas of surfaces, and the volumes of solids.

CXLVII.

1. A merchant imports goods and pays 3 per cent., for freight and duty, on the cost price; he sells to the retailer at a profit of 15 per cent. on his whole outlay; the retailer sells to the consumer at a profit to himself of 25 per cent. Find the cost to the merchant of goods sold to the consumer for £2369.

2. Find by the aid of tables—

$$(1)\ \sqrt[3]{\cdot 08765};\ (2)\ \frac{256\cdot 5}{\cdot 045}\{(1\cdot 045)^{14}-1\}.$$

3. Prove that—

$$\frac{2(x+2+\sqrt{x^2-4})}{x+2-\sqrt{x^2-4}}=x+\sqrt{x^2-4}.$$

4. Solve the equations—

$$\left\{\begin{array}{c} x^{-1}+y^{-1}+z^{-1}=13 \\ y^{-1}-x^{-1}=1 \\ x^{-1}y^{-1}-2z^{-1}=0 \end{array}\right\}$$

5. The first 4 terms of a geometrical progression are together equal to 45, and the first 6 to 189. Find the series.

6. Show how to find the coefficient of any assigned power of x in the expansion of $(a+bx+cx^2+\&c.)^n$ when n is a positive integer; and find the coefficient of x^4 in the expansion of $(1+3x+4x^2)^3$.

7. Given the sides of a triangle a, b, c, find an expression for $\sin^2 \frac{1}{2}A$; show that the formula is always positive, and that the numerator is less than the denominator.

8. When is it unadvisable to use the formula for $\sin \frac{1}{2}A$ in solving a triangle from the data of the sides?

9. An observer standing on a horizontal plane, observes

the angle of elevation of the top of a mountain to be $63° 25'$; walking 1 mile towards the mountain, on an ascent that makes an angle of $30°$ with the horizon, he finds the elevation of the mountain top to be $74° 25'$. Find the height of the mountain above the horizontal plane.

10. The areas of two regular polygons, of the same number of sides, inscribed in a circle, and described about it, are as 3 : 4. Find the form of the polygon.

11. What is the angle of inclination of two faces of a tetrahedron?

12. If A B C be the section of a right cone, made by a plane passing through its vertex and its axis, B C the common section of this plane with the base of the cone; prove that the common section of any plane, perpendicular to A B C with the base, will be perpendicular to B C.

13. Find the differential coefficient in respect to x of x^m.

14. Find the equation to the tangent drawn to any point of the locus $x^2+y^2=c^2$, and prove that it is always perpendicular to the radius vector.

15. If ϕ be the angle which the normal to a curve makes with the radius vector r, from the pole at the point (r, θ), prove that $\sin\phi=\frac{dr}{ds}$, $\cos\phi=\frac{rd\,\theta}{ds}$.

16. Obtain the integral $\int_0^\pi r \sin\theta\, d\,\theta$ in the curve where $r=a\,\theta$.

17. Integrate the functions—

(1) $\int\frac{dx}{a+b\cos x}$; (2) $\int\frac{dx}{\sqrt{x^2+a^2}}$; (3) $\int\theta\sin\theta\, d\,\theta$.

18. Investigate an expression for the volume of an ellipsoid.

CXLVIII.

1. Reduce $\dot{2}8571\dot{4}$ and $\cdot 2\dot{1}4285\dot{7}$ to vulgar fractions in their lowest terms.

2. If the population of London be 2500000, and each family be taken to consist of 5 persons, and to consume $\frac{1}{4}$ of a ton of coals per week, what would be the cost of the coals consumed in London in a year of 52 weeks, at 25*s.* per ton?

3. Find, by the rule of Practice, the value of 57689 things at £2 13*s.* $7\frac{1}{2}$*d.* each.

4. Find the greatest common measure of x^5+5x^3+6 and $x^4+40x+39$.

5. How many sheep must a farmer buy at 20*s.* each, that losing 5 and selling the remainder at 25*s.* each, he may gain £15?

6. Prove that if the difference of the sums of the even and odd digits of a number be divisible by 11, the number itself will be a multiple of 11.

7. Given $x^3+9x-16=0$, find an approximate value of x to 5 places of decimals.

8. Solve the equation $x^4-3x^3-6x^2+28x-24=0$, which has equal roots.

9. Of a plane triangle, given $a=4107$, $b=1866$, $C=61° 53'$, find A, B, and c.

10. The mortar for a rod of brickwork requires $1\frac{1}{2}$ cubic yards of lime, at 12*s.* per yard, 3 cubic feet of sand at 3*s.* 6*d.* What is the cost of the lime and sand required for building a circular tunnel whose internal diameter is 6 feet, and which is $3\frac{1}{2}$ bricks thick and 10 feet long?

11. Standing on the paddle-box of a steamer, steaming 14 miles an hr., on a straight course, I see an object across the bows; $\frac{1}{2}$ an hr. afterwards I see it in a line with the funnel. Find the distance of the object from each point of

observation: the spot on which I stand being 90 ft. from the funnel, and 150 ft. from the bows; and the funnel and bows being 190 ft. apart.

12. Find the number of square yards in a regular pentagon measuring 20 feet round.

13. Investigate the conditions under which a function of a single variable becomes a maximum or a minimum.

14. A rectangular plot of ground of given area is to be enclosed from the waste by a wall and divided into three equal areas, by partition walls parallel to one of its sides. What must be the dimensions of the rectangle that the length of walling may be a minimum?

15. Expand, by Maclaurin's theorem, $\tan^4 x$ in terms of x to three terms.

16. Integrate the functions:—

$$(1) \int a^{m+x}dx\,; \quad (2) \int \frac{dx}{x\sqrt{a^2+x^2}}\,; \quad (3) \int x^3(1+x^2)^{-\frac{1}{2}}dx.$$

17. Find the differential coefficient of the length of the arc of a plane curve referred to rectangular coordinates.

18. The equation to the cycloid from the vertex being $\frac{dy}{dx}=\sqrt{\frac{2a-x}{x}}$, show that its whole length is $8a$.

CXLIX.

1. Reduce to its simplest form $\frac{1\frac{1}{8}}{1\frac{1}{2}}$; and find how many French mètres, each measuring 39·37079 inches, there are in a mile.

2. If 48 men, working 8 hrs. a day for 1 week, can dig a trench 135 feet long, 40 wide, and 28 deep, in what time can 12 men, working 10 hrs. a day, form a railway cutting containing 131250 cubic yards?

3. A book is published at a cost of 5*s*. a copy, and sold at 10*s*. a copy. Now, if it had been sold at 6*s*. 8*d*. a copy the annual sale would have been 200 copies more, the edition

would have been sold off 2 years sooner, and it would have begun to pay profit 1 year later. How many copies were published? Interest of money to be neglected.

4. A government contractor imported a number of mules, which he bought for delivery at 10 guineas per head under the contract price, at a total cost of £350, and although 7 of them died on the passage he realised a profit of £14 on the transaction. How many mules did he sell to the government?

5. Two trains set out at the same time, one from L to P, the other from P to L; the latter travelled (on the average, stoppages included) 2 miles an hour faster than the former from P to E, and 4 miles an hour faster from E to B. The two trains arrive at the same instant at B. At what rates did they respectively travel? The distance from L to B being 118 miles, that from B to E 75 miles, and that from E to P 53 miles.

6. Find the coefficient of x^5 in the expansion of $(2-3x)^9$.

7. State and prove the rules for finding the logarithm of a number not set down in the tables, or the number corresponding to a logarithm not exactly to be found in the tables.

8. Has the equation $x^4-18x^3+36x^2+418x+363$ equal roots? Solve it.

9. A B, A C, are two railroads inclined at an angle 50° 20′; a locomotive engine starts from A along A B at the rate of 30 miles an hr.; after an interval of 1 hr., another locomotive engine starts from A along A C, at the rate of 45 miles an hr.: find the distance between the engines 3 hrs. after the first started.

10. The perimeter of a right-angled triangle is 24 feet, and its base is 8 feet. Find the other sides.

11. Looking down from the top of a hill, the foot of which is 125 feet below the level of his eye, a man sees a statue *40 ft.* high, and the column 60 ft. high upon which it stands,

subtending equal angles at his eye. How far off is he in a horizontal direction from the object?

12. A regular hexagon, each side of which is 10 ft., revolves about a line which joins the points of bisection of two opposite sides; find the whole surface of the solid thus generated.

13. Determine the geometrical signification of the equations—

$$(1)\ 10x^2-xy-21y^2-9x-y+2=0.$$
$$(2)\ x^2+y^2+6x-10y+34=0.$$

14. Prove that if $u=\frac{z}{y}$, $\frac{du}{dx}=\frac{y\frac{dz}{dx}-z\frac{dy}{dx}}{y^2}$.

Write down the first and second differential coefficients of

$$(1)\ x^x;\ (2)\ xe^{\tan x};\ (3)\ \tan^{-1}x.$$

15. Prove the formula of integration by parts.

16. Find the area of a portion of a common parabola.

17. Prove that the greatest rectangle which can be inscribed in an ellipse is half of that contained by the axes.

18. If r be the radius vector at any point of a curve, and p the perpendicular from the pole upon the tangent, prove ρ the radius of curvature equal to $\frac{rdr}{dp}$.

In the ellipse, whose semi-axes are a, b given $p^2=\frac{b^2r}{2a-r}$, find the radius of curvature drawn at the extremity of the major axis.

CL.

1. Explain the rule for multiplying a number consisting of several digits by a like number, by means of the example 356×125.

2. What is meant by a proper fraction $\frac{a}{b}$? Explain the rule for dividing it by a whole number c.

3. Simplify $\dfrac{a^2-b^2}{1-\dfrac{a}{b}+\dfrac{a^2}{b^2}} \times \dfrac{1-\dfrac{b}{a}+\dfrac{b^2}{a^2}}{1-\dfrac{b^2}{a^2}}$.

4. Solve the equations—

$$(1)\ x^4+8x=4\,(x^2+1); \qquad (2)\ \begin{Bmatrix} 3xy-y^2=8 \\ 4x^2-xy=33 \end{Bmatrix}$$

5. Find 3 fractions converging to $\sqrt{26}$.

6. Show that impossible roots enter equations in pairs. What assumption is here made respecting the coefficients of the equations? In what case will irrational roots enter in pairs?

7. If a be an arc of a circle subtending an angle θ, and r the radius of the circle, show that

$$\theta=\frac{2\text{ right-angles}}{\pi}\times\frac{a}{r}\ \text{where } \pi=3{\cdot}14159.$$

What will this become if we take an angle one-sixth of a right angle as the angular unit?

8. If $\tan 2\theta=\sqrt{3}$, express all the positive values of θ which satisfy the equation.

9. If A, B, C be the angles of a spherical triangle, and $A+B+C=2S$, show that $\tan\frac{a}{2}=\sqrt{\dfrac{-\cos S\cos(S-A)}{\cos(S-B)\,(S-C)}}$; a being the side opposite the angle A.

Show that this expression always expresses a possible quantity.

10. Differentiate $y=\dfrac{(x+1)^2}{x^2+1}$; $y=e^{\sin x}$; $y=\log_a \sin^{-1}x$.

11. Determine the maxima and minima of $f(x)$, when

$$f(x)=(x-2)^4\,(x-4)^2.$$

12. Determine the ratio between the height and radius of the base of a cylindrical quart cup so that its surface may be a minimum.

13. Show that if $y'=f(x')$ be the equation to a curve, the *equation to* the tangent is $y-y'=f'(x')\,(x-x')$.

14. Expand as far as x^4—

$$(1)\ \log\left(x+\sqrt{x^2+a^2}\right);\quad (2)\ (e^x+e^{-x})^n.$$

15. Prove the following differential formulæ which occur in the theory of plane curves:—

$$(1)\ \text{Subtangent}=y\frac{dx}{dy};\quad (2)\ \text{Subnormal}=y\frac{dy}{dx};$$

$$(3)\ \frac{ds^2}{dx^2}=1+\frac{dy^2}{dx^2}.$$

16. Integrate with respect to x, the functions:—

$$(1)\ \frac{1}{x^2+a^2};\quad (2)\ \frac{x}{x^4+a^4};\quad (3)\ x^3\cos x.$$

17. Prove (1) that $\int_1^0 \frac{1}{x}\log(1-x)\,dx=\frac{\pi^2}{6}$; (2) if m and n be positive integers, $m \mathrel{7} n$, $\int_0^\infty \frac{x^{2m}dx}{1+x^{2n}}=\frac{\pi}{2n\sin\frac{2m+1}{2n}\pi}$.

18. Find the relation between a and b, that the envelope of $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$ may be $xy=c^2$.

MIXED MATHEMATICS.

FORMULÆ IN STATICS.

R = resultant of P and Q acting at an angle θ,

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta; \text{ if } P=Q,\ R=2P\cos\tfrac{1}{2}\theta.$$

$$\theta=0,\ R=P+Q;\quad \theta=\pi,\ R=P-Q.$$

If P, Q, R are in equilibrium at a point O,

$$P : Q : R = \sin QOR : \sin POR : \sin POQ.$$

R = resultant of any number of forces acting upon a body in one plane.

$$R^2 = \{\Sigma(X)\}^2 + \{\Sigma(Y)\}^2 \ :\ \tan\theta = \frac{\Sigma(Y)}{\Sigma(X)};$$

in case of equilibrium $\Sigma(X)=0 : \Sigma(Y)=0.\ \Sigma(Yx-Xy)=0.$

R = resultant of any number of forces acting upon a body in any directions,

$$R^2 = \{\Sigma(X)\}^2 + \{\Sigma(Y)\}^2 + \{\Sigma(Z)\}^2;$$

$$\cos\theta = \frac{\Sigma(X)}{R} : \cos\phi = \frac{\Sigma(Y)}{R} : \cos\xi = \frac{\Sigma(Z)}{R} :$$

in case of equilibrium $\Sigma(X)=0,\ \Sigma(Y)=0 : \Sigma(Z)=0;$

$\Sigma(Zy-Yz)=0;\ \Sigma(Xz-Zx)=0;\ \Sigma(Yx-Xy)=0.$

If $\bar{x}$, $\bar{y}$, be the coordinates of the centre of gravity of a system of bodies,

$$\bar{x} = \frac{\Sigma(P.x)}{\Sigma(P)} : \ \bar{y} = \frac{\Sigma(P.y)}{\Sigma(P)}.$$

In the lever,

$$\frac{P}{W} = \frac{\text{perpendicular from fulcrum on W's direction}}{\text{perpendicular from fulcrum on P's direction}}.$$

Pressure on fulcrum $=[\text{P}^2+\text{W}^2-2\,\text{PW}\cos(\alpha+\beta)]^{\frac{1}{2}}$.

Direction of pressure $=\theta$; $\tan\theta=\dfrac{\text{P}\sin\alpha+\text{W}\sin\beta}{\text{P}\cos\alpha-\text{W}\cos\beta}$.

In the wheel and axle $\dfrac{\text{P}}{\text{W}}=\dfrac{\text{rad. of axle}}{\text{rad. of wheel}}$.

Single moveable pulley, $\dfrac{\text{W}}{\text{P}}=2\cos\alpha$; $2\alpha=$ angle between the strings.

System of pulleys each hanging by separate string—

$$\text{P}=\frac{1}{2^n}\{\text{W}+(2^n-1)w\},\ n \text{ being no. of moveable pulleys.}$$

System of pulleys, same string passing round all the pulleys, $\text{W}+\text{B}=n\,\text{P}$. $\text{B}=$ weight of block.

System of pulleys, when all the strings are attached to the weight, $\text{W}=(2^n-1)\,\text{P}+(2^n-n-1)\,w$; n being no. of strings attached to weight.

Inclined plane, smooth; $\dfrac{\text{P}}{\sin\alpha}=\dfrac{\text{W}}{\cos\epsilon}=\dfrac{\text{R}}{\cos(\alpha+\epsilon)}$.

Inclined plane, rough, $\text{P}=\dfrac{\text{W}\sin\alpha\mp\mu\,\text{W}\cos\alpha}{\cos\epsilon\mp\mu\sin\epsilon}$.

Screw, smooth—

$$\frac{\text{P}}{\text{W}}=\frac{\text{vertical distance between two threads}}{\text{circumference of circle described by P}}.$$

Screw, with friction—

$$\frac{\text{P}}{\text{W}}=\frac{b(\sin\alpha-\mu\cos\alpha)}{a(\cos\alpha+\mu\sin\alpha)}=\frac{b}{a}\tan(\alpha-\epsilon),\ \text{if } \mu=\tan\epsilon.$$

In the catenary $y+c=\sqrt{c^2+s^2}$;

$$x=c\log\frac{y+c+\sqrt{y^2+2yc}}{c};\quad y'=\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right);$$

$$t=y+c;\quad \sqrt{(\lambda^2+k^2)}=c\left(e^{\frac{h}{2c}}-e^{-\frac{h}{2c}}\right).$$

FORMULÆ IN DYNAMICS.

Motion uniformly accelerated—

$$v=ft;\ s=\tfrac{1}{2}ft^2;\ v^2=2fs;\ u=v\pm ft;$$
$$s=vt\pm\tfrac{1}{2}ft^2;\ u^2=v^2\pm 2fs.$$

When the body moves freely $f=g=32{\cdot}2$: on inclined plane $f=g\sin\alpha$: if P moving on an inclined plane draw Q up an an inclined plane $f=\dfrac{P\sin\alpha-Q\sin\beta}{P+Q}g$.

Impact: A moving with velocity a, impinges on B whose velocity is b.

Velocity of A after impact$=\dfrac{Aa\pm Bb-Be(a\mp b)}{A+B}$.

Velocity of B after impact$=\dfrac{Aa\pm Bb+Ae(a\mp b)}{A+B}$.

Projectiles in vacuo—

Range at end of time $t=vt\cos\alpha$;
height$=vt\sin\alpha-\frac{1}{2}gt^2$.

Equation to path $y=x\tan\alpha-\dfrac{x^2}{4h\cos^2\alpha}$ where $h=\dfrac{v^2}{2g}$.

Time of flight$=\dfrac{2v\sin\alpha}{g}$: horizontal range$=\dfrac{v^2\sin 2\alpha}{g}$.

Greatest height$=\dfrac{v^2\sin^2\alpha}{2g}=h\sin^2\alpha$.

Range on inclined plane$=\dfrac{4h\cos\alpha\sin(\alpha-i)}{\cos^2 i}$:

time of flight$=\dfrac{2v\sin(\alpha-i)}{g\cos i}$.

Simple pendulum: $t=\pi\sqrt{\dfrac{l}{g}}$.

A body moving in a circle: accelerating force in direction of normal$=\dfrac{v^2}{r}$; moving force$=\dfrac{mv^2}{r}$.

Conical pendulum $v^2=\dfrac{gl\sin^2\alpha}{\cos\alpha}$; $t=2\pi\sqrt{\dfrac{AC}{g}}$.

FORMULÆ IN HYDROSTATICS.

Normal pressure on area a^2, the depth of whose centre of gravity below surface of fluid is h=weight of volume a^2h of the fluid.

Specific gravity of a substance

$$=\frac{\text{weight of any volume of substance}}{\text{weight of same volume of water}}.$$

In a compound body: $G=\frac{V_1\rho+V_2\rho+V_3\rho+\&c.}{V_1+V_2+V_3+\&c.}$; when there is no change of volume.

Density of air in receiver of air pump after n strokes $=\left(\frac{A}{A+B}\right)^n\rho$.

Bramah's press, $\frac{P}{W}=\frac{b\,r^2}{a\,R^2}$.

In diving-bell tension of string $=$ weight of bell $-g\rho Ax$ where $A=$ area of top of bell, $x=$ length occupied by air, is found from the equation $hb=(h+a)\,x+x^2$, $b=$ length of bell, $a=$ depth of its top. (*Besant.*)

FORMULÆ IN PRACTICAL MECHANICS.

Unit of work $=$ pressure of 1 lb. exerted through a space of 1 foot.

Work done in moving resistance of m lbs. through n feet $=m\,n$ units.

Unit of horse power $=$ 33000 units of work.

Modulus of a machine $=\frac{\text{work yielded}}{\text{work expended}}$.

Work done in moving a body on a horizontal plane $=f\,.\,Ws$ where $f=$ coefficient of friction, $W=$ weight of body, $s=$ space described.

Work done in moving a body up an inclined plane $=$ work due to friction $+$ work due to force of gravity $=Wh+f\,.\,Wl$: h being the height, l the length of plane, whose inclination is very small: otherwise work done $=Wh+f\,.\,W\cos a\times l$.

Work accumulated in body, moving with velocity v, $=\frac{Wv^2}{2g}$.

Work done in upsetting a heavy body $=$ work requisite to

raise the body vertically through the height which its centre of gravity is raised.

Work done in raising material of given form=weight of material in lbs. × number of feet through which C. G. is raised.

Moment of inertia of a system of bodies

$$=m_1r^2_1+m_2r_2^2+m_3r_3^2+\&c.=\text{I}.$$

Radius of gyration

$$=k=\sqrt{\frac{\text{I}}{m}}=\sqrt{\frac{m_1r_1^2+m_2r_2^2+m_3r_3^2+\&c.}{m_1+m_2+m_3+\&c.}}$$

If I=moment of inertia about an axis passing through C. G. I_1=moment about an axis at a distance a from former, $\text{I}_1=\text{I}+a^2m$; and on similar suppositions $k_1^2=k^2+a^2 \therefore k_1^2-a^2$ is constant.

Radius of gyration of wheel$=\dfrac{\text{radius of wheel}}{\sqrt{2}}$.

Radius of gyration of rod revolving about its middle point $=\dfrac{l}{2\sqrt{3}}$.

Radius of gyration of rod revolving about its extremity $=\dfrac{l}{\sqrt{3}}$.

WOOLWICH PAPERS.

I.

1. Prove the '*Triangle of Forces.*'

2. A B C is an isosceles triangle, A and B the equal angles, C D a perpendicular from the vertex on the base, take G D$=\frac{1}{3}$C D; then if G A, G B represent two forces acting on a point at G, G C will represent the force that will keep the point G at rest.

3. Three forces in the plane of a triangular board act

upon its sides in directions perpendicular to them: prove that if the forces are in equilibrium, they are proportional to the sides on which they act.

4. Three forces, represented by 3, 4, and 5 pounds respectively, keep a particle in equilibrium: determine the angles at which they act.

5. Forces act on a triangle which are represented in magnitude and direction by the straight lines drawn from the vertices to the points of bisection of the opposite sides; show that they will maintain the triangle in equilibrium.

6. The resultant of two forces is 100 lbs., and the angles which it makes with the directions of the forces are 20° and 30°; find the component forces.

7. A heavy body is drawn up an inclined plane by means of a string, which being parallel to the plane, and passing over a pulley, has another heavy body hanging vertically at the end of it; find the accelerating force. If the plane be inclined at an angle of 30°, and the accelerating force be $\frac{1}{2}g$, find the ratio of the masses of the two bodies.

8. A weight Q is drawn along a smooth horizontal table by a weight P hanging vertically: find (1) the accelerating force on P, (2) the accelerating force on the centre of gravity of P and Q.

9. A body is projected with a velocity v in a direction making an angle of 30° with the horizon; find its velocity and direction of motion when it has reached a height $2g$. What is the horizontal space described in this time?

10. A body is projected horizontally with a velocity $4g$ from a point whose height above the ground is $16g$; find the direction of motion (1) when it has fallen half way to the ground, (2) when half the time of flight has elapsed.

11. Two particles are projected simultaneously from a given point with different velocities and in different directions, but in the same vertical plane; find the path described by their centre of gravity.

12. A lump of metal weighs 59 oz. in water and 61 oz. in alcohol, whose *s. g.* is ·8; find the weight and *s. g.* of the metal.

II.

1. Three forces act upon a particle in a plane; show how to find their resultant.

2. Three forces of 3, 4, 5 lbs. act on a particle in the centre of a square in directions towards three of the angles of the square: find the magnitude and direction of the force which will keep the particle at rest.

3. Can the forces 3, 5, 11 acting on a particle keep it at rest?

4. A force of 40 lbs. acting on a point between two forces of 20 lbs. and $20\sqrt{3}$ lbs. respectively, makes an angle of 60° with the former and 30° with the latter; find the magnitude and direction of the force which will keep the point at rest.

5. Two forces, P and Q, act upon a point; find the greatest and the least forces which can keep the point at rest. How will the directions of the forces in each case be adjusted?

6. At what angle must two forces, P and Q, act so that a single force which keeps them at rest shall be a mean proportional between the greatest and least values, as determined in *Quest.* 5?

7. A ball A impinges directly upon a ball B, in motion: show that the velocity of their centre of gravity is not affected by the impact.

8. Show that the time of a projectile's describing any arc of its path is the same as would be occupied in moving uniformly along the chord with the velocity which it has when its motion is parallel to the chord.

9. Find the direction in which a body must be projected with a given velocity to hit a given mark, and show generally that there are two directions that will satisfy the conditions.

10. A body oscillates once in $2{\cdot}5''$; find the distance between its centres of suspension and oscillation.

11. A cube of wood floating on water descends $\frac{1}{2}$ in. when a weight of 15 oz. is placed upon it; find the size of the cube, supposing a cubic foot of water to weigh 1000 oz.

12. A B C D is a parallelogram whose diagonals A C, B D intersect in E, and A B is in the surface of a fluid; show that when the fluid is homogeneous the pressures on the triangles A E B, B E C, C E D are as 1, 3, 5. Compare the pressures when the parallelogram is a square and the density of the fluid varies as the depth.

III.

1. Show how forces may be represented geometrically by straight lines. Define a moment, and show how it may be represented geometrically, in accordance with the above representation of a force.

2. Deduce the principle of equality of moments from that of the parallelogram of pressures.

3. There is a uniform bar, weighing 10 lbs., and 5 ft. long: weights of 9 lbs. and 5 lbs. are suspended from its extremities: on what point will it balance?

4. A uniform beam A B, whose weight is W, rests with one end on a smooth horizontal plane A C, and the other end B on a smooth plane C B inclined to the horizon at 60°. Find the tension of a string C A=C B which keeps the beam from slipping.

5. Two planes of equal altitude are inclined at angles of 60° and 45° to the horizon; required what weight resting on the latter will balance 20 lbs. on the former, the weights being connected by means of a string passing over the common vertex parallel to the planes.

6. The mouth of a harbour is $\frac{3}{4}$ mi. wide, and the tide is running out of it at the rate of $2\frac{1}{2}$ mi. an hour; in what direction must a man, who can row in still water at the rate

of 5 mi. an hour, point the head of the boat in order to make for a point directly opposite to him?

7. In *Qu.* 6, find the length of the man's course when he keeps the head of the boat pointed to the opposite shore, and determine the length of time employed in making this course, and compare it with that employed in making a course direct across the harbour.

8. A body has fallen from rest through a feet at the moment when a second body begins to fall, b feet below the point which the first then occupies; find the distance traversed by the second before it is overtaken by the first, the force of gravity being *constant*, but unknown in amount.

9. Find the inclination of a gradient in a railway that a carriage descending down it, by its own weight, may move through $\frac{1}{2}$ a mile in $30''$; find the space it will describe in the next $30''$.

10. Describe the *force-pump*, and explain its action.

11. The specific gravity of gold being taken as 19·3 and of copper as 8·6, find the specific gravity of an alloy containing 2 oz. of copper to 23 oz. of gold.

12. Find the depth at which a hemisphere floats with its base downwards, when the specific gravities of the fluid and solid are as 16 : 11.

IV.

1. Show that if θ be the angle between the directions of two given forces, their resultant is greatest when $\theta=0$; least when $\theta=\pi$; and intermediate for intermediate values of θ.

2. Find the resultant of two equal forces acting on a point.

3. Three forces, 99, 100, 101 lbs. respectively act upon a point in directions making an angle of 120° with each other, successively; find the magnitude of their resultant, and the angle which it makes with the force 100.

4. An endless band A B C D passes over three pulleys at

A, B, C, and a weight of 100 lbs. is attached to D. Supposing A B, C B each to make angles of 45°, and A D, C D angles of 30°, with the vertical, and A and C to be in the same horizontal line, find the magnitude and direction of the pressures on B, A, and C.

5. Four bodies weighing 1, 2, 4, 7 lbs., respectively, are placed with their centres of gravity in a straight line at the respective distances of 3, 5, 7, 9 ft., from a given point in that line; find the common centre of gravity.

6. Find the conditions of equilibrium of any system of forces acting in one plane. Account for only one of these conditions being necessary, in the case of a lever attached to a fulcrum.

A uniform beam A B, whose weight is W, rests in equilibrium between a vertical wall B C and the horizontal plane A C, both smooth; C E is a string without weight attached to a point E in the beam. If B A C$=\alpha$, and A C E$=\beta$, show that the tension of the string$=\dfrac{W \cos \alpha}{2 \sin (\alpha-\beta)}$.

7. If a bullet be fired from a level ground in a direction making an angle i with the horizon, and the horizontal range be R, find the time of flight.

8. If a body be projected from A in the direction A B C, and from C any point in that line a vertical line C D be drawn, meeting the curve described by the projectile in D, and if B, the middle point of the finite line A B C, be joined to D, show that B D will be the direction of the motion at D, and that the velocity at D will be to that at A as BD : A B.

9. How long will 10 men be in pumping dry the hold of a ship which contains 30,000 cubic ft. of water, the centre of gravity of the water being 14 ft. below the point of discharge, and each man yielding 1500 units of effective work per minute?

10. A uniform rod of length l and weight w resting vertically upon one extremity on a horizontal plane, is allowed to fall over. On what point must it strike a nail in

the plane, that the whole accumulated work may take effect upon the nail; and how far would it drive the nail, supposing the resistance constant?

11. Explain the construction of the '*Hydrometer.*'

12. A cylindrical pontoon floats with $\frac{3}{4}$ of its diameter out of water, but sinks to the water level when loaded with a weight of 32 cwt.: supposing the diameter of the external surface to be 3 ft., and a cubic foot of water to weigh 1000 oz., find the length of the pontoon.

V.

1. Prove the principle of the parallelogram of forces.

2. The two extremities of a string are fastened to two nails in a vertical wall, and the string is stretched tight by a weight tied to a given point in it: show how the tensions of the two parts of the string may be determined by geometrical construction.

3. If two equal forces act upon a point at an angle of 150°, what will be the magnitude of the resultant?

4. Two men pull, each with a force of 80 lbs., in different directions, making an angle of 60° with one another, at the end of a rope, passing freely round a fixed point. Find the pressure on the point.

5. A B C D is a square. A force of 3 lbs. acts from A to B, a force of 4 lbs. from B to C, a force of 6 lbs. from D to C, and a force of 5 lbs. from A to D. Find the magnitude and position of the resultant force.

6. Explain what is meant by the '*modulus of a machine.*'

7. A tank is 12 feet long, 7 feet wide, 8 feet deep; the surface of water in a well from which it is filled by a pump is 56 ft. below the bottom of the tank, the man working the pump performs 2600 units of work per minute, and fills the tank in 30 hrs.; find the modulus of the pump, a cubic *foot* of water weighing 1000 oz.

8. State the 'Third Law of Motion,' and show how it is applicable to the direct collision of elastic balls in determining their motions after impact.

9. A ball, $3m$, strikes a ball m at rest, the modulus of elasticity being $\cdot 6$; compare the velocities of the striking ball before and after impact.

10. A particle being projected with a velocity of 1610 ft. a second and an elevation of 30°, find the *latus-rectum* of its path, and the distance of the focus from the point of projection.

11. When a body floats in water, show that the weight of the body is equal to that of the fluid displaced.

12. A hemisphere whose weight is 5600 grs. floats with its vertex downwards, and $\frac{1}{3}$ of its vertical radius immersed; what weight must be placed on the hemisphere so that $\frac{2}{3}$ of the vertical radius may be immersed?

VI.

1. State fully the principle of the equality of moments. What other condition is necessary to the equilibrium of any number of pressures acting in the same plane?

2. O P, O Q represent two equal forces acting upon O, at right angles to each other, and O R represents the third force which keeps O in equilibrium; show that the triangle formed by the intersection of the lines drawn perpendicular to O P, O Q, O R, through P, Q, and R, is isosceles and right angled.

3. The resultant of two forces is 9 lbs., one of them is 6 lbs., and the other is inclined to the resultant at an angle of 30°; find the other force and the angle between the two.

4. A beam 45 ft. long balances itself on a point $\frac{1}{3}$ of its length from the thicker end, but when a weight of 15 lbs. is suspended from the smaller end, the prop must be moved 3 ft. towards it, to maintain equilibrium. Find the weight of the beam.

5. How is the nature of the equilibrium of a body or of

a system of bodies affected by the circumstance of the centre of gravity being in its highest or its lowest position ?

Examine the nature of equilibrium when P supports W on the common wheel and axle.

6. The mean section of a stream acting on the floats of a water-wheel is $5\frac{1}{3}$ by $1\frac{1}{2}$ ft. ; its mean velocity is 40 ft. a minute, it has a fall of $17\frac{1}{2}$ ft. ; the modulus of the wheel is ·7 ; it is required to raise water by means of the wheel to a height of 300 ft. How many cubic feet will it raise per minute ?

7. Two shots are fired from the same point with the same velocity, the elevation of one being a, that of the other $a+\delta$, where δ is a very small quantity ; find the intersection of their paths.

8. A body is projected upwards over a plane making an angle of 15° with the horizon ; find the direction of projection when the range is a maximum.

9. Two billiard balls in contact are struck by a third, moving in a direction perpendicular to the line joining their centres ; find the velocity of the latter after impact, v being the velocity of the impinging ball before impact, e the modulus of elasticity.

10. What must be the length of a pendulum to beat 9 times in 1′ at a place where the seconds' pendulum is 39·14 inches long ?

11. Show how to compare the specific gravities of two fluids by weighing the same solid in each.

12. The diameters of the tube and bellows being respectively $\frac{5}{8}$ in. and $1\frac{7}{8}$ in. in a hydrostatic bellows, how much will the weight upon the lid be raised above its original position when 5 lbs. of water is poured in through the tube, a cubic foot of water weighing 1000 oz. ?

VII.

1. Prove the '*Polygon of forces.*'

2. Three pressures, of 10 lbs. each, act on a point. The

first two are inclined at an angle of 60° to each other, and the third is inclined at an angle of 30° to the second; find their resultant.

3. A bar, each foot of which weighs 7 lbs., rests upon a fulcrum distant 3 ft. from one extremity; what must be its length, that a weight of $71\frac{1}{2}$ lbs. suspended from that extremity may just be balanced by 20 lbs. suspended from the other extremity?

4. A string with a weight W at one end, and running through a small loop C at the other, passes over two fixed points A, B, at the extremities of a line 4 ft. long and making an angle of 15° with the horizon; find the angles between this line and the oblique portions of the string when there is equilibrium. The length of the portion A C B A of the string being 12 ft.

5. Two forces act at a right angle, their resultant is double the less; what is the ratio of the forces?

6. A B C is a right-angled isosceles triangle, C being the right-angle. A force of 4 lbs. acts along A B from A to B; a force of 3 lbs. acts along C B from C to B; and a force of 2 lbs. acts along A C from A to C. Determine the magnitude and direction of the resultant.

7. If a body be projected up a given smooth inclined plane, show how to determine the space it will describe in a given time.

8. An inclined plane rises 1 ft. vertically for every 100 ft. in length; with what velocity must a body be projected up the plane that it may describe 183·9 ft. in 10 seconds, gravity being taken as 32·2 feet?

9. Define the terms, '*work*,' '*labouring force*,' '*unit of work*,' '*horse-power*.'

10. If a pressure of 1 ton is exerted through 20 yds., how many units of work are done? And if it is done by a steam engine in half a minute, what is the horse-power of the engine?

11. Describe the '*barometer*;' explain how it is used to determine the height of a mountain.

12. State the conditions of equilibrium of a floating body.

The weight of a cubic foot of water is 1000 oz.; a cone, whose specific gravity is half that of water, floats with its vertex downwards and axis vertical: find the weight of water which it displaces, the height of the cone being 30 in., the diameter of its base 9 inches.

VIII.

1. Find the condition of equilibrium in the system of pulleys where the same string passes round all the pulleys.

2. A rectangular table stands on a rough inclined plane, and has two sides horizontal; if the coefficient of friction between the lowest feet and the plane be μ, and between the highest feet and the plane μ_1, find the inclination of the plane when the table is on the point of moving.

3. Two heavy spheres, the radii of which are 2 and 1·5 in. respectively, are suspended at the opposite extremities of a horizontal straight rod $7\frac{7}{12}$ ft. long, where does the direction of their resultant cut the rod?

4. A uniform heavy string has its ends fixed; determine the form of the string when in equilibrium. If the length of the string be 100 ft., and the two fixed points be in the same horizontal plane and 90 ft. apart, compare approximately the tension at the lowest point with the weight of the string.

5. When an isosceles wedge is urged by a given pressure between two obstacles, show how to find the resistance of the obstacles.

6. An isosceles wedge whose angle is 120°, is urged by a pressure of 2 cwt. between two obstacles, and is in a state bordering on motion forwards; if 30° be the limiting angle of resistance, find the resistance of each obstacle.

7. Prove that the portion of the centrifugal force at the earth's surface varies as the square of the cosine of the *latitude*.

8. Two bodies are let fall at an interval of $1''$, how far will they be apart after an interval of $5''$, from the fall of the first?

9. The plane of a circle being vertical, prove that the time of descent down all chords drawn from either extremity of the vertical diameter will be the same. Find the line of quickest descent from a given point to a given circle.

10. A body moving on a rough inclined plane is subject to a constant resistance equal to $\frac{1}{3}$ of its weight: how long will it continue in motion, and how far will it go, if started with a velocity of 30 ft. a second?

11. A weight of 12 oz. draws up another of 6 oz., by means of a string passing round a fixed pulley; determine the pressure on the pulley.

12. A cylindrical tube, 2 ft. long and closed at one end, contains mercury which occupies 16 in. of its length, the tube is then inverted (without any of the air escaping), and 2 in. of its length from the open end introduced into a basin of mercury: determine the space which the air will occupy in this position. The height of the mercury in the standard barometer being 30 inches.

IX.

1. Show that every system of particles has one centre of gravity, and only one whilst in the same position.

2. If in the system of pulleys in which each string is attached to the weight, these strings are fastened to a bar at a distance of 2 inches from each other, this being the radius of each pulley; find from what point the weight must be suspended in order that the bar may be kept horizontal, the weights of the pulleys being neglected.

3. What is meant by the '*limiting angle of friction?*' Show that when a particle is placed on a rough horizontal plane and acted upon by a pressure towards the plane, the condition of its moving or remaining at rest depends upon the direction of the pressure, and not upon its magnitude.

4. A uniform beam, whose length is $2b$ and weight w, rests upon a given rough cylinder, of radius r, in a vertical plane; determine the weight w that must be suspended from one extremity of the beam so that it may just be on the point of sliding off the cylinder, the coefficient of friction being μ.

5. Two equal weights are made to balance on two inclined planes with a common vertex, by means of a string passing freely over a fixed pulley situated above the vertex: compare the angles which the string makes with the planes, the length of one plane being double that of the other.

6. Prove the formulæ $s=ut-\frac{1}{2}ft^2$; $v^2=u^2-2fs$.

7. With what velocity must a body be projected vertically upwards that it may just rise to the height of 1600 ft.? and what time will it take in rising, supposing $g=32$ ft.?

8. A shot is fired with a velocity v towards a tower whose height is h, and whose horizontal distance is one-half of the greatest range attainable with the given velocity. Find between what limits the elevation must lie in order that the shot may hit the tower.

9. A particle describes a circle with uniform velocity, show that the force on the particle is directed towards the centre, and determine its magnitude.

10. Find the time of oscillation of a simple pendulum in a small circular arc.

Taking the force of gravity at a station to be 32·2 ft., find the length of the pendulum which will oscillate in 1·5″.

11. A diving-bell is lowered in water until its mouth is 18 ft. below the surface, and the water 2 ft. high in the bell; supposing the bell when full to contain 1000 lbs. of water, find how much the tension of the rope is diminished when air is forced into the bell until it is free from water.

12. A triangle A B C is immersed in fluid in such a position that the point A is in the surface, and the lines A B, A C *equally inclined* to it, B C being produced to meet the surface

in E; show that the pressures on the triangles A B C, A C E are as $AB^2 - AC^2 : AC^2$.

X.

1. If two surfaces are in contact, how is the condition of their being (1) smooth, (2) rough, expressed in mechanical problems?

2. If two rough bodies rest on an inclined plane at a distance from each other equal to the length of a string connecting them, and if the coefficients of friction for the two bodies be different, find the greatest inclination the plane may have without their sliding.

3. A frustum of a right cone has a height h, the radii of its ends, r and r_1, $r > r_1$; determine the distance of its centre of gravity from the centre of its smaller end.

4. Find the power which will just support a weight of 100 lbs. by means of a system of 4 pulleys, the strings being parallel and all attached to the weight, each moveable pulley weighing 1 lb.

5. A rod without weight is 2 ft. long; at one end a force of 4 lbs. acts, at the other end a force of 8 lbs., and these forces are parallel. Find the magnitude and point of application of a single force which will keep the rod at rest.

6. At the earth's surface $g = 32{\cdot}2$ ft., and the space through which a body falls in 1″ from rest is 16·1 ft., explain and account for this relation. Find the velocity acquired by a body falling freely 6440 feet.

7. Supposing the force of gravity to vary inversely as the square of the distance from the earth's centre, find the space through which a body would fall in 1 hour at a distance of 240,000 miles from the earth's centre, the radius of the earth being 4,000 miles.

8. Find the line of quickest descent from a given point to a given fixed straight line in the same vertical plane.

9. A ball of 4 lbs. weight moving from left to right with

a velocity of 8 yds. per second, impinges directly upon a ball of 10 lbs. weight moving in the same direction with a velocity of 2 yds. per second: the first ball is reduced to rest by impact, find the modulus of elasticity.

10. In the conical pendulum the length of the string is 6 ft., and the weight describes a circle in $\cdot 5''$: compare the weight and the tension of the string.

11. The *s. g.* of gold is 19·25, that of copper, 8·9. How many cubic inches of copper must be mixed with 12 cub. in. of gold so that the *s. g.* of the compound may be 18·45?

12. A cubical box filled with 1000 oz. of water, is supported in such a position that one edge is horizontal, and one of the edges, perpendicular to it, inclined at an angle of 45° to the horizon: find the total pressure on the six faces of the cube.

XI.

1. Define the '*centre of gravity*;' and find the centre of gravity of a pyramid.

2. Prove that the centre of gravity of the perimeter of a triangle is the centre of the circle inscribed in the triangle formed by joining the middle points of the given triangle.

3. A string fastened to a nail, and hanging vertically, supports the apex of a uniform isosceles triangle of weight w, which rests with its base upon a smooth horizontal plane; find the tension of the string and the pressure on the plane.

4. A bar of iron 15 inches long, which weighs 12 lbs., and is of uniform thickness, has a weight of 10 lbs. suspended from one extremity; where must a fulcrum be placed that it may just balance upon it?

5. Illustrate the statement, 'The number of units of work yielded by any agent in a given time is a measure of its efficiency.'

6. The diameter of the piston of a steam engine is 80 in., the mean pressure of the steam 24 lbs. per square inch, the length of stroke 10 feet, the number of strokes per minute 11.

How many cubic feet of water will it raise per minute from a depth of 25 fathoms, the modulus of the engine being ·6, and a cubic foot of water weighing 62·5 lbs.?

7. Define the terms, '*centre of oscillation*,' '*centre of percussion*,' '*angular velocity*.'

8. A uniform rod of given length and weight rests by one extremity upon a horizontal plane, and is allowed to fall over (without slipping) from a vertical to a horizontal position. Find the angular velocity which it will acquire.

9. The monument in London is said to be 210 feet high; (1) with what velocity must a shot be fired from its base, vertically, to reach the top in ·75″? (2) what will be its velocity on passing the top? (3) how high will it rise?

10. In what time will a shell range 4500 feet at an elevation of 40°, neglecting the resistance of the air?

11. If α be the angle between the two tangents at the extremities of any arc of the parabolic path of a particle acted on by gravity, v and v' the velocities at these extremities, u the velocity at the vertex; show that the time of describing the arc is $\dfrac{v\,v' \sin\alpha}{g\,u}$.

12. A cylindrical pontoon 12 feet long and 3 feet in diameter, floats with $\frac{2}{3}$ of its diameter immersed; what additional weight in pounds will it bear without sinking, taking 1000 oz. as the weight of 1 cubic foot of water?

13. Describe Smeaton's air-pump, and find the density of the air in the receiver after n ascents of the piston.

XII.

1. The centre of gravity of a body being given, and also that of a portion of it, show how to find the centre of gravity of the remainder.

2. The top of a triangle is cut off by a straight line parallel to its base, at a distance of $\frac{2}{3}$ of the altitude of the triangle; find the centre of gravity of the remaining trapezoid.

3. Find the centre of gravity of a shell bounded by two nonconcentric surfaces whose radii are r and r_1.

4. How many horses, each exerting a traction of 200 lbs., would be required to draw a waggon weighing 5 tons up a hill whose slope is 1 in 18, the traction on the road, if level, being estimated at $\frac{1}{20}$ of the gross load?

5. In an isosceles roof, when there is no tie beam, show how to determine the amount and direction of the pressure of the roof on the side walls which support it.

6. If each rafter be inclined to the horizon at an angle θ, the span remaining the same, show that the pressure is least when $\tan\theta = \frac{1}{\sqrt{2}}$.

7. A and B are two equal elastic spheres, A moving with a velocity v impinges upon B at rest, the direction of A's motion before impact making an angle of 60° with their line of centres; determine the directions and velocities of A and B after impact.

8. There are three equal elastic balls A, B, C; A is let fall from a given point; at the moment that it reaches the horizontal plane, B is let fall from the same point; and at the moment that A in returning meets B, C is let fall. Show that B will meet C for the second time where it first met A.

9. An imperfectly elastic ball A falls down a circular arc through a height h, and at the lowest point impinges upon another imperfectly elastic ball B. Show that if A be reduced to rest by impact it will drive B up a height e^2h, and that when B returns it will drive A up a height e^2h, following it through a height $e^2(1-e)^2h$.

10. A hollow sphere is half filled with mercury, and then filled up with water; compare the pressures on the upper and lower hemispheres.

11. Find the depth of the centre of pressure of a parallelogram immersed in a fluid, one side of the area coinciding with the surface of the fluid.

12. A given cylinder floats vertically with $\frac{1}{m}$ of its axis

immersed in one fluid, and $\frac{1}{n}$ of its axis immersed if placed in another; supposing p parts of the first fluid mixed with q parts of the second, what portion of the cylinder will be immersed when it floats in the mixture?

XIII.

1. Show how to find the centre of gravity of any number of bodies in the same plane.

2. A body cannot be in stable equilibrium upon a horizontal plane if it rests on less than three points of support.

3. Two weights, connected by a string passing over the common vertex of two inclined planes, balance each other; if they are set in motion, show that their centre of gravity will move in a horizontal straight line.

4. A right cone whose weight is W, is suspended by two vertical cords attached to the extremities of a side which is horizontal; determine the tensions of the cords.

5. Weights of 1, 3, 5, 7 ounces are placed in order in the angles of a square; find the distance of their centre of gravity from that of the square, which is without weight and has a side of 10 inches.

6. Find the resultant of four parallel forces of 1, 2, 3, 4 lbs. acting at the angles of a square.

7. A body moves under the action of a uniform force; show that the spaces described from the commencement of motion are as the squares of the times.

8. A body describes 300 feet in the fifth second from the beginning of motion; if it be acted on by a uniformly accelerating force, how many feet will it describe in the eighth second?

9. In what time will a shell range 3000 feet at an elevation of 30°?

10. If R and R_1 be the ranges of a shot fired at the same horizontal angle and with the same velocity up and down an inclined plane, whose angle of inclination is θ, and r the

range on a horizontal plane with the same angle of projection and the same velocity, prove $(R+R_1)\cos\theta=2r$.

11. What assumptions are made in the theory of projectiles which are not true in practice?

12. A pontoon is in the form of a cylinder with hemispherical ends, its extreme length is 22 feet, its diameter $2\frac{2}{3}$ feet, its weight with superstructure 1550 lbs.; what additional weight will be required just to immerse the pontoon in water of which 1 cubic foot weighs 62·5 lbs.?

XIV.

1. Find the ratio of the power to the weight in the screw, with friction.

2. A uniform ladder is placed between a rough horizontal plane and a rough vertical wall, at an angle of 45°, the coefficient between the wall and the ground being $\frac{5}{8}$; a man whose weight is half that of the ladder ascends: find what the coefficient of friction must be between the ladder and the wall, that when the man reaches the top of the ladder it may just begin to slip.

3. Find the least number of turns that may be given to a thread upon a cylinder 4 ft. long and 1 ft. in circumference, in order that the weight w may be supported by friction alone, the coefficient of friction being ·5.

4. A line is drawn cutting off $\frac{1}{4}$ of a square; find the distance of the centre of gravity of the remainder from the centre of gravity of the square, (1) when the line is parallel to a side of the square, (2) when it is parallel to a diagonal.

5. Four bodies, weighing 3, 4, 5, 7 lbs. respectively, are placed with their centres of gravity in a straight line at the respective distances of 2, 4, 6, 8 feet from a fixed point in that line. Find the position of the centre of gravity of the bodies.

6. A stone is let fall from the top of a tower at the same instant that another is projected vertically upwards from the

bottom of the tower; find the velocity of projection if the stones meet half way, the height of the tower being h.

7. Two bodies are projected from the top of a tower with the same velocity, one horizontally, the other at an elevation of 45°; they reach the ground t'' apart: find the height of the tower.

8. How is a cycloid described? Show that if a body oscillate in a cycloid, the time of a vibration is independent of the length of the arc of vibration.

9. A seconds' pendulum is carried to the top of a mountain, and there loses 8″ a day. Determine approximately the height of the mountain.

10. A hollow cone resting on its base is filled with fluid; what is the direction of the pressure at any point of its surface? Find the total pressure on the base; height of cone 12 in., radius of base 3·25 in., *s.g.* of fluid$=2\cdot56$.

11. State and investigate the conditions of equilibrium of a floating body.

12. A solid cylinder is of a substance whose density is $\frac{2}{5}$ of that of the fluid in which it floats; how much of the cylinder is immersed when it floats (1) with its axis perpendicular to the surface of the fluid, (2) when its axis is parallel to the surface?

XV.

1. Show that the coefficient of friction between two substances is the tangent of the angle of inclination of a plane formed of one of the substances, when a body formed of the other is about to slide down it.

2. Two bodies resting on two given inclined planes with a common vertex are connected by a cord parallel to the planes: find the relation between the weights of the bodies, and that between the pressures on the planes.

3. A circular arc of 150°, supposed without weight, rests on a horizontal plane with its concavity upwards, and with

weights of 8 lbs. and 12 lbs. suspended from its extremities. Find the position of equilibrium.

4. In the funicular polygon, find the tensions of the several strings, and the ratio between any two suspended weights.

5. A power P sustains a weight W on a system of weightless pulleys in two blocks, each block having 3 sheaves, and the string as it passes from one block to the other being vertical. Find the ratio of P to W.

6. Prove that the principle of virtual velocities is true in the system in *Qu.* 5. Show that this principle here expresses the fact that when a weight is raised by this system of pulleys the useful work yielded is the same as the work applied by the moving power. Is this generally the case with machines in practice? Why?

7. Distinguish between '*moving force*' and '*accelerating force.*' If a body be projected with a velocity u in the direction of the force of gravity (g), and if v be the velocity acquired, s the space described in the time t, show that $t=\frac{v-u}{g}=\frac{2s}{v+u}$.

8. Two bodies are projected, at the same instant, in the direction of gravity, the velocity of one being 5 times that of the other, after 5·5″ their velocities are respectively 277·1 and 677·1 ft. a second; find the initial velocity of each and the force of gravity.

9. A body is projected up an inclined plane whose inclination is 45°, it reaches the top of the plane with $\frac{1}{8}$ of its original velocity, where will it strike the horizontal plane passing through the foot of the inclined plane.

10. Find the radius of gyration of a circular plane 10 ft. in radius round an axis through a point in its circumference perpendicular to its plane.

11. When a body is immersed in a fluid, show that the weight lost by the body is to its whole weight as the $s.g.$ of the fluid is to the $s.g.$ of the solid.

12. The $s.g.$ of gold is 19·25; a vessel filled with water

alone weighs 8·5 oz.; when a nugget of gold is put into the vessel and it is filled up with water, it weighs 44·75 oz.: determine the weight of the nugget.

XVI.

1. Investigate the conditions of equilibrium of a wedge.

2. ABCD is a quadrilateral, and is acted upon by forces represented in magnitude and direction by AB, AD, CB, CD: find their resultant.

3. A weight P when suspended successively at each end of a uniform rod, one point of which is fixed, is balanced by weights Q, R respectively: find the position of the fixed point, and the weight of the rod.

4. A weight just rests on a given inclined plane, and it is found that a force equal to half the weight, if applied in the most favourable direction, is just sufficient to move the weight up the inclined plane; determine the coefficient of friction.

5. A heavy body rests with its spherical end upon a convex sphere, show how to determine whether the equilibrium is *stable* or not, sliding being prevented.

6. If the resting body be a hemisphere, determine the extreme value of its radius within which equilibrium will be possible, the radius of the fixed sphere being 20 ft.

7. If a heavy particle is allowed to fall from the edge down the interior of a smooth hemispherical bowl, prove that its pressure on the surface at the lowest point is 3 times the weight of the particle.

8. If the moving force upon a body varies directly as its distance at any time from a given point towards which it falls, the whole time of the body's falling to that point will be the same whatever may be the distance from which it falls. Apply this principle to prove the principle of the cycloidal pendulum.

9. A railway train running at the rate of 40 miles an hr.

is detached from the engine at the foot of an incline of 1 in 50: how far, and how long would it move up the plane, friction being neglected?

10. Explain the formula $t=2\pi\sqrt{\frac{h}{g}}$ applicable to the motion of a conical pendulum; and apply it to determine the number of revolutions per minute which will cause the suspending bars, each 1·5 ft. long, of the governor of a steam-engine to stick out at an angle of 30° with the vertical.

11. A body projected from a point in the circumference of a circle returns to the point it started from, after two reflections from the circumference, supposing the coefficient of elasticity to be ·9, find the angle between the line along which the body starts, and that in which it returns to the starting-point.

12. A rectangular sluice-gate 12 ft. square is placed vertically in water, and the pressure on the half of it cut off by a horizontal line is $\frac{1}{15}$ greater than on the half bounded by a diagonal: how high has the water risen above the top of the gate?

XVII.

1. State the principle of virtual velocities, and apply it to determine the relation between the power and the weight, in '*the wheel and axle.*'

2. A body weighing 2 cwt. is supported on an inclined plane, making an angle of 70° with the vertical, by means of a force parallel to the base of the plane. Find the force and the pressure on the plane.

3. Supposing that $5\frac{1}{2}$ turns cause the head of a screw to advance $\frac{2}{3}$ of an inch in a straight direction, what power applied at the extremity of an arm 18 inches long will be required to produce a pressure of 1000 lbs. upon the head of the screw?

4. Define the term '*centre of gravity*;' and show how to

find practically the centre of gravity of an irregularly-cut piece of cardboard, proving the proposition on which your method depends.

5. Find the centre of gravity of a triangle.

6. If a piece of uniform wire, bent into the form of a triangle A B C, be suspended freely from the angular point A, prove that it will only rest with the side B C horizontal when the angle A B C is equal to the angle A C B.

7. If a body be projected with a given velocity down a smooth inclined plane, find the space it will describe in a given time.

8. With what velocity must a body be projected down an inclined plane of given length whose inclination is 30°, so that the time down the plane may be equal to the time of a body's falling freely through the vertical height?

9. A given body would just rest on an inclined plane when inclined at an angle of 30° to the horizon: find the velocity which the body would acquire in sliding down the plane for 2″, when it is inclined at an angle of 60° to the horizon. If the body be projected up the plane with a velocity of 100 ft. per second, how far will it rise?

10. A train of 75 tons ascends an incline of 1 in 800, with the uniform speed of 40 miles an hour: what is the horse-power due to friction and gravity, assuming the friction to be 6 lbs. a ton?

11. Show how to compare the specific gravity of a solid body with that of a fluid of greater density.

12. The apparent weight of a piece of marble when weighed in water is 9 oz., when a piece of elm weighing 6 oz. is tied to the marble, the apparent weight of the two is 5 oz.: find the specific gravity of the elm, that of the water being unity.

XVIII.

1. If four couples be represented by the four faces of a triangular pyramid, prove that they will be in equilibrium.

2. The upper hinge of a gate sustains only a horizontal strain, the whole weight being borne by the lower hinge. Show how the direction and magnitude of the thrust on the lower hinge may be determined: and find it in the case of a rectangular gate whose weight is 56 lbs., whose centre of gravity is in the intersection of its diagonals, whose hinges are at the angles, and whose length is 8 ft., height 4 ft.

3. Four forces P, Q, R, S, act along the sides of a rectangle and are proportional to those sides; find the direction of their resultant.

4. Describe the common steelyard, and show how to graduate it.

5. ABCD is a square. A force of 3 lbs. acts from A to B, a force of 4 lbs. from B to C, and a force of 5 lbs. from C to D; find the single force which will preserve equilibrium.

6. If the square in *Qu.* 5 have its centre fixed, what force acting along AD will keep it in equilibrium?

7. Explain how variable velocity is measured. Prove the formula $v = V + ft$.

8. A particle is projected vertically upwards with a velocity of 100 ft. per second, how long will it continue to rise?

9. Explain the principle of '*composition of velocities*;' and describe in general terms how the motion of a body may be determined.

10. A and B are perfectly elastic balls, which move in opposite directions, and impinge directly: the mass of A is double that of B, and B's velocity is treble that of A; determine the motion of each after impact.

11. Find the weight of a hydrometer which sinks as deep in a fluid of specific gravity ·86 as it does in water when loaded with 60 grains.

12. A ring is composed of gold, and has a diamond set in it: the whole weighs 69·5 grains in air, and 64·5 grains in water. The specific gravity of the gold is 19, and that of the diamond $3\frac{1}{2}$. Find the weight of the diamond.

XIX.

1. Determine the conditions of equilibrium in the case of the lever, the forces acting at any angles. Find the magnitude and direction of the pressure on the fulcrum.

2. A and B are the extremities of a uniform straight lever 4 ft. long, B C, D A are each taken equal to 1 ft., from B a weight of 25 ounces is suspended, and from D a weight of 30 oz., the same weight P is suspended both from A and C; find P so that the fulcrum may be at the same point in case of equilibrium, whether the lever has weight or not.

3. A uniform beam rests with its extremities upon two given smooth inclined planes, determine its inclination to the horizon.

4. Two beams, A C, B C, whose weights are W_1, W_2, are connected together at a given angle α, and turn about a horizontal axis at their point of junction: find the position of equilibrium which they will take by the action of their weight.

5. Describe and graduate the Danish steelyard.

6. If a beam A B is supported in a horizontal position at A and B, and the weight be uniformly distributed along A B, find the moment of the strain at any point C of the beam.

About what point will this moment be greatest? determine its magnitude in this case.

7. If two elastic spheres impinge in direct impact with given velocities, show how to find the velocity of each sphere after impact, and explain what assumptions are made in the investigation.

8. Two balls are projected at the same instant from two points in the same horizontal plane, and in opposite directions, so as to describe the same parabola; if the mass of one ball be double that of the other, and if the modulus of elasticity be ·5, show that after impact one ball will descend vertically, and the other return in its original parabola.

9. Explain what is meant by '*centrifugal force.*' A stone weighing 1 lb. is attached to the end of a string 3 ft. long, of which the other end is fixed; the stone describes a horizontal circle revolving 3 times a second : find the inclination of the string to the vertical, and its tension.

10. A seconds' pendulum is lengthened ·01 inch: find how many seconds it will lose in 24 hrs.

11. Define a fluid. Show that the pressure round an internal point in a fluid is the same in all directions.

12. A cylinder partially filled with a fluid revolves round its axis with a uniform velocity, show that the form which the surface of the fluid assumes is a paraboloid.

XX.

1. Investigate the relation of the power to the weight in the case of a straight horizontal lever, taking into account the friction at the cylindrical axis upon which it works.

2. Find the power necessary to sustain 6 cwts. in a system of pulleys 3 in number, each weighing $2\frac{1}{2}$ lbs.; each string being attached to the weight.

3. The sides of a uniform heavy triangle are $3a$, $4a$, $5a$ respectively: find the centre of gravity of the remainder after the inscribed circle is removed.

4. A heavy triangle ABC is suspended successively from the angles A and B, and the two positions of any side are at right-angles to each other; show that $5c^2 = a^2 + b^2$.

5. Find the position of the centre of gravity of 4 heavy particles P, Q, R, S, situated in one plane, with reference to two straight lines, in the same plane, at right-angles to one another.

6. P and 2P, weights, are placed in two adjacent angles of a square, find the weights that must be placed in the other angles so that the common centre of gravity of the 4 weights may be at the centre of the square.

7. State the '*third law of motion.*'

8. A given weight P descends drawing up a weight Q, by means of a string over a fixed smooth pulley; find the accelerating force and the tension of the string. Find the relation of P to Q, so that P may descend from each rest 16·1 feet in 2 seconds.

9. If a body start from the bottom of a given inclined plane with a velocity v, find the space through which it will ascend (1) when the plane is perfectly smooth; (2) when the plane is rough, stating how the friction in the formula obtained is to be estimated.

10. The path of a projectile *in vacuo* is a parabola; its velocity at any point is that due to the height of the direction above the point: prove the latter statement, assuming the former. Find the focus of the parabola when the elevation is α, and the velocity of projection 64·4 ft. a second.

11. Define the '*centre of pressure*,' and find it for any rectangular area one side of which is horizontal.

12. If a square whose side is 20 inches be immersed vertically in a liquid with one side horizontal, find the depth of the centre of the square when the distance between it and the centre of pressure is $1\frac{2}{3}$ inches.

XXI.

1. Show that a single force acting on a point may be replaced by two forces at right-angles to each other, one of which may have any direction with respect to the single force. The single force is 100 lbs., one of the two makes an angle of 60° with it: find the two equivalent forces

2. If 4 forces acting upon a point be represented in magnitude and direction by 4 of the sides of a regular pentagon taken in order, show from the algebraical expression for determining their resultant, that it will be represented by the fifth side.

3. When a door is supported by two hinges, show how to

find the horizontal pressures at those hinges, and prove that the vertical pressures are indeterminate. If the door weigh 100 lbs. and be 4 ft. wide, find the horizontal pressure when the distance between the hinges is 6 ft.

4. A number of penny pieces are cemented together so that each just laps over the one below it by the nth part of its diameter: find how many may be thus piled without falling, when the lowest stands on a horizontal plane.

5. From a circular disc, of radius r, a circular disc of radius r_1 is removed, the distance between the centres being d: determine the centre of gravity of the remainder.

6. Show that the ratio of the relative velocities of two balls before and after impact depends only on the modulus of elasticity.

7. A particle is projected from a point in a horizontal plane, and after describing a parabolic arc impinges again upon the plane which is imperfectly elastic; compare the height (h_1) and the range (r_1) of the second parabolic arc with h and r those of the first; and determine the whole space described before the particle comes to rest.

8. In the conical pendulum determine the inclination of the string to the vertical when its length is 10 ft. and it makes 30 revolutions in one minute.

9. Show how to find the velocity of a cannon ball by observing the angle of recoil of a ballistic pendulum.

10. A body being wholly immersed in a fluid, find the moving force tending to cause it to ascend or descend.

11. A cylinder rests in a fluid A, with $\frac{1}{3}$ of its axis immersed, in another B with $\frac{3}{5}$ of its axis immersed; how deep will it sink when floating in a fluid which is a mixture of equal quantities of A and B?

12. A block of ice, the volume of which is a cubic yard, is observed to float with $\frac{2}{25}$ of its volume above the surface, and a small piece of granite is seen imbedded in the ice; find the volume of the stone, the *s. g.* of ice being ·918 and that of granite 2·65.

XXII.

1. Find the relation between the power and the weight on the screw.

2. Four forces act along four edges A C, A D, B C, B D of a triangular pyramid A B C D, and are proportional to these lines; find their resultant.

3. What is the height from the ground of the centre of gravity of a pyramid formed of 4 8-inch shells touching each other?

4. The third part of a circular hoop, at liberty to roll on a horizontal plane, is kept at rest by two weights of 20 lbs. and 30 lbs. respectively, hanging freely from its extremities; determine the angle subtended at the centre between the point of contact with the plane and either extremity, neglecting the weight of the hoop.

5. Find the ratio of the power to the weight in a system of pulleys in which each pulley hangs from a fixed point by a separate string, and all the strings are parallel, taking the weights of the pulleys into account. If P be the power, W the weight, w the weight of each pulley, n the number of moveable pulleys, show that

$$P - w = \frac{1}{2^n}(W - w).$$

6. When a balloon is half a mile high, and rising at the rate of 16·1 ft. per second, a heavy stone is released from the hand of the aeronaut; when will the stone begin to fall towards the ground, and in what time will it reach it?

7. From the top of a tower 170 ft. high a shot is fired vertically with a velocity of 1000 ft. per second: find the height to which it will rise, and the velocity with which it will reach the ground.

8. Projectiles are discharged *in vacuo* from a given point with a constant elevation, what is the locus of the vertices of the parabolas which they describe?

9. A shot is fired with a given velocity from the point A

up an inclined plane, so as to have a maximum range, and strikes the plane at a point A′: with what velocity and elevation must it be fired back from A′ so as to return to A by the same course?

10. A vessel of given weight, moving with a velocity v, is brought to rest by the tension of a rope of length l: supposing the modulus of elasticity of the rope to be e, find its elongation when the vessel stops.

11. Describe the siphon, and the manner in which it acts.

12. Show that a homogeneous rectangular parallelopiped with a square base, immersed with its base horizontal in any fluid of greater *s. g.* than itself, will float in a position of stable equilibrium therein, if the square of its height $< \frac{2}{3}$ the area of its base.

XXIII.

1. Prove '*the parallelopiped of forces.*'

2. Apply the principle of virtual velocities to determine the relation between the power and the weight in a system of 4 moveable pulleys, with parallel strings, all attached to the weight.

3. A two-wheeled carriage, weighing W, is just capable of being set in motion on a horizontal plane by a force H. The radii of the wheel and axle being r and ρ respectively. Find the coefficient of friction between them.

4. Determine generally the conditions of the stability of a pier of masonry sustaining on its summit a given pressure acting in a given direction.

5. Find the centre of gravity of three uniform beams forming a right-angled isosceles triangle, the greatest side of the triangle being 20 feet.

6. Three weights 4, 6, 8 lbs. are suspended on a straight horizontal lever without weight, and at equal distances of 12 inches from each other, where must the fulcrum be placed for equilibrium?

7. Define the '*unit of work*;' and show that when a

pressure of m lbs. is exerted through n ft., the work done is represented by $m \times n$.

8. Find the horse-power of a fall of 1000 cubic feet of water per minute, to a depth of 20 feet.

9. A train of 200 tons weight is travelling at the rate of 20 miles an hr. How far would it run on a horizontal line before it came to rest, if the steam were thrown off, supposing the friction of the rails to be 8 lbs. a ton?

10. A body which weighs 18 lbs. is put in motion on a horizontal plane so as to gain 5 ft. of additional velocity every second. Supposing the friction of the plane to be 2 lbs.; what must be the amount of pressure acting parallel to the plane?

11. A cubical vessel is filled with fluid, compare the pressures on its sides with that on its base.

12. When the cube in *Qu.* 11 has one of its diagonals vertical, find the pressure on one of its lower faces: edge of cube, a in., $s. g.$ of fluid 1·75.

XXIV.

1. Determine the relation between the power and the weight on the inclined plane (1) when the plane is smooth; (2) when it is rough.

2. What force is necessary to support a weight of 50 lbs., on a plane inclined at an angle of 15° to the horizon, the force acting horizontally?

3. A heavy body is drawn along a horizontal plane subject to friction by a force inclined at an angle to the plane. Determine generally the magnitude of the force, and show at what angle it is least.

4. Find (1) what amount of force is requisite to sustain, and (2) what amount of force is just insufficient to push a weight of 250 lbs. up a plane inclined at an angle of 45° to the horizon, the direction of the force being supposed to make

an angle of 30° with the plane, and the coefficient of friction being $\frac{1}{8}$.

5. If a force, acting at an angle of 45° to the horizon, can just support a weight on a smooth plane, inclined at an angle of 15° to the horizon, what is the greatest angle of inclination of a rough plane to the horizon on which the same force acting in the same direction can just support the same weight, the coefficient of friction being $2 - \sqrt{3}$.

6. Assuming 32·2 to represent the constant force of gravity upon the foot and second scale, what number will represent the same force upon the mile and minute scale?

7. A heavy body is observed to be 2·25″ in falling down a well; what would be the depth of the well, neglecting the resistance of the air?

8. Prove that 45° is the angle of projection which gives the maximum horizontal range for a projectile *in vacuo*. Show also that the diminution of the range when this angle is increased or diminished by the same quantity, is the same.

9. At what inclination to the horizon must a wooden way be laid in order that a block of iron may just slide down by its own weight, the coefficient of friction being ·625?

10. Find the ratio between the radii of a wheel and axle which shall cause a heavy weight to draw up another, $\frac{1}{3}$ of itself in magnitude, through a given space in the shortest time: neglecting inertia and friction.

11. Two equal weights of 50 lbs. are suspended over a fixed pulley, what weight must be taken from one and added to the other to make it descend through 20 ft. in 2·5″?

12. Investigate the conditions which must be satisfied when a body floats in an incompressible fluid.

XXV.

1. Define '*accumulated work*;' and investigate an expression for the work accumulated in a body (w) moving with a velocity v.

2. Find the work accumulated in a ball weighing w lbs. and describing n revolutions a minute, in a circle of r ft. radius.

3. Explain generally the conditions on which the stability of an arch depends.

4. A gun is made to move in the circumference of a circle traced upon the deck of a ship, show that the common centre of gravity of the ship and gun will thereby be made to describe a circle.

5. Describe the '*Common Balance.*' What is the '*sensibility*' of a balance? Show how it may be increased.

6. A body is projected vertically upwards with a velocity of 80 ft. a second, to what height will it ascend?

7. A body weighing 6 cwts. moves from rest, under a constant pressure, upon a perfectly smooth horizontal surface, and acquires a velocity of 8 ft. per second at the end of 1 minute. Find the pressure.

8. A body is projected obliquely on a smooth inclined plane; determine its path.

9. A weight of 20 lbs. raises another of 112 lbs. by means of a wheel and axle, whose radii are $2\frac{1}{2}$ ft. and 4 in. respectively. Find the space through which the lifting weight descends in $10''$: neglecting friction and the inertia of the machine.

10. Define the term '*specific gravity.*' The volumes and specific gravities of two liquids being given, determine the specific gravity of the fluid resulting from their admixture.

11. In a cylinder $\frac{3}{4}$ filled with water, a hydrometer, whose weight w is given, is observed to sink to a certain depth. If the cylinder be now filled with a fluid of 3 times the *s. g.* of water, find the weight with which the hydrometer must be loaded to cause it to sink to the same depth as before.

12. If an air-pump have a single exhausting barrel whose content is $\cdot 01$ of that of the receiver, compare the pressure of the air in the receiver, after 10 double strokes of the piston, with the pressure of the atmosphere.

XXVI.

1. Find the centre of gravity of a uniform rod bent so as to form a quadrilateral figure.

2. Find the centre of gravity of the surface of a right cone.

3. The lengths of the arms of a balance being unequal, how may it be used so as accurately to ascertain the weights of articles?

4. Divide the base A B of a triangle A B C into segments A D, D B, having to one another the ratio $m : n$; and join C D: prove that if forces be represented in magnitude and direction by m . C A and n . C B their resultant will be represented by $(m+n)$ C D.

5. On a plane whose inclination is α, P, acting in a direction making an angle ϵ with the plane, is just on the point of moving W up the plane; if ϕ be the limiting angle of resistance, prove that $\frac{P}{W}=\frac{\sin(\alpha+\phi)}{\cos(\epsilon-\phi)}$.

6. The measure of the force of gravity being 32·2 when a second is the unit of time, what will be its measure when ten seconds is the unit of time?

7. Two particles are simultaneously projected with velocities u, v at angles of inclination to the vertical α and β: determine the time which will elapse before they are moving in parallel lines.

8. What is meant by the proposition that the centres of oscillation and suspension are reciprocal? Prove that this proposition is true for a system of material particles.

9. Find the centre and time of oscillation of 3 weights 4, 5, 6, placed at distances of 4 in. from each other on a rod 12 in. long without weight, the weight 6 being at the lower end of the rod.

10. Show how to determine the $s. g.$ of a substance by means of Nicholson's hydrometer.

11. A substance when put in the upper cup of the

hydrometer requires 110 grs. to be placed with it, and when put in the lower cup, 135 grs. must be put in the upper cup, to sink the instrument to the same point in the stem to which 560 grs. in the upper cup alone would sink it; find the *s. g.* of the substance.

12. By what experiments can it be proved that air has weight? Assuming that 100 cubic inches of air weigh 31 grains, and that a cubic foot of water weighs 1000 oz.; compare the *s. g.* of air with the *s. g.* of water.

XXVII.

1. If any number of forces not in equilibrium act upon a point in a plane, show that they will have a single resultant, and find it.

2. Triangles are inscribed in a circle having equal chords for their bases and a common vertex: show that the locus of the centre of gravity of the triangles is a circle.

3. A uniform beam is placed between two rough inclined planes, the inclinations of which are complementary to each other, find either of the limiting positions of equilibrium: if θ, θ_1, be the greatest and least values of the inclination of the beam to the horizon for the limiting positions of equilibrium, and if the coefficient of friction be tan ϕ, prove

$$\phi=\tfrac{1}{4}(\theta-\theta_1).$$

4. Find the equation of '*the common catenary*.'

5. A rigid body stands upon a horizontal plane upon three supports, how do you determine whether it will stand or fall? If it stands, estimate the pressure upon each point of support.

6. Explain '*the second law of motion*.'

7. A particle is projected with a velocity of 161 feet per second in a direction making an angle of 30° with the horizon: find its velocity and the direction of its motion when it has reached a height of 64·4 ft. Find also the horizontal space described in this time.

8. Find the range of a projectile *in vacuo*.

Ex. Gravity=32·2 ft.; velocity of projection = 3220 ft. per second; sine of angle of elevation = ·6.

9. Particles are projected from the same point simultaneously with different velocities and in different directions, but all in the same plane: find the path of the common centre of gravity of the particles.

10. If the atmosphere be divided into thin strata of equal thickness, show that their densities decrease in geometrical progression.

11. Show how to determine the difference of the heights of two stations by means of a barometer and a thermometer.

12. A tube closed at one end and filled with air has its open end stopped by a piston capable of sliding into it freely. To what depth must the tube be immersed in a horizontal position in water that the piston may be driven in through $\frac{1}{3}$ of the tube's length, the pressure of the atmosphere being equal to that of a column of water 34 feet high?

XXVIII.

1. Explain how a plane area is said to have a centre of gravity. Find the centre of gravity of two material plane isosceles triangles on the same base, their vertices being on opposite sides of the base at distances h_1, h_2 from it.

2. Distinguish between stable and unstable equilibrium, give an example of neutral equilibrium.

3. A homogeneous elliptical spheroid rests with its smaller end on a concave hemisphere, what must be the radius of the hemisphere that the equilibrium may be stable?

4. A force acting parallel to a rough inclined plane just supports a weight upon it, find the inclination of a smooth plane, such that the same force acting along it shall support the same weight.

5. Determine the amount and the direction of the least

force which will draw a body weighing W lbs. up a rough plane inclined at $\alpha°$ to the horizon.

6. A smooth rod slides between two fixed rings B, C, so as to be constrained to move in a vertical line. Its lower end A rests on a fixed plane inclined at 45° to the horizon. If the rod weighs 4 lbs., and if $AC=2AB$, find the pressure on each ring.

7. Calculate the length of a pendulum beating seconds in London, assuming $g=32{\cdot}19$.

8. A heavy body suspended by a flexible cord revolves in a vertical circle: under what circumstances may the strain upon the cord vanish?

9. Determine the motion of a '*conical pendulum.*'

10. Find the velocity with which a cannon ball must impinge upon a bank of earth that, sustaining a given uniform resistance, it may enter it a given depth.

11. Describe and explain the common '*suction pump*:' and investigate a formula for determining the height to which the water is raised by the first stroke of the piston.

12. A hollow spherical iron shell, of 6 in. radius, floats in water so as to be exactly half immersed: what must be its thickness, the *s.g.* of iron being 7·78?

XXIX.

1. Two forces P and Q have a resultant R which makes an angle α with P: if P be increased by R, whilst Q remains unchanged, show that the new resultant makes an angle $\frac{1}{2}\alpha$ with P.

2. A rod without weight is 1 ft. long: at one end a force of 2 lbs. acts, at the other a force of 4 lbs., and at the middle a force of 6 lbs.; these forces are all parallel. Find the magnitude and direction of a single force which will keep the rod at rest.

3. A plane is inclined to the horizon at an angle of 30°: find the magnitude and direction of the least force which

will prevent a body weighing 100 lbs. from sliding down the plane, the coefficient of friction being ·25.

4. Apply the principle of virtual velocities to find the relation of the power to the weight in the case of a bent lever.

5. A ladder, 10 ft. long, rests with one end against a smooth vertical wall, and the other on the ground, the coefficient of friction being ·5: find how high a man, whose weight is 3 times that of the ladder, may ascend before the ladder begins to slip, the foot of the ladder being 6 ft. from the wall.

6. Prove that a body on a horizontal table will stand or fall as the vertical line through its centre of gravity falls within or without its base. Is the statement true if the plane be rough?

Two books, similar in every respect, each 10 inches long, lie one exactly upon the other on a table, over the edge of which they project 3 in. How much farther may the upper book be pushed out before they fall over?

7. P hanging vertically draws Q along a smooth horizontal table, find the accelerating force, and the space described in t seconds.

Ex. P$=2$ oz., Q$=8$ oz., the given time$=2{\cdot}5''$.

8. Find the range of a projectile on an inclined plane passing through the point of projection; and find what the direction of projection must be that with a given velocity this range may be the greatest.

Ex. Velocity of projection 1610 ft. per second, angle of projection 45°, inclination of plane 30°.

9. A body is projected vertically upwards with a velocity of 966 ft. a second; find the time in which it will attain a height of 2753·1 ft. from the point of projection; and explain the double result.

10. In what time will the velocity of the body in *Qu.* 9 be reduced one half?

11. Show that the common surface of two fluids which do not mix is a horizontal plane.

12. The volume of the receiver of an air-pump is 20 times that of the barrel, and a piece of bladder is placed over a hole in the top of it. The bladder can bear a pressure of 3 lbs. to the square inch. Determine the number of strokes that can be made without bursting the bladder.

XXX.

1. Explain what is meant by the following terms in the theory of roofs:—'*truss*,' '*tie-beam*,' '*king-post*,' '*queen-post*,' '*strut*.' In the simple isosceles truss-roof, find the tension of the tie-beam.

2. A weight of 1000 lbs. is placed upon a rough inclined plane: the inclination of the plane to the horizon is 45°, and the coefficient of friction $\frac{1}{\sqrt{3}}$; find the least force which will keep the body from sliding down the plane, and the least which will draw it up.

3. Ten cannon balls are piled on a horizontal plane, in the form of a triangular pyramid: find the height of their common centre of gravity.

4. A heavy string is stretched over a smooth plane curve, find the tension at any point, and the pressure on the curve.

5. A chain of given weight is suspended from two equal vertical posts of given height, and the angle which the chain makes with either post is measured: find the moment of the force which tends to overturn one of the posts.

6. Define the terms—'*pressure*,' '*velocity*,' '*accelerating force*,' '*moving force*,' '*impressed*' and '*effective forces*,' '*angular velocity*,' '*moment of inertia*.'

7. A stone is projected vertically upwards with a velocity of 1000 ft. a second, how far will it ascend?

8. A block of stone of given weight and dimensions

stands upon a railway truck which is suddenly stopped. It is prevented from *sliding* upon the truck, it is required to determine the speed of the train so that it may just be overturned: and if it be allowed to slide and the coefficient of friction be μ, what must be the velocity if it slides a feet?

9. A perfectly flexible rope, part of which rests on a table, is dragged off it by the weight of the other part, which hangs over the side; determine a general expression for the velocity when μ is the coefficient of friction.

10. Two rectangular planes, whose lengths are 4 in. and 6 in. respectively, and their breadths 3 in. and 4 in., are immersed horizontally 4 ft. and 2 ft. respectively beneath the surface of a fluid. Compare the pressures upon them.

11. A cubical box is filled half with mercury and half with water. Compare the pressure on the sides with that on the base, the specific gravity of mercury being 13·5 times that of water.

XXXI.

1. Explain the different kinds of levers, giving examples of each. Where must the fulcrum be placed in order that a man who can lift a weight of 120 lbs., may, with a heavy crowbar weighing 30 lbs. and 5 ft. long, just raise 5 cwt.?

2. When a body is placed on a horizontal plane, examine the conditions of its standing or falling.

3. A right cone and a hemisphere are cemented to the same circular base: find the height of the cone, so that if the hemispherical portion of the solid be placed in any position, on a smooth horizontal plane, it will remain at rest; distance of centre of gravity of hemisphere from that of sphere being $\frac{3}{8}$ of the radius.

4. What is the '*key-stone*' of an arch? what the '*intrados*' and the '*extrados*?' Describe the manner in which an arch ordinarily gives way: and explain the principles upon which *the point* of rupture may be approximately found.

5. If $\tan\beta$ represent the coefficient of friction between the weight W and the inclined plane, α being the angle of inclination of the plane, ϵ the angle which the force P makes with the plane, show that the limiting values of the ratio P : W for equilibrium are

$$\frac{\sin(\alpha+\beta)}{\cos(\epsilon-\beta)} \text{ and } \frac{\sin(\alpha-\beta)}{\cos(\epsilon+\beta)}.$$

6. Determine the space described in a given time by a body which moves in a straight line, and whose motion is uniformly accelerated or retarded.

7. Find the time in which a body will descend by gravity down a smooth inclined plane 200 ft. long, the height of the plane being 3 ft.

8. A train weighing 10 tons descends a plane of the above length and height: find the velocity acquired, friction being 10 lbs. a ton.

9. A projectile shot with a velocity of 1600 ft. a second, is to strike the top of an object 60 ft. high at a distance of 500 ft. Find its elevation.

10. Each additional 100 tons weight depresses a vessel 3 in. in the water: how far can her magnitude be estimated from these data?

11. If w_1, w_2, w_3 be the apparent weights of a body when weighed in three fluids whose densities are respectively ρ_1, ρ_2, ρ_3; show that

$$w_1(\rho_3-\rho_2)+w_2(\rho_1-\rho_3)+w_3(\rho_2-\rho_1)=0.$$

12. Find the height to which the water will rise in a diving-bell when its lower extremity has sunk d feet below the surface of the water.

XXXII.

1. A horizontal lever is 12 in. long, a weight of 8 oz. hangs from one extremity, where must the fulcrum be placed in order that a force of $11\frac{3}{7}$ oz. acting at the other end at an angle of 150°, may keep the lever in equilibrium?

2. A fourth of the area of a triangle is cut off by a straight line parallel to the base, find the centre of gravity of the remaining trapezoid.

3. Describe the '*compound wheel and axle*' known as the '*differential wheel and axle*:' and investigate the relation between the power and the weight in the case of equilibrium.

4. Define a '*couple*;' and prove that its effect is not altered by turning its arm through any angle in the plane of the forces.

5. If the algebraic sums of the moments of a system of forces, in one plane, about any two points in that plane, be separately zero, what inference can be drawn?

6. Three uniform bars jointed together form an isosceles triangle A B C, the angle C being a right-angle. The frame is hung in a vertical plane with the angle C downwards, and the side A B horizontal, by a string attached to the middle point of A B. Find the inclination to the horizon of the direction of the mutual action of the bars at the joint A or B.

7. Prove the formulæ $v^2=2fs$; $u^2=v^2\pm 2fs$.

8. Find the number of feet described by a falling body between the fourth and seventh seconds.

9. P and Q are two balls at rest on a horizontal table, find in what direction P must impinge on a vertical plane in order that after reflexion it may strike the ball Q.

10. Find the relation between the work necessary to drag a body up an inclined plane, subject to friction; and the work necessary to drag it along the base of the plane and to lift it vertically through its height.

11. Describe the '*Hydrostatic Press.*'

12. A ship sailing into fresh water is observed to sink through a small space, a, and then on being discharged of a weight, w, to rise through a small space, b. Find the weight of the ship, the density of salt water being $\frac{1}{40}$ greater than that of fresh, and the area of the plane of floatation between the limits of the observations being constant.

XXXIII.

1. Determine generally the relation between the power and weight in the wheel and axle, taking into account the friction of the axis: (1) when the power and the weight act in parallel lines; (2) when they are inclined to one another.

2. Supposing the tenacity of iron wire ·1 in. in diameter to be 41 tons per square inch; how many such wires must be put together to sustain a strain of 3 tons?

3. A cylindrical bar of iron, 2 in. in diameter and 40 ft. long, is subjected to a strain of 12 tons. By how much will it be elongated supposing every additional ton of strain per sq. in. of the section of such bar to produce an elongation of one ten-thousandth part?

4. Find the centre of gravity of a trapezoid.

5. If the mass of each one of a system of particles be multiplied by the square of its distance from a given point, the sum of the products is least when the given point is the centre of gravity of the system.

6. If equal particles, the mass of each being $3m$, are placed in the angles of a triangle, whose sides are a, b, c, show that the sum of the least products, taken as stated, will be $m(a^2+b^2+c^2)$.

7. Investigate an expression for the work done by the steam on the piston of an engine at every stroke: diameter of piston a in., length of stroke b in., mean pressure of steam m lbs. to the sq. in.

8. A man weighing 120 lbs., pulls himself up in a bucket weighing 20 lbs. by means of a system of pulleys in which the weight is to the power as 3 : 1; what force must he exert to raise himself from the ground, neglecting the weight and friction of the cords and pulleys?

9. A gun being fired at an elevation of 45°, the shot reaches the ground in 30″ at a distance of 5000 yds. Find

the velocity of projection, and the greatest height to which the shot rose.

10. A body falls without friction, under the action of gravity, down a plane curve, show that the velocity at any point will be the same as if the body had fallen freely through the same vertical distance.

11. Define the '*meta-centre*;' and show how its position may be determined.

12. How much cork of specific gravity ·25 does a man require, whose *s. g.* is $1\frac{1}{9}$ and weight 12 stone, to enable him to float in water? How deep will a sphere of elm-wood of two feet diameter sink in hydrochloric acid of *s. g.* 1·2, that of elm being ·7?

XXXIV.

1. Find the relation of the power to the weight in the system of pulleys where each pulley hangs by a separate string.

2. A weight of 21 cwts. 1 qr. 20 lbs. is to be lifted by means of a crane, of which the axle for receiving the rope is a foot in diameter, and carries a wheel of 50 teeth worked by a pinion of 6 teeth. What force must a man apply to the handle of the winch by which the pinion is worked, and of which the arm is 16 in. long, in order to raise the weight?

3. To determine the pressure which applied in a given direction to the inner edge of the top of a pier of uncemented stones, will be just sufficient to overturn it.

4. A uniform beam A B, whose weight is W, rests with its end A on a smooth horizontal plane A C, and with its end B on a smooth plane B C, inclined 60° to the horizon. Find the tension of a string C A = C B which keeps it from sliding.

5. A cube has its base attached to a horizontal table. What is the greatest portion of it that can be cut off by a plane through one of its upper edges that this portion may

not slide upon the lower one when the coefficient of friction is $\frac{1}{4}$?

6. What is a '*simple pendulum?*' Find the time of oscillation of a simple pendulum in a small circular arc.

7. If a seconds' pendulum loses 10 beats in 24 hrs. by being taken to the top of a mountain, find approximately the height of the mountain: gravity varying inversely as the squares of the distances from the earth's centre, and the radius of the earth being 4000 miles.

8. How is the '*mass*' of a body estimated? Show how to find the acceleration of the motion of a given body by a given pressure.

9. A weight of 8 oz. hanging vertically draws a weight of 12 oz. by means of a string along a smooth horizontal table: find the space described by either body in the third second of their motion, and determine the tension of the string.

10. Distinguish between liquids and gases, between elastic and non-elastic fluids. What is the characteristic property of fluids in respect to their transmission of pressure?

11. The mean section of a stream is 4 ft. by 2 ft., its mean velocity 40 ft. per minute, it has a fall of $17\frac{1}{2}$ ft.; it is required to raise water by means of a water-wheel whose modulus is ·7; to what height will it raise 196 cubic ft. per minute?

XXXV.

1. Find the thrust at the ends of the rafters of a simple triangular roof without a king-post, in terms of their weight, their mutual inclination, and the horizontal span.

2. Two forces of 6 and 8 lbs. respectively, act at the ends of a rigid rod, without weight, 10 ft. long: the forces are inclined at angles of 60° and 30° to the rod: find the magnitude and position of the force which will keep the rod in equilibrium.

3. ABCD is a four-sided figure, and the diagonals AC, BD intersect in O; find the distance of the centre of gravity of the figure from O in terms of OA, OB, OC, OD and the angle between the diagonals.

4. On a uniform straight lever weighing 5 lbs., and 5 ft. in length, weights of 1, 2, 3, 4 lbs. are hung at the distances 1, 2, 3, 4 ft. respectively. Find the position of the fulcrum on which the whole will rest.

5. Four parallel forces P, -3P, -5P, and 6P act on a rigid rod at distances from one of its ends proportional to their respective magnitudes. Find the point in which the resultant meets the rod.

6. A weight of 1 oz. hanging over the edge of a smooth horizontal table, draws a weight of 1 lb. along the table by means of a string. Find the tension of the string, and the space described in $5''$.

7. Assuming $g=32{\cdot}19$, find the length of a seconds' pendulum: and if a seconds' pendulum loses 10 beats in 24 hrs., find how much it must be shortened to keep correct time.

8. A shot is fired *in vacuo* at an elevation $\sin^{-1}\frac{3}{5}$, with a velocity of 6440 ft. per second: find its height after $5''$ have elapsed, and the greatest height it attains.

9. Show that the time of a projectile's describing any arc of its path varies as the difference of the tangents of the angles which the tangent lines at the extremities of the arc make with the horizon.

10. If a heavy particle slide down the convex arc of a vertical circle from its highest point, show where it will quit the circle, and find the *latus-rectum* of the parabola which it afterwards describes.

11. How is the pressure at any point of a fluid estimated? In what direction does the fluid pressure at any point of the surface of a vessel take place? If the atmospheric pressure be 15 lbs. on the sq. in., and the steam in a boiler have an elastic force of 4 atmospheres, with what weight must a

circular safety-valve be loaded whose diameter is 3·5 inches, so as to prevent the escape of any steam?

12. Given that the surface of a sphere is four times the area of its greatest section, and that the centre of gravity of a hemispherical surface bisects the radius, find the pressure on a hemispherical bowl filled with water, the radius of the bowl being 12 inches. Weight of 1 cubic foot of water = 1000 oz.

XXXVI.

1. Two heavy bodies, P and Q, are situated at distances p and q from a plane, on the same side of it; find the distance of their centre of gravity from the plane.

2. The edge of a square is 2 ft., and coincides with the foot of a plane inclined at an angle of 15° to the horizon; how high may a parallelopiped be constructed upon the square, as a base, without toppling over?

3. Two beams 12 ft. and 16 ft. long respectively are connected by a tie-beam 20 ft. long, which rests upon a horizontal surface, and at their junction support a weight of 112 lbs.; find the pressure upon each prop and the strain on the tie-beam.

4. Seven bodies of equal weight are placed so that their centres of gravity coincide with as many angles of a cube, of which the diagonal measures a ft. Find the distance of their common centre of gravity from the unoccupied angle of the cube.

5. Three equal smooth balls of given weight lie in contact on a horizontal table, and a string passes round them in the plane of their centres. If a fourth equal ball be laid upon them so that the four centres are the angular points of a regular tetrahedron, find (t) the tension of the string.

6. State some cases in which *pressures* do no work. Find the work accumulated in a train weighing 80 tons and moving 60 miles an hour.

7. A right pyramid, with a square base of 16 ft. side, has an altitude of 24 ft., and stands on a horizontal plane, find the '*work*' necessary to overturn it round one of its edges, a cubic foot of its material weighing 100 lbs.

8. A train weighing 60 tons has a velocity of 40 miles an hour; if the steam be turned off, how far will it ascend an inclined plane rising 1 ft. in 100, supposing the friction to be 8 lbs. a ton?

9. A cylinder that weighs 100 lbs. turns upon its axis, which is horizontal; if motion be communicated by a weight of 10 lbs. attached to a perfectly flexible string without weight, which is coiled round the surface of the cylinder in a vertical plane, find the space through which the weight will descend in 10″.

10. Show how to determine the *s. g.* of a substance by weighing it in air and in water. A pound weight of gold, *s. g.* 17·5, is coined into $46\frac{29}{40}$ sovereigns; how many grains will a sovereign weigh in water?

11. A hollow cone, whose vertical angle is 2θ, is filled with water, and placed with its base downwards; determine the place where a small orifice must be made in its side so that the issuing fluid may just strike the bottom of the cone.

12. In the vertical side of a vessel is a rectangular orifice having one side horizontal. This aperture is closed by a rectangular board just fitting it, and capable of turning in one direction only round a horizontal axis in its own plane. Determine the position of this axis so that no fluid may escape from the vessel until it has risen a certain height above the aperture.

XXXVII.

1. If two parallel forces act on a straight rod, without weight, tending to twist it in opposite directions, find the

magnitude and point of application of the force which will keep it at rest.

2. If the parallel forces 7 and 5 act at the opposite ends of a rod 12 in. long, making an angle of 30° with the direction of the rod, where and how must a single force act for equilibrium? Find the perpendicular pressure on the rod.

3. The angle at the vertex of an isosceles triangle is 120°, from this angle and the middle points of the equal sides perpendiculars are drawn to the base: supposing the perimeter of the triangle and these perpendiculars to be uniform rods, find the centre of gravity of the framework thus formed.

4. Given the distances of the centres of gravity of any system of heavy bodies from a given plane, to find the distance of their common centre of gravity from the same plane.

5. A B is a rod, without weight, turning about its middle point C; A D is a heavy uniform rod jointed to the former at A, whilst its end D is attached to the point B, of that rod by a string D B, so that D A B is a right angle. Draw a figure exhibiting the position of equilibrium, and mark upon it the direction of the mutual action at the joint A.

6. A body weighing 6 lbs. slides from rest down a rough inclined plane through 20 ft. in 4 seconds. What force acting along the plane would sustain the body in equilibrium?

7. An elastic ball impinges obliquely on an immoveable plane; given the velocity and direction of motion before impact, find the motion after impact.

8. Show that if a perfectly elastic ball be let fall upon an inclined plane, and rebound striking the plane again, the interval between the collisions is independent of the inclination of the plane.

9. If a ball A impinge upon B at rest, and the modulus of elasticity be $\frac{2}{3}$, what must be the ratio of the masses in order that A may be reduced to a state of rest by the impact?

10. The elevation of a projectile being 15°, what must be the velocity of projection in order that the horizontal range may be a mile? And to what height will it rise in its flight?

11. The capacity of the barrel of an air-pump is half that of the receiver; if a barometer that stands at 30 in. be placed under the receiver, after how many strokes of the piston will the barometer fall below 3 inches?

12. Find the quantity of water discharged in a given time through an orifice made in the bottom of a vessel kept constantly full. If a circular orifice be 2 in. in diameter, the head of the water 4 ft., and the discharge $\frac{1}{8}$ of a cubic foot per second, will the discharge agree with the theoretical result? What is meant by the coefficient of velocity? Find it in the preceding example.

XXXVIII.

1. Define '*statical friction*;' and state the laws which apply to it.

2. A uniform beam rests with its ends upon two given inclined planes; find its inclination to the horizon when the planes are rough, the coefficients of friction being μ and $\mu_{,}$.

3. Find the altitude of the largest cone upon a base of given diameter ($2r$) that will just stand upon a rough plane, inclined at an angle $a°$ to the horizon. Find the highest point in its side to which a weight may be attached without upsetting the cone.

4. If a solid cone whose height is twice the diameter of its base be suspended from a point in the circumference of its base, prove that in the position of equilibrium its axis will be inclined at an angle of 45° to the horizon.

5. If an inch in length of a catenary weighs 2 oz., and the tension at the lowest point is 20 oz., find the tension at a point

of catenary whose vertical height above the lowest point is 5 inches.

6. A particle moves over 14, 22, and 34 ft., respectively, in the 1st, 3rd, and 6th seconds of its motion. Is it subject to the action of a uniform force?

7. A body falls from the top of a tower 330 ft. high. How many feet does it pass over in the last $3''$?

8. Two weights of 25 and 6 lbs., respectively, are connected by a cord passing over a pulley. In what time will the former descend through 166 ft. from rest?

9. A ball is projected upwards with a velocity v, and when it has attained $\frac{3}{4}$ of its greatest altitude another ball is projected upwards from the same point with the same velocity; determine when and where they will meet.

10. Determine the velocity with which a particle must be projected so as to strike the top of an object 50 ft. high and 1000 yds. distant, the angle of projection being 45°.

11. At what rate would a locomotive engine working at 60-horse power draw a train up an inclined plane having a rise of 1 in 100, supposing the train and engine to weigh 60 tons, and the friction to be 6 lbs. a ton?

12. The *s. g.* of quartz is 2·62, that of gold 19·35; a nugget of quartz and gold weighs 11·5 oz. and its *s. g.* is 7·43; find the weight of gold in the nugget.

XXXIX.

1. Prove the property of Guldinus for determining the volume of a solid of revolution, and apply it to determine the volume of a ring whose section is an equilateral triangle.

2. A weight of 5 lbs. is suspended freely from a fixed point by a perfectly flexible string; find what horizontal force applied to the string will draw the upper portion of it 30° out of the perpendicular.

3. The ends of a cord whose length is $2a$ are fastened at

the points A and B whose distance is $2c$; a smooth ring running along the cord sustains a weight W: determine the position of equilibrium and the tension of the cord, A B being inclined at α° to the horizon.

4. A bar of iron of uniform thickness, 10 ft. long and weighing $1\frac{1}{2}$ cwt., is supported at its extremities in a horizontal position, and carries a weight of 4 cwt. suspended from a point distant 3 ft. from one extremity. Find the pressures on the supports.

5. Each side of a square A B C D is 18 inches, A C and B D the diagonals intersect in O, the triangle B O C being removed; find the centre of gravity of the remainder, and show that its distance from A D is 7 inches.

6. How are *moving* forces and *uniformly accelerating* forces respectively estimated in dynamics? Give an example of each kind of force, and show how the relation of these forces to each other is expressed.

7. If two unequal weights P and Q, $P > Q$ connected by a string are suspended over a fixed pulley, find the space through which the heavier will descend in t seconds.

8. If two equal weights, each of 50 oz., are suspended over a fixed pulley, what weight must be added to one of them to make it descend 64·4 feet in 5 seconds?

9. If the string in *Qu.* 8 be cut after an interval of n'' from the commencement of motion, how far will the bodies be apart after a second interval of m''?

10. Describe the construction and action of the air-pump.

11. If the mercury in a barometer placed in the receiver of an air-pump stand at 30 in., and if after 12 barrels have been pumped out the mercury sinks 13 in., compare the capacities of the receiver and the barrel.

12. Show how to find the pressure on any plane area immersed in a liquid. If an isosceles triangle be immersed vertically in a liquid with its base horizontal and its vertex coincident with the surface of the liquid, and if the perpendicular from the vertex on the base be bisected by a line

parallel to the base, show that the pressure upon the upper portion of the triangle is to the pressure on the lower as 1 to 7.

XL.

1. Show how to determine the centre of gravity of the frustum of a pyramid, the ends being parallel.

2. A body P suspended from one end of a lever without weight is balanced by a weight of 1 lb. at the other end of the lever; when the fulcrum is removed through half the length of the lever, it requires 10 lbs. to balance P: determine the weight of P.

3. A heavy beam rests upon a peg with one end against a smooth vertical wall: find the position of equilibrium.

4. A uniform lever 4 ft. long weighs 10 lbs., and weights of 30 lbs. and 40 lbs. are appended to its extremities; where must the fulcrum be placed to produce equilibrium?

5. If three parallel forces P, Q, R, act at given distances from each other *perpendicularly* on a straight lever without weight, find where the fulcrum must be placed to maintain equilibrium; would it affect the position of the fulcrum if the word in italics were omitted?

Ex. $P=5$, $Q=7$, $R=9$; distance between P and $Q=3$ feet, distance between Q and $R=4$ feet.

6. Explain why the force of gravity at the earth's surface is represented by 32·2 feet.

7. How is velocity estimated in dynamics when variable? Investigate the formula for descending bodies $s=vt+\frac{1}{2}gt^2$. What change must be made if the body is ascending?

8. A body is projected vertically upwards with a velocity of 644 ft. per second; find the time at which it will be at the height of 2817·5 ft. Explain the double result.

9. How is fluid pressure estimated at any point of the surface of a vessel with which the fluid is in contact?

10. Show how to find the pressure on the whole surface of any vessel of known form containing liquid.

11. A cylinder whose radius is 9 in., and height 24 in., is filled with water; find the pressure on the internal concave surface of the cylinder, the weight of a cubic foot of water being 1000 oz. avoirdupois.

12. How is it shown that the elastic force of the air at a given temperature varies inversely as the space it occupies?

XLI.

1. Find the relation of P to W in the system of 5 pulleys in which each string is attached to the weight. When P = 100 lbs., find (1) W; (2) the strain on the fixed pulley, each pulley weighing 5 lbs.

2. The arms of a bent lever are at right-angles to one another, and one of them is twice as long as the other: a weight of 18 lbs. is suspended from the shorter arm, and 12 lbs. from the longer: find the angle which the latter makes with the horizon in the position of equilibrium.

3. An iron wire, $\frac{1}{6}$ of an inch in diameter and 12 ft. long, is securely attached to two fixed points 10·5 ft. apart; what weight suspended in the middle point of the wire will break it, assuming that it will bear a strain of 75000 lbs. to the sq. inch?

4. If a system of forces acting at a point O and keeping it at rest be represented in magnitude and direction by OL, OM, ON, &c., prove that O will be the centre of gravity of a system of particles of equal weights placed at L, M, N, &c.

5. Explain what is meant by '*friction*,' and by the '*limiting angle of resistance*.'

6. A heavy beam rests with one end on the ground, and the other end in contact with a vertical wall. Having given μ, μ_1, the coefficients of friction for the wall and the ground respectively, a and b the distances of the centre of gravity of the beam from its upper and lower ends: find its least possible inclination to the ground.

7. Find the tension of the string in the conical pendulum.

8. A weight of 2 lbs. revolves at the extremity of a string 2 ft. long, and makes two revolutions in a second: find the tension of the string in pounds.

9. Show how to find the centre of oscillation of any number of material particles.

Four particles, whose weights are as 1, 3, 7, 5, are situated in a straight line at distances, measured in the same direction, 1, 2, 3, 4 from a horizontal axis perpendicular to this line; find the centre of oscillation of the weights.

10. If the axis be shifted and made to pass through the centre of oscillation thus found, where will the new centre of oscillation be?

11. In the air-pump the capacity of the barrel is $\frac{1}{10}$ that of the receiver; show that, after 3 strokes of the piston, the air in the receiver will have lost $\frac{1}{4}$ of its density nearly.

12. Three observed readings of an imperfect barometer are h_1, h_2, h_3, are respectively a_1, a_2, a_3 less than they would be were the barometer true; prove

$$\frac{h_2-h_3}{a_1}+\frac{h_3-h_1}{a_2}+\frac{h_1-h_2}{a_3}=0.$$

XLII.

1. Find the magnitude and direction of any number of parallel forces acting upon a body.

2. Equal forces act in parallel directions at the angles of a regular hexagon: determine the line of action of their resultant.

3. A uniform rod rests partly within and partly without a fixed smooth hemispherical bowl: determine the position of equilibrium.

4. Find the centre of gravity of weights 3, 4, 5, placed in the angles of a triangle whose sides are 3, 4, 5: the least weight being placed in the least angle, and the greatest in the greatest angle.

5. Prove that the distance of the centre of gravity of n equal particles from any plane is $\frac{1}{n}$th part of the sum of the distances of the particles from the same plane.

6. Find the distance from the ground of the centre of gravity of a square pyramid formed of 12-inch spherical shot, the number of shot in a side of the base being 15.

7. In how many seconds would a stone fall, *in vacuo*, through a mile? What is the momentum acquired by a 24 lb. shot when it strikes the earth after falling from the top of a tower 100 ft. high? With what velocity must the shot be thrown to reach the ground in $2''$?

8. Given the moment of inertia of a body or system of bodies about an axis passing through its centre of gravity, find its moment of inertia about any axis parallel to the first.

9. A shaft 8 ft. in diameter is to be sunk 60 ft. in chalk, whose *s. g.* is 2·315. How many days' work would be expended in raising the chalk to the surface by a man working with a windlass, and yielding 70000 units of work daily?

10. Two weights, P and 2P, are connected by a flexible cord passing over a pulley: find how far 2P will descend in two seconds, and determine the tension of the cord.

11. A cannon ball weighing 68 lbs. after traversing 4 ft. of the bore of a cannon, leaves it with a velocity of 1000 ft. per second; what mean pressure must have been exerted upon it to give it this velocity, neglecting friction and the resistance of the air?

12. The lever of a hydrostatic press is 3 ft. long, and the distance from the fulcrum to the point at which it acts on the smaller piston is 8 in., the diameter of the greater piston is 1 ft., and that of the less $\frac{1}{2}$ inch.; with what pressure does each pound applied to the extremity of the lever tend to raise the greater piston?

XLIII.

1. Prove, that if a body be suspended freely from a fixed point, it will not be in equilibrium, unless the line joining the point of suspension with the centre of gravity be vertical.

2. Show that the principle of virtual velocities holds good in the case of the lever.

3. The arms of a false balance are to one another as 7 to 8, and the weight is put into one scale as often as into the other: what will be the gain or loss per cwt. to the seller?

4. A triangular slab of uniform thickness is supported at its 3 angular points. Show that the pressures on the supports are equal to one another.

5. Find the dimensions of the strongest rectangular beam that can be cut out of a given cylinder.

6. A body, of weight W, is placed on a rough plane which is inclined at an angle α to the horizon. If the coefficient of friction be $\tan\beta$, show that the body cannot be drawn up the plane by a force less than $W\sin(\alpha+\beta)$.

7. A body is projected upwards with a velocity u, and, after passing through a space s, has a velocity v; show that $v^2=u^2-2gs$. If the velocity of projection be $5g$, find the time in which the particle will rise to the height $8g$.

8. A weight of 100 lbs. is drawn along a smooth horizontal table by means of a weight of 20 lbs. hanging vertically: find the space described in $5''$.

9. Show that the centre of gravity of the weights in *Qu.* 8 moves in a straight line, and find its velocity at the end of the time t.

10. One perfectly elastic ball strikes another equal ball at rest, the direction of the motion of the moving ball making an angle of 45° with the line joining their centres: find the direction of the motion of each ball after impact.

11. A circular plate has a radius of 4 in., find the moment of inertia of the plate about an axis perpendicular to its plane at a distance of $\frac{1}{2}$ in. from its centre.

12. Describe the diving-bell. If the bell be cylindrical and sustained by a rope in the direction of its axis, find the tension of the rope when the bell has descended to a given depth.

XLIV.

1. Prove the parallelogram of couples.

2. There is a homogeneous cylinder, whose height is 12 in., and radius of base 6 in.; it is placed upon its end on a horizontal plane which is gradually inclined. At what inclination will the cylinder fall over? What is the coefficient friction if the cylinder be on the point of overturning and slipping at the same instant?

3. If a tetrahedron $M\,M'\,M''\,M'''$ be formed by joining the centres of gravity of 4 masses m, m', m'', m''', these masses will be respectively proportional to the tetrahedron, whose common vertex is the centre of gravity of the four masses, and whose bases are respectively the opposite faces of the tetrahedron $M\,M'\,M''\,M'''$.

4. Two uniform beams of unequal lengths are placed in the interior of a smooth spherical surface, with their lower ends in contact: determine the position of equilibrium?

5. Weights of 1, 2, 3, 4, 5, 6 lbs. are placed in order at the angular points of a hexagon: find their common centre of gravity.

6. Show that the principle of virtual velocities holds in the case of a body in equilibrium on a smooth inclined plane.

7. Prove that centrifugal force is directly proportional to the radius of the circle described, and inversely proportional to the periodic time. Also that the earth's attraction is about 289 times the centrifugal force at the equator.

8. A body is projected in a direction making an angle of

15° with the horizon, and a velocity of 50 ft. a second; find its range, greatest altitude, and time of flight.

9. A shell being discharged at an angle of 45°, its explosion, which took place at the moment it reached the ground, was heard at the mortar 3·5″ after the discharge. Required the horizontal range, the velocity of sound being $35g$ ft. a second.

10. A weight is placed against the inner surface of a hollow cylinder which revolves round its axis, which is vertical. The coefficient of friction being μ, determine the angular velocity so that the weight w may, by its centrifugal force, be just prevented from slipping.

11. Describe briefly the '*Hydrostatic Bellows.*'

12. A cylindrical pontoon 9 ft. long and $2\frac{3}{4}$ ft. radius floats with $4\frac{1}{8}$ ft. of its vertical diameter out of the water; what weight will be necessary to sink it to the water level?

XLV.

1. What is the axis of a statical couple? Three couples act upon one body in three coordinate planes at right angles to each other: find the magnitude and position of the resultant couple.

2. If the 4 faces of a triangular pyramid be equilateral triangles, each having a side equal to a, and if h be the distance of the centre of gravity from any angle of the pyramid, prove that $h^2 = \frac{3}{8} a^2$.

3. A pyramid is cut from a cube by a plane which passes through the extremities of three edges that meet in a point. Find the distance of the centre of gravity of the remainder from the centre of gravity of the cube.

4. How is work estimated mathematically? In a machine is the work applied always equal to the effective work?

5. A cistern is 20 ft. long, 7 ft. wide, and 8 ft. deep, the height of the bottom of the cistern from the surface of the water in a well is 56 ft.; in what time will a man pump the

cistern full of water, allowing that he performs 2600 units of work per minute, and that the modulus of the pump is ·66 ?

6. Two velocities are impressed simultaneously upon a body; show how to determine its motion. A ship is sailing at the rate of $10\frac{10}{11}$ miles an hour; across the deck perpendicular to the direction of the ship's course a ball is bowled with a velocity of 12 ft. a second; find its path and velocity.

7. A given weight is fastened to one extremity of a string, and whirled round the other end in a horizontal plane with a uniform velocity describing the circumference in a given time; find the tension of the string.

8. A string 3 ft. long is capable of sustaining a weight of 6 lbs.; if a weight of 1 lb. be attached to the end of the string, find the greatest number of revolutions it can make in a minute without breaking the string.

9. Define the terms, '*neutral line*,' '*neutral surface*.' Show how to find the point of rupture of a rectangular beam.

10. Three weights, 3, 4, 5 lbs., attached to rigid rods without weight of 1, 2, 3 ft. in length, respectively revolve round a vertical axis in 30″; find the accumulated work in the system, and its radius of gyration.

11. For what surfaces when immersed in a fluid will the fluid pressures have a single resultant? For what surfaces will the directions of the pressures all pass through one point ?

12. If a right-angled triangle, having one of its oblique angles double the other, be immersed vertically in a fluid with its hypothenuse coinciding with the surface of the fluid, show that the pressures on the two triangles formed by joining the right angle with the middle of the hypothenuse will be equal.

XLVI.

1. (1) If four forces act upon a point to keep it at rest, *state* generally how any one of the forces must be related to the resultant of the other three.

(2) If three forces acting on a point are in equilibrium, show that each force is proportional to the sine of the angle contained by the other two.

(3) If OP, OP_1, represent two equal forces acting upon O at right angles to each other, and OQ represent the third force which keeps O in equilibrium, show that the triangle formed by the intersection of the lines, drawn perpendicular to OP, OP_1, OQ, through P, P_1, Q, is isosceles and right-angled.

2. Find the conditions of equilibrium of any system of forces acting in one plane. Account for one of these conditions *only* being necessary in the case of a lever attached to a fulcrum.

3. A uniform beam AB, whose weight is W, rests in equilibrium, between a vertical wall BC and a horizontal plane AC, both smooth. CE is a string without weight, attached to a point E in the beam. If $\text{BAC}=\alpha$, $\text{ACE}=\beta$, show that the tension of the string$=\dfrac{\text{W} \cos \alpha}{2 \sin (\alpha-\beta)}$.

4. Explain the construction of the Screw, considered as a mechanical power. Find the relation between the power P and the weight W, when the screw is rough and the friction opposed to P.

5. If a beam AB is supported in a horizontal position at A and B, and the weight is uniformly distributed along AB, find the moments of the strain at any point C of the beam. About what point will this moment be greatest?

6. When is the impact of two bodies said to be direct? An imperfectly elastic ball A impinges directly with a given velocity upon another ball B, of the same material, at rest; investigate the velocity of each after impact.

7. If B in *Qu.* 6 impinge upon another ball C at rest, find the velocity of C, and show that it is greatest when B is a mean proportional between A and C.

8. A projectile has a velocity of 1,600 ft. a second, and is projected at an angle of 30° ; find its horizontal and vertical distances from the point of projection after 12″, gravity being taken at 32 ft.

9. Three equal balls are projected at the same instant, from the same point, with the same velocity, 1,600 feet in a second, and at angles 30°, 45°, 60°, respectively. Find the height of the common centre of gravity of the balls above the horizontal plane after 12′.

10. If O be the centre of oscillation, K the centre of gyration, G the centre of gravity, of a body suspended from a horizontal axis passing through C, prove $CG \times CO = CK^2$.

11. Find an expression for the time of a small oscillation of an equilateral triangle, oscillating in its own plane about a horizontal axis, through an angle perpendicular to the plane of the triangle.

12. How is it shown that the elastic force of the air at a given temperature varies inversely as the space it occupies ?

> A cylindrical tube 2 ft. in length, and closed at its upper end, is immersed vertically in sea-water, and it is found that the water has risen in the tube $1\frac{1}{2}$ ft. ; find the depth to which the tube has been sunk, supposing 32 ft. of sea-water measures the atmospheric pressure.

XLVII.

1. (1) Define the terms '*Force*,' '*Tension*,' '*Velocity*.' (2) If two men pull the ends of a short cord in opposite directions, each exerting a force of 20 lbs., what is the tension of the cord ? (3) When a cricket-ball is flying in the air, what forces are acting upon it ?

2. A uniform heavy bar, 2 ft. long, can turn about a

fixed point. A weight of 10 lbs., hung at its ends successively, is balanced by the weights $4\frac{1}{2}$ lbs. and 21 lbs. hung at the other end. Find the weight of the bar, and the lengths of the portions on each side of the fixed point.

3. Employ the principle of virtual velocities to find the weight which 1 lb. will support by means of two blocks of pulleys, of which the upper one is fixed, and the lower one, weighing 1 lb., is moveable, each block containing 4 pulleys or sheaves, and the portions of string between the blocks being all vertical.

4. A weight of 1 lb. rests on a rough plane, whose inclination to the horizon is 30°, and a string, attached to this weight, passing along the plane and over its lower edge. sustains a weight of 4 oz. hanging by it. Find the ratio of the friction to the pressure on the plane.

5. Two uniform rods AB, CD, are connected by the lower end B of one being jointed to a given point B of the other, and the points A and D being united by a string, so that ABD is a right angle. The system hangs from the point A, about which it can turn. Find the position of equilibrium, and the action at the joint B; having given that, $AB=BC=4$, $BD=8$; and that the weight of AB is double the weight of CD.

6. Three elastic balls A, B, C, have their centres in a straight line, and lie at rest. A has a motion impressed upon it, with which it strikes B directly, and B then strikes C. If after impact, A and B are stationary, prove that the masses of the balls are in geometric progression.

7. A body projected at an elevation of 45° strikes an object 20 yds. high, at a horizontal distance of 400 yards. Find the other angle of elevation at which the body may be projected, with the same velocity, to strike the same object. Compare the times of flight in the two cases.

8. A cylindrical tumbler of 3 in. diameter is filled with water to the height of 2 in. Find the whole pressure on

the cylindrical surface, taking the weight of 1 c.ft. of water to be 1,000 oz.

9. Prove that the resultant pressure of a liquid on a solid floating in it, is equal to the weight of the liquid displaced.

10. What is '*Specific Gravity?*'

11. A body weighs 56 oz. in a vacuum, and 49 oz. when immersed in water; required the weight in pounds of a cubic foot of the substance of which the body is composed.

12. Explain the principle of the diving-bell.

A hemispherical bell is sunk in water until the surface of the water under the bell bisects the vertical radius; find the depth to which the bell is sunk.

XLVIII.

1. Show the propriety of representing statical forces geometrically, and mention any advantages which belong to this mode of representation.

2. What is meant by the '*Coefficient of Friction?*' How is it related to the '*limiting angle of resistance?*'

3. A uniform ladder is just supported by the friction of the ground, resting at an angle of 45°, between a smooth vertical wall and a rough horizontal plane; find the coefficient of friction.

4. Find the centre of gravity of three heavy particles placed in the angles of a right-angled triangle; and show that if the weights of the particles be proportional to the opposite sides of the triangle, their centre of gravity will coincide with the centre of the inscribed circle.

5. Distinguish between '*Stable*' and '*Unstable Equilibrium.*'

Two convex surfaces are spherical at their point of contact, the upper surface is displaced by rolling through a small angle; find the condition that the equilibrium may be stable.

6. A heavy right cone rests with its base on a fixed

rough sphere of given radius, find the limit to the height of the cone consistent with stable equilibrium.

7. If material of given weight and form is raised through a given height, how is the work estimated?

A reservoir 120 ft. long, 60 ft. wide, 22 ft. deep is to be filled from a river whose surface is 50 ft. below the bottom of the reservoir; find the horse-power of an engine that will fill it in 24 hrs. (*Wt. of* 1 *c.f. of water* = 62·5 *lbs.*)

8. Distinguish between '*Accelerating Force*' and '*Moving Force*,' and show how they are respectively estimated in Dynamics.

9. A body is projected in the direction of an accelerating force with a velocity u; after a time t it has acquired a velocity v, prove that s the space described $= \frac{u+v}{2} \times t$.

10. A body projected in the direction of gravity with a velocity of 64 ft. a second, describes 241·1 ft. in the nth second; find n; $g = 32{\cdot}2$ ft.

11. How may the force of gravity be determined by means of a simple pendulum? Assuming the expression for the time of an oscillation; show how the height of a mountain may be determined by means of a pendulum.

A seconds pendulum taken to the top of a mountain loses 10 oscillations in 24 hrs.; find the height of the mountain.

12. A weight of W lbs. is pulled up an inclined plane by a pressure of P lbs. acting parallel to the plane; show that the accelerating force is equal to $\left(\frac{P}{W} - \frac{\sin(\alpha+\phi)}{\cos\phi}\right)g$, in which α is the inclination of the plane, ϕ the limiting angle of resistance.

XLIX.

1. A string ABC hangs vertically from the fixed point A with weights B, C, of 7 and 4 oz. respectively, attached to it at given points. State the forces which act on the upper

weight B and are in equilibrium, and find the tension of the parts of the string AB, BC.

2. If three men keep a body at rest by pulling it by cords with equal forces, prove that the directions of the cords include angles of 120°. If the direction of one of the cords is slightly altered in the plane of the forces, the other two remaining unchanged; prove that the men who pull these two must both alter the force with which they pull if equilibrium is to be maintained.

3. A weight of 100 lbs. is carried by two men on a horizontal hand-barrow, the handles which they hold being 6 ft. and 4 ft. respectively, from the weight. What weight does each man sustain? Is the result altered if the barrow be not horizontal.

4. State the conditions necessary and sufficient for equilibrium when forces act in one plane on a rigid body.

5. AB, CB are uniform rods of given weights, whose upper ends are capable of turning about given points A, C in the same horizontal line. If their lower ends are jointed together at B, find the mutual action between them at that point. Explain the result when the rods are together equal in length to AC.

6. In the case of an isosceles roof supporting a given weight, when there is *no tie beam*, show how to determine the magnitude and direction of the pressure, tending to overturn the side walls that support the roof.

7. Find the centre of gravity of a trapezium in which AB, CD are parallel sides, and AC is perpendicular to them, AB being double AC or CD.

8. State how the velocity of a body regarded as a point is measured.

9. If two bodies of masses m, m', moving in the same straight line, have at any instant the velocities v, v', prove that the velocity of their centre of gravity at the same instant is $\frac{mv+m'v'}{m+m'}$.

10. When a body moves from rest under the action of a constant accelerating force, prove the formula $s=\frac{1}{2}ft.^2$.

If a body moving as above describes 50 ft. in 50 seconds, what is the velocity produced in it?

11. Prove that the open surface of a liquid at rest under the action of gravity is a plane.—Why does the upper surface of the mercury in a barometer take a curved form?

12. Explain how a '*siphon*' draws liquid out of a vessel.

L.

1. Three given forces are in equilibrium on a point; show how to determine the angle between the directions of any two of the forces.—The magnitudes of the forces are as 2, $\sqrt{6}$, $\sqrt{3}+1$; determine their inclination to each other.

2. Is the tension of a perfectly flexible string affected by its passing over (1) a smooth surface, (2) a rough one? A and B are given fixed points in a horizontal line, BC is a string of given length carrying a small ring at C, which is without weight; a string ACW fixed at A, passes through the ring C, and hanging vertically supports a weight W; find the position of the ring when there is equilibrium.

3. Explain the term '*Virtual Velocity*.' Show that the principle of virtual velocities holds in the case of two forces in equilibrium on a bent lever.

4. When a horse-power is said to be measured by 33,000, what units are referred to?— What must be the horse-power of a locomotive engine which moves at the rate of 20 miles an hour upon a level rail, the weight of the train being 50 tons, and the resistance of friction 8 lbs. a ton?

5. In the impact, whether direct or oblique, of two smooth balls, show that the motion of the centre of gravity is unaffected by impact.—In the case of direct impact, when is the centre of gravity *at rest* both before and after impact?

6. Find the range and time of flight of a projectile on an inclined plane passing through the point of projection; and show, for a given velocity, how the body must be projected that the range shall be a maximum.

7. Find the inclination of the plane in *Qu.* 6 when the maximum range upon it is $\frac{2}{3}$ of the maximum range on a horizontal plane, with the same velocity.

8. Find the centre of oscillation of a right-angled isosceles triangle, about an axis through its vertex and perpendicular to its plane, in terms of the altitude of the triangle.

9. Explain the construction, action and graduation of a common barometer.

10. The mercurial column stands at 30 inches, the *s.g.* of mercury is 13·568; determine in lbs. the pressure of the atmosphere.

11. If a vertical wall, of given thickness and material, sustains the pressure of water in a reservoir, show how to determine the conditions that the wall shall just not be overturned.

12. If the section of an embankment of brickwork supporting the vertical side of a reservoir be of the form of a trapezoid whose parallel sides are horizontal, the upper side being 5 ft. and the lower 10 ft. and the whole height of the embankment 50 ft.; find the height to which the water must rise against the vertical wall so as just to overturn it, the *s.g.* of the brickwork being 2·17.

LI.

1. Show that the conditions necessary to render a force, acting on a particle, determinate may be represented geometrically.

2. A heavy particle W is placed on a smooth horizontal plane, a given weight P hangs over a smooth peg, at a given height above the plane, by means of a string the other end

of which is fastened to W, find the condition that P may just raise W off the plane: if P be not sufficient to raise W, will W remain at rest?

3. How does the friction of a surface modify the direction of its resistance? At what inclination of a plane will a particle be just in the act of moving down it, when the coefficient of friction is $\frac{1}{2}$?

4. If P be the force necessary to support a heavy particle on a smooth inclined plane, P_1 the force acting when the plane is rough and the particle is in a state bordering on motion *up* the plane, and P_2 the force acting on the particle when it is in a state bordering on motion *down* the plane, prove P an arithmetic mean between P_1 and P_2: P, P_1, and P_2 all acting parallel to the inclined plane.

5. Define the centre of gravity of a heavy body. Show that the centre of gravity of an equilateral triangle must coincide with the centre of the circle inscribed in it.

O is the centre of a given circle, AB a chord, OC bisects AB in C, take $OQ = \frac{2}{3}$ OC. Prove Q the centre of a circle, which is the locus of the centres of gravity of all triangles standing on AB as a base, and having their vertices in the circumference of the given circle.

6. Show that two equal and opposite parallel forces acting on a rigid body cannot be kept at rest by a single force.—Define the '*axis of a couple*,' and show that any couple acting on a rigid body may be replaced by three couples in planes at right angles to each other.

7. If a heavy, uniform and flexible string be suspended from two fixed points, determine the tension of the string at any point of the curve which it forms. Hence show that if a uniform chain hangs freely over two fixed points the extremities of the chain will be in the same horizontal line, in the case of equilibrium.

8. A weight P hanging freely over the edge of a horizontal table draws Q along the table; find the accelerating force and the tension of the string as P descends (1) when the

table is *smooth*, (2) when it is *rough*, the friction of the string against the table being neglected. If the table be rough and the coefficient of friction $\frac{1}{3}$, find the space described from rest in $3\frac{1}{2}$ seconds, when the weights of P and Q are 4 and 9 oz. respectively.

9. A weight W attached to the end of a string of length r revolves uniformly in a horizontal plane about the other end of the string; show that if T be the tension of the string and P the time of one revolution $\frac{T}{W}=\frac{4\pi^2 r}{P^2 g}$.—A string 3 ft. long is capable of supporting a weight of 6 lbs.: how many revolutions per minute may be made by a weight of 2 lbs., which is attached to one end, whilst it revolves uniformly round the other end, without breaking the string?

10. Find the centre of oscillation of any number of compound bodies when rigidly connected, the distances of the centre of oscillation and the centre of gravity of each separate body, from the point of suspension being given.

Find the centre of oscillation of 3 particles 8, 10 and 12, situated in the same straight line at distances 3, 5 and 7 ft. respectively from an axis perpendicular to this line.

11. Show how to find the *s.g.* of a solid that will not sink in water.

A piece of cork weighs *in vacuo* 2 oz.; when a piece of lead which weighs 8 oz. in water is attached to the cork, the united mass weighs in water $1\frac{2}{3}$ oz.; find the *s.g.* of the cork.

12. Show how to find an expression for determining the difference of altitude of two stations by means of a barometer, the temperature during the observations being constant. In determining the heights of mountains, what other considerations affect the problem?

LII.

1. Explain the meaning of the terms '*Action*' and '*Reaction*.'

A man, weighing 150 lbs., supports a weight of 50 lbs. by a vertical cord passing over a fixed pulley. What pressure does he exert on the ground?

2. Define the '*Moment*' of a force about a point.

Assuming the parallelogram of forces, prove that the algebraic sum of the moments of two forces about a point in the space between their directions is the same as the moment of their resultant.

3. A uniform rod weighing 10 lbs. passes through two fixed rings, and is horizontal. The rings are 3 ft. apart, and the lengths of the parts of the rod which project beyond them are 2 ft. and 11 ft. Find the pressure on each ring.

4. A cube of given weight resting on a horizontal table is gradually raised round one edge by a smooth wedge pressed under it. When the cube has been raised through an angle, which is half the angle of the wedge, find the friction between it and the table at the edge on which it turns.

5. Find the ratio between P and W on a single moveable pulley without weight, when the strings are not parallel. Under what circumstances will P be less than W?

6. When a ball of given elasticity impinges obliquely on a smooth plane, find its subsequent velocity, and show whether this is greater or less than its velocity before impact.

7. Prove that a projectile *in vacuo* will strike the horizontal plane through the point of projection, at an angle equal to that at which it was projected.

8. A body is projected with a velocity of 2,000 ft. per second, at an elevation of 10°; find its greatest altitude.

9. Find the pressure of a liquid upon a plane exposed to it.

A river-wall is inclined at 75° to the horizon. The depth of water being 6 ft., find the pressure on a rod of length of the wall, taking 1,000 oz. as the weight of a cubic foot of water.

10. When the barometer stands at the height of 30 in., find the ratio of the pressure of the air to its density having given that a cubic inch of mercury has a weight 10,462 times that of a cubic inch of air.

11. If a lighter fluid rests upon a heavier, and their specific gravities be s and s', and a body whose *s.g.* is σ, rests with its volume V in the upper fluid and V′ in the lower. Prove $V : V' :: s'-\sigma : \sigma-s$.

12. If a cylindrical vessel containing liquid be whirled round its vertical axis with a given angular velocity, show that the form of the surface of the fluid in a state of equilibrium is a paraboloid.

If the cylinder, open at the top, be filled, find the angular velocity so that half the liquid may be thrown out when the form of equilibrium is attained.

LIII.

1. Two forces, each of given magnitude, P, act perpendicularly on the rod ACB, at the points A, B. Replace them by two equivalent forces, one of which, of given magnitude, Q shall act perpendicularly on the rod at the point C, BC being double AC.

2. Find the position of the centre of gravity of a plane triangle.

Deduce, by Guldin's properties, the volume of a right circular cone.

3. Define a '*Vertical*' line. If a body is capable of swinging freely about a fixed point; prove that the positions in which it rests are those in which its centre of gravity is in a vertical line with the fixed point.

4. A uniform smooth bar, ABC, of weight W, is supported

horizontally at the ends A, C. Another uniform bar DB, of weight W′, can turn about its upper end D, which is vertically above C, and presses with its lower end B on the former bar. Given that $AB = \frac{1}{3} AC$; find the pressures on the points of support A, C, D.

5. If a body describes a straight line with a uniformly accelerated motion, prove that its motion resolved in the direction of any other straight line is also uniformly accelerated. Is any one of the three laws of motion required in the proof?

6. A body shot vertically upwards is 240 ft. above the point of projection at the end of 4″. How high does the body rise ($g=32$)?

7. Two unequal weights are connected by a perfectly flexible string without weight, passing over a smooth fixed pulley, so that one descending draws up the other. Prove that the tension of the string remains the same throughout the motion.

8. If a body slides from rest down a smooth plane, inclined to the horizon at the angle 23°, in what time will it gain a velocity of 150 ft. per second ($g=32{\cdot}2$)?

9. A body is projected *in vacuo* with a velocity of 1,610 ft. in a second, at an angle of 45°; find the range, greatest height, and area of the parabola described.

10. A particle descends the arc of a quadrant of a circle, the radius of which is 16 ft.; find its velocity at the lowest point.

11. A straight tube 4 in. in diameter has its lower end hemispherical. If it be held in a vertical position, and water be poured into it to stand at a height of 6 in. above its lowest point; find the resultant pressure on the hemispherical end (1 *c.ft. of water weighing* 1,000 *oz.*).

12. A uniform cylinder floating vertically in water has 6 in. of its length immersed. How far will it sink in a fluid whose *s.g.* is ·8?

LIV.

1. Explain why a single force acting on a point may be resolved into two others, in any directions at right angles to one another.

Find the resultant of three or more forces acting on a point in a plane.

2. ABC is a triangle, bisect AC in E, bisect AE, CE in G, and F respectively; if BA, BG, BE, BF, BC, represent, in magnitude and direction, forces acting on the point B; prove that their resultant is equal to 5 BE.

3. When any number of forces are in equilibrium on a rigid body in one plane, three conditions are generally necessary for equilibrium; state them, and explain the kind of motion to which each condition applies.

In the case of forces acting in one plane on a lever, account for one condition being sufficient for equilibrium.

4. A heavy equilateral triangle is hung upon a smooth peg by a string, the ends of which are attached to two of its angular points; if the distance of the centre of gravity of the triangle from the peg is equal to a side of the triangle, show that the triangle has a side either in a horizontal or a vertical position.

5. If a given pressure act on a given wall, in a vertical plane and in a given direction, how is the stability of the wall determined?

A wall of brickwork 3 ft. thick and 35 ft. high, sustains on the inner edge of its summit a pressure on every foot of its length, the direction of the pressure making an angle of 60° with the horizon: find its amount when it will just not overthrow the wall, the weight of 1 c.ft. of brickwork being 112 lbs.

6. Find the centre of gravity of a sector of a circle.

7. A ball impinges obliquely on a fixed smooth plane; find the motion of the ball after impact, the elasticity between the ball and the plane being given.

A ball, whose modulus of elasticity is $\frac{1}{3}$, is let fall from a given height h above an inclined plane whose inclination is 30°; after striking the plane the ball rebounds and strikes it again: show that the range on the plane between the two points of impact is $\frac{8h}{9}$.

8. If a heavy body be dragged along an inclined plane, show that the units of work expended will be equal to the numbers that would be expended in dragging it along the base, supposed equally rough, and in lifting it up the perpendicular height.

What distance will an engine of $92\frac{2}{11}$ horse-power drag a train in 10 minutes with a uniform velocity up an incline of 1 in 200, the weight of the train being 80 tons and the friction 7 lbs. per ton?

9. What are the '*Centres of Gyration*' and '*Oscillation*' of a material body?

A heavy particle is placed at a distance of 36 in. from the centre of suspension; at what distance must another heavy particle of double the weight be placed, in order that the two when connected may vibrate seconds?

10. If a plane surface be exposed to the pressure of any fluid, what is understood by the '*Centre of Pressure?*'

A rectangle is immersed vertically in a fluid, one side coinciding with the surface of the fluid; find the depth of the centre of pressure.

11. Describe '*Nicholson's Hydrometer*.'

The hydrometer weighs 250 grs. and requires 72 grs. to sink it to the required depth in water and 9 grs. in alcohol; required the *s.g.* of alcohol.

12. Describe the construction and action of the '*Forcing Pump*:'

Find the force necessary to overcome the resistance when the piston is in any position during its descent, and the water stands at a given height in the ascending tube.

LV.

1. Six given forces act on a point in directions of the successive sides of a regular hexagon given in position. Find the magnitude and direction of their resultant.

2. Find the relation between two weights which balance one another on a given wheel and axle.

Prove that the principle of Virtual Velocities is true in this instance.

3. A body weighing 10 lbs. lies on a rough plane between which and itself the coefficient of friction is $\frac{1}{6}$. The plane is inclined at an angle of 5° to the horizon. What pressure, acting along the plane, will the body bear before it begins to slide upwards?

4. Three smooth discs of equal size lie in contact upon a table, and are kept in position by a fixed ring, which circumscribes them all, so that they exert no pressure on it or on one another. If one of them is now acted upon by a given force P in the line joining its centre and the centre of the ring, find the pressures produced between the other discs and their pressures on the ring.

5. If a plane area revolves about a given axis, show that the content of the solid generated is equal to the area that revolves multiplied by the length of the path described by its centre of gravity.

Apply this property to find the content of the cone generated by the revolution of a given right-angled triangle.

6. A bar, 4 ft. long, of 6 lbs. weight, has weights of 3 and 6 lbs. hung from its ends. Find the position of a fulcrum on which the bar will rest in equilibrium.

7. A plane figure is formed by an isosceles right-angled triangle, and by the squares described upon the three sides of the triangle; find the centre of gravity of the whole plane area.

8. A ball impinges with a given velocity on a plane,

between which and itself the coefficient of elasticity is $\frac{1}{9}$, and it is reflected in a direction perpendicular to its direction at incidence. Find the velocity with which it is reflected.

If the velocity of projection be 500 yds. per second, in what second of its flight will the body have a direction inclined at 45° to the horizon? Show that when the body attains this position it will have travelled horizontally twice as far as it has fallen vertically.

9. Find the time in which a body will descend by gravity down a smooth inclined plane 200 ft. long, the height of the plane being 3 ft.

A train of 10 tons descends a plane of the above length and height; find the velocity acquired when the friction is 8 lbs. a ton.

10. Find the pressure on a given plane area under water, taking account of the pressure of the atmosphere on the surface of the water.

When a cube is held under water, state whether the resultant pressure of the water on it will change or not as the cube takes different positions.

11. A straight vertical tube closed at both ends is divided into two portions by a thin moveable disc without weight. The upper, 6 in. long, is full of water, of which a cubic foot weighs 1,000 oz. The lower part is full of air of such pressure as to keep the disc in equilibrium. Compare the density of the air in the tube with that of atmospheric air exerting a pressure of 15 lbs. on the square inch.

12. Draw figures exhibiting, without description in words, the action of (1) the forcing pump, (2) the air pump, (3) the condenser.

LVI.

1. When a weight hangs by a string, what is meant by the statement that the tension of the string is the same throughout? Is the tension the same if the string has weight?

A and B are two points in the same horizontal line, AC and BC are strings without weight, tied at the point C to a weight W, AB and AC are equal to each other, and the angle BAC is 36°. If T and T_1 are the tensions of BC and AC, prove $T - T_1 = \frac{W}{2 \cos 18°}$.

2. What is meant by the coefficient of friction between two substances? If the coefficient of friction between a rough horizontal plane and a moveable particle be $\sqrt{\frac{1}{3}}$, within what angle may a pressure be applied to the particle without causing it to move?

A rough hemispherical bowl, with the coefficient of friction $\sqrt{\frac{1}{3}}$, has a radius of 12 in.; find, approximately, the extreme height above the lowest point of the bowl, at which a particle may be placed on it so as to remain at rest.

3. If a body of any form be placed on a smooth horizontal plane; find the conditions that it may stand or fall.

4. If 2α be the angle of a right cone, standing on its base on an inclined plane whose inclination to the horizon is ϕ; show that when $4 \tan \alpha = \tan \phi$, the cone is on the point of turning over.

5. In the case of an isosceles roof, explain how the thrust on the tie-beam acts.

Two rafters, AB and AC, are each 20 ft. long; the tie-beam is 35 ft.; a weight of 1 ton is suspended from A; determine the strain on the tie-beam, neglecting the weight of the rafters.

6. Show how to find the position of the centre of gravity of any solid of revolution.

If a and b are the radii of the ends of the frustum of a paraboloid, nearer to and further from the vertex respectively, h the height of the frustum, find the distance of the centre of gravity of the frustum from the less end.

7. A heavy body W is drawn up an inclined plane by means of a string attached to a weight P that hangs over

the top of the plane; find the accelerating force, (1) when the plane is smooth, (2) when it is rough.

If the inclination of the plane is 30°, P = 25 lbs., W = 15 lbs., find the space described by P in 7·5″ when the plane is rough, the limiting angle of resistance being 30°.

8. What is a '*Simple Pendulum?*' Show that the time of a small oscillation in a circle whose radius is l will be equal to $\pi\sqrt{\frac{l}{g}}$.

9. A pendulum whose length is L makes m oscillations in one day; its length is diminished by a small quantity, and it is found to make $m+n$ oscillations in a day; show that the diminution of its length is $\frac{2n}{m}$ L nearly.

10. What is meant by the '*work accumulated*' in a moving body?

A train moving at the rate of 15 mi. an hour, comes to the foot of an incline of 1 in 300, the resistances being 8 lbs. a ton: how far will the train move up the incline before stopping?

11. Find the internal pressure on a sphere filled with water, the radius of the sphere being 6 inches and the weight of 1 c.f. of water 62·5 lbs.

12. Give reasons for the statement that the mercury in the barometer tube is supported by the pressure of the air on the surface of the mercury in the basin.

The mercury in a barometer which stands at 30 in. is placed under the receiver of an air pump, and after 12 turns the mercury sinks to 15 inches. Compare the capacities of the receiver and the barrel of the pump.

LVII.

1. The directions of two forces, represented by 3 lbs. and 5 lbs. respectively, include an angle of 60°. Find the

magnitude of their resultant, and the tangent of the angle which its direction makes with the less force.

2. If three forces P, Q, R, applied to the same point O are in equilibrium, prove that

$$P : Q : R :: \sin QOR : \sin ROP : \sin POQ.$$

Three forces are represented in direction by the perpendiculars drawn from the vertices of a triangle to the opposite sides, and are inversely proportional to those perpendiculars; prove that they are in equilibrium.

3. Find the magnitude and direction of the resultant of two parallel forces acting on a rigid body.

Explain the result when the forces act in opposite directions.

4. A rigid body is acted on by any number of forces in one plane; find the conditions of equilibrium.

Prove that a force and a couple in the same plane cannot be in equilibrium.

5. ABC is a uniform heavy lever of which B is the fulcrum, and of which the arms BA, BC include an angle of 120°. If, when the lever is left to itself in a vertical plane, the arm BA is horizontal, determine the ratio of BA to BC.

6. Determine the centre of gravity of a homogeneous pyramid having a triangular base. Show that it coincides with the centre of gravity of four equal weights placed at the corners of the pyramid.

7. State Newton's third Law of Motion, and give examples in illustration of it.

8. Prove that a body in falling from an upper point to a lower acquires just that velocity which would be requisite to carry it back from the lower point to the upper one.

9. If a circle be placed in a vertical plane, show that the time of descent down all chords of the circle, terminating at the lowest point of the circle, is the same.

Down which of these chords must a body fall in order to acquire at the bottom the greatest horizontal velocity?

10. Two equal weights P and Q are connected by a thread; P is placed at the bottom of a smooth plane, inclined at an angle of 30° to the horizon: the thread is passed over a pulley at the top of the plane, and Q hangs freely at its extremity. Compare the time in which P will be drawn to the top of the inclined plane, with the time in which, if separated from Q, it would fall down the plane, friction being neglected in both cases.

11. If two imperfectly elastic balls impinge directly, determine the subsequent motion of each.

A ball of given mass moving with a velocity of 1 ft. per second impinges on an equal ball at rest; if the common elasticity of the two balls is ·5, find the amount of *vis viva* lost in the impact.

12. A cone floats in water with its vertex downwards; and the centre of gravity of the cone is level with the surface of the fluid: what is the specific gravity of the cone?

LVIII.

1. Show that any system of forces acting on a rigid body can be reduced to a force and a couple.

2. Investigate a formula for finding the centre of gravity of any solid of revolution, and apply the formula to find the centre of gravity of the hemisphere.

3. A heavy string, of uniform density and thickness, is suspended from two given points; find the equation to the curve in which the string hangs when it is in equilibrium.

If the two points of support are nearly in a horizontal line, and the distance between them nearly equal to the length of the string, determine approximately the tension at the lowest point.

4. A system of bodies is under the action of no forces except their weights, mutual pressures, and pressures upon smooth fixed surfaces; show that the system will be in

equilibrium if placed so that the centre of gravity is in the highest or lowest possible position.

Two equal uniform heavy beams are connected by a hinge, and placed over two smooth pegs in the same horizontal line, the hinge being uppermost; determine the position of equilibrium.

5. Prove that the path of a projectile *in vacuo* is a parabola.

What must be the angle of projection in order that the horizontal range may be m times the greatest height?

6. In any machine moving uniformly and without friction, the mechanical work expended by the power which moves the machine is equal to the mechanical work done by the machine. Explain this principle, and apply it to the Screw and to the Hydrostatic Press.

7. A particle is constrained to move on a smooth vertical curve; find the velocity of the particle and the pressure on the curve.

8. Explain D'Alembert's principle.

A heavy body is capable of motion round a horizontal axis; find the time of a small oscillation.

Ex. Suppose the body to be a cube, and the axis coincident with an edge of the cube.

9. Show how to determine the centre of pressure of a plane surface immersed in a fluid.

Ex. A triangle with one side in the surface.

10. Investigate an approximate formula for finding the height of a mountain by means of a barometer.

11. Explain what is meant, in fluid motion, by the hypothesis of '*Steady Motion*,' and by the hypothesis of '*Parallel Sections*.'

12. What is meant by the '*Vena Contracta*;' and to what extent does it modify theoretical results deduced without regard to it?

LIX.

1. Assuming the theorem of the parallelogram of forces for two equal forces, prove it for two unequal forces.

2. A uniform beam is supported by a cord, which is fastened to both ends of the beam, and passes over a small fixed pulley-wheel: show that the beam can remain at rest only in either a horizontal or a vertical position. Examine whether in the former position the equilibrium is stable or unstable: and supposing the cord to be twice the length of the beam, compare the tension of the cord, and the horizontal strain on the beam, with the weight of the beam.

3. How would you ascertain, without actual measurement, whether the arms of a balance are equal or unequal; and if they are unequal, in what ratio they are unequal?

4. Two inclined planes, of which one is smooth and the other has $\frac{1}{2}$ for its coefficient of friction, have a common altitude and are placed back to back. If the smooth plane is inclined at an angle of 30° to the horizon, determine the inclination of the other plane, so that two equal weights placed upon the inclined planes, and connected by a cord passing over a pulley-wheel at the top, may just remain at rest.

5. Determine the centre of gravity of a body.

Ex. Find the centre of gravity of the plane figure bounded by two circles of given radii, one of which touches the other internally.

6. Define '*Velocity*,' '*Angular Velocity*,' '*Momentum*,' '*Vis Viva*,' '*Mechanical Work*.' What are the units in terms of which each of these quantities is measured?

7. 'The accelerating force of gravity at the sea level is 32·2 nearly.' Explain this statement, and show how it is proved to be true.

8. If a heavy body projected in any direction *in vacuo* crosses twice the same horizontal plane, it crosses it each time with the same velocity, and in directions equally inclined to the plane.

9. Two weights, of 80 lbs. each, are suspended by a cord 80 ft. long, which passes over a fixed pulley, and to the extremities of which they are attached. If, when the weights are at rest in the same horizontal plane, a weight of 1 lb. is added to one of them, in what time and with what velocity will the other reach the plane?

10. A, B, C, are equal spheres whose common elasticity is $\frac{1}{2}$, and whose centres are in a straight line. B and C are contiguous and at rest: A, moving with a velocity of 1 ft. a second, impinges upon B: what will be the positions and velocities of the spheres one second after impact?

11. Show that the direction of gravity is everywhere perpendicular to the surface of a fluid at rest.

12. A cubical vessel is filled with fluid: compare the whole pressure on the inner surface of the vessel with the weight of the fluid.

LX.

1. Define '*Force*' generally with respect to both Statics and Dynamics; and explain the necessity, practically, of expressing specifically the measure of forces with reference to the nature of the problems, statical or dynamical, into which they may enter.

2. Define the properties of bodies designated as '*Solid*,' '*Viscous*,' or '*Fluid*,' with reference to the mutual mechanical action of the component particles on each other, when acted on by extraneous forces. What is the distinction, mechanically considered, between a flexible and an elastic body? How is the elasticity measured (1) when acting statically, and (2) when acting dynamically?

3. Define a '*Couple*,' and point out the necessity of introducing the idea of a couple, or some equivalent idea, into the investigation of many mechanical problems. If 1 lb. be assumed as the unit of statical force, and 1 ft. as that of *length*, what will be the unit of couple?

4. A body, which cannot roll, is sustained on an inclined plane by a string: find the direction of the string, in which the least tension will support the body, there being friction between the body and the plane, of which the coefficient is μ, and the inclination of the plane to the horizon being a.

5. Two equal uniform beams are placed with their lower ends on two equal piers, and their upper extremities lean against each other like two rafters: if a heavy weight W be placed symmetrically on their upper ends, what will be the horizontal stress on each pier?

6. A chain of length $2b$ and weight W is suspended from two fixed points in a horizontal plane, and distant $2a$ from each other: find the tension at any point.

7. If the chain in *Qu.* 6, instead of being fastened to two fixed points of support, pass freely over small pulleys in the same positions, the two ends thus hanging vertically, and supporting, by their weight, the middle portion of the chain, what must be the length of the vertical portions of the chain, not to disturb the equilibrium of the *catenary* between the pulleys? What would be the consequence of having these vertical parts of the chain too long or too short?

8. Two inclined planes are so placed that the upper boundary of one coincides with the lower boundary of the other along a horizontal line; the inclination of the higher plane is a, and greater than β, that of the lower one; also, the upper plane is smooth, whilst there is friction, of which the coefficient is μ, on the lower one. A body *slides* from rest at a point, on the upper plane, distant b from the common line of junction of the planes, by the action of gravity: find the velocity of the body at any point on the lower plane, assuming it, on account of its form, to be incapable of *rolling* down the lower plane, and omitting any possible *impulsive* effect of friction at the instant when the descending body first comes in contact with the lower plane.

9. A U-shaped tube of uniform bore, of which the curved part is a semicircle 12 inches in length, is placed in an upright position, and contains as much mercury as fills 15 in. of its length, and as much water above the mercury on one side as fills 41 in. more. If the difference of level between the two mercurial surfaces is 3 in. nearly, what is the *s.g.* of mercury, and what is approximately the height of a column of water, which will produce a pressure equal to that at the bottom of the tube?

10. Prove that, when a body floats in a fluid, the weight of the fluid it displaces is equal to the weight of the body.

11. Show that the fluid pressure on any surface immersed in a uniform fluid at rest under the action of gravity, will be equal to that on a plane surface of equal area immersed in the same fluid, in a horizontal position, and at the same depth as the centre of gravity of the proposed surface.

12. A hollow cube, with four of its sides vertical, is filled with water. If one of the vertical sides were to become entirely detached from the base and the two adjoining sides, find the magnitude, direction, and point of application of the single force which would preserve the unattached side in its position.

APPENDIX.

NAVAL CADETSHIP PAPERS.

I. (*a.*)

1. The moon's distance from the earth is 240,000 miles, and the sun's is 95,000,000 miles; how much farther is the sun from the earth than the moon is?

2. (1) Reduce 540,697 inches to miles, furlongs, &c.
   (2) Multiply 15 lbs. 9 oz. 17 dwts. 18 grs. by 23.

3. If 4 cwt. 1 qr. 26 lbs. of tobacco cost £43 18*s.* 6*d.* what quantity can be purchased for £4 19*s.* 9*d.*?

4. What quantity of silver can be bought for £36, when 14 lbs. 6 oz. 15 dwts. cost £45?

5. A person's income is £598 10*s.*, his weekly expenses are £7 10*s.*, and he gives away £20 annually in charity; how much will he have remaining?

6. When coals cost 11*s.* 6*d.* a ton, what will be the annual expense of fuel for an engine whose daily consumption averages $2\frac{1}{4}$ tons?

7. How many days of 15 hrs. each would 60 men take to perform a piece of work, if 45 men can do it in 30 days of 12 hrs. each?

8. If 1 lb. troy of gold be coined into £46 14*s.* 6*d.* what is the weight of a sovereign?

9. (1) Reduce $\frac{6409}{7395}$ to its lowest terms.
(2) From $10\frac{2}{21}$ take $7\frac{5}{14}$.
(3) Multiply together $\frac{5}{6}$, $\frac{9}{10}$ and $2\frac{2}{3}$.

10. (1) Multiply 7·51 by 0016.
(2) Divide 201·4095 by ·703.
(3) Reduce $100\frac{4}{125}$ to a decimal.

11. How many square ft. of glass are required to glaze 5 windows, each containing 14 panes of glass, each measuring 18 in. by 6 in.?

12. A man mixes 20 gal. of ale which cost him 16*d.* a gallon, with 65 gal. of beer which cost him 9*d.* a gallon; at what price must he sell the mixture to gain 20 per cent.?

II. (*a.*)

1. Express in figures five hundred and nine millions six thousand and one.

2. Divide 7,890,567 by 42, first, by long division, and secondly, by its factors 6 and 7.

3. If a land mile consists of 1,760 yds., and a knot, or sea mile, of 6,080 feet, how many knots correspond to 1,400 land miles?

4. (1) Divide 121,260 c. yds. 22 c. ft. 392 c. in. by 584.
(2) Find the value of 17 pieces of cloth each 41 yds. 1 qr. long, at £1 8*s.* $0\frac{1}{2}$*d.* for 2 yds. 3 qrs.

5. A coach-wheel 7 ft. 8 in. in circumference revolves 3,762 times in performing a certain journey; what would be the circumference of a wheel which made 3,496 revolutions in the same distance?

6. Standard gold being worth £3 17*s.* $10\frac{1}{2}$*d.* per oz. troy, what will be the price of 2 lbs. 9 oz. 6 dwts. 16 grs.?

7. If 11 men can reap a field of corn of $7\frac{1}{2}$ ac. in 3 days of 12 hrs. each, how long will it take 8 men to reap 9 ac. working 16 hrs. a day?

8. A can perform a piece of work in 4 days, which B can

perform in 5 days; in what time can they perform it together ?

9. (1) From $\frac{11}{14} \times \frac{9}{112}$ take $\frac{1}{56}$.

(2) Divide $5\frac{1}{3}$ of $4\frac{1}{8}$ by $7\frac{1}{3}$.

(3) Reduce 14*s*. $10\frac{1}{2}$*d*. to the fraction of a pound.

10. (1) Multiply ·1 by ·1, and divide the product by ·00625.

(2) Reduce $\frac{321}{625}$ to a decimal.

(3) Find the sum of ·125 of 30*s*. and ·375 of 13*s*. 4*d*.

11. How much water must be added to a cask containing 80 gal. of spirits at 14*s*. a gal. to reduce the price to 10*s*. 6*d*. a gallon ?

12. If by selling a chronometer for £40, I lose 20 per cent., what ought I to sell it for to gain 10 per cent. ?

13. Add together $\dfrac{15\frac{2}{3}}{7\frac{5}{6}}$ of £2, $\frac{4}{15}$ of £140 10*s*. 6*d*. and $\frac{3}{5}$ of 4*s*. 2*d*.

III. (*a*.)

1. What number divided by 48,762 will give 6,285 for a quotient and 26,108 for a remainder ?

2. How many dollars are there in £289 17*s*. at 4*s*. 3*d*. per dollar ?

3. A stationer buys 36 reams of paper at 16*s*. 6*d*. per ream, and 84 reams at 17*s*. 9*d*. per ream; find his entire outlay.

4. If 2 cwt. 1 lb. cost £116 19*s*. $0\frac{3}{4}$*d*. what is the cost of 1 lb.?

5. An equal number of men, women, and boys, earned £8 5*s*. in 6 days; each man earned 1*s*. 4*d*., each woman 10*d*., and each boy 4*d*., a day; how many were there of each ?

6. If the duty on hops at $1\frac{1}{2}$*d*. per lb. amounts to £26,357 9*s*. $10\frac{1}{2}$*d*., on what quantity is it paid ?

7. (1) What will 1 ton 11 cwt. 2 qrs. 14 lbs. cost at £1 13*s*. 7*d*. per cwt. ?

(2) Extract the square root of 22,071,204.

8. In what time will 25 men do a piece of work, which 12 men can do in 15 days ?

9. (1) From $\frac{2}{5}+\frac{5}{6}$ take $\frac{3}{8}+\frac{7}{12}$.

 (2) Find the value of $\frac{3}{7}$ of $1\frac{2}{5}$ of $12\frac{1}{2}$ of $\frac{3}{20}$.

 (3) Reduce $\frac{4}{7}$ lb. to the fraction of a cwt.

10. (1) Divide ·0068 by 340, and 314 by ·0005.

 (2) Extract the square root of 3·026.

 (3) Reduce 16 cwt. 1 qr. 21 lbs. to the decimal of a ton.

11. I bought 40 oxen for £500, and sold 12 at a loss of 3 per cent. ; how must I sell the rest per head so as to be free from loss on the whole?

12. Five hundred boys are distributed in 3 houses; the smallest house contains $\frac{7}{25}$ of the whole number, and the largest contains $\frac{19}{14}$ of the smallest: what is the number in each ?

13. Standard gold is formed by mixing 11 parts of fine gold with one part of copper; how many sovereigns will be coined from 4 oz. avoirdupois of fine gold, when 1 lb. troy of standard gold is coined into $46\frac{29}{40}$ sovereigns? (1 *lb. av.*=7,000 *grs. troy.*)

IV. (*a.*)

1. What number divided by 9,612 gives for a quotient 58,647 ?

2. A gold watch and chain cost £20, the watch being £11 10*s.* dearer than the chain ; find the price of each?

3. (1) Required the cost of 25 cwt. of tobacco at £7 15*s.* 8*d.* per cwt.

 (2) What is the weight, in lbs., of ten anchors, which weigh altogether 46 tons 1 qr. 17 lbs. ?

4. If A has £3 3*s.* $3\frac{3}{4}$*d.*, and B has £2 0*s.* $10\frac{1}{4}$*d.* ; how many farthings have they between them, and how much money must A give to B that the latter may have 2,000 farthings ?

5. A farmer buys 14 sheep at £1 11*s.* 4*d.* each: three of them having died, at what price must he sell the rest to gain 38*s.* by the bargain?

6. What quantity of silver may be bought for £72, if the cost of 14 lbs. 6 oz. 15 dwts. be £45?

7. Find the cost of 2 qrs. 24 lbs. at the rate of 20 guas. a ton.

8. If the carriage of 6 cwt. 2 qrs. for 124 miles cost £3 4*s.* 8*d.*, what weight will be carried 93 miles for £3 0*s.* $7\frac{1}{2}$*d.*?

9. (1) Add together $\frac{3}{7}$, $\frac{5}{21}$ and $\frac{17}{24}$.
   (2) Divide $5\frac{5}{14}$ by $12\frac{1}{2}$.
   (3) What is the value of $2\frac{2}{7}$ of £6 3*s.* 4*d.*?

10. (1) Add together 14·1254, 11·5, 16·0004 and 17·112.
   (2) Multiply ·0625 by 1·74.
   (3) Reduce $1\frac{1}{28}$ to a decimal.

11. Having sold 12 yards of cloth for £5 14*s.*, I thereby gained 14 per cent.; what was the prime cost of a yard?

12. What number is that to which if $\frac{2}{7}$ of $\frac{5}{9}$ be added, the sum will be 1?

13. How many 3-inch squares can be cut out of a 12-inch square?

V. (*a.*)

1. From one hundred tons subtract 10 cwt. 3 qrs. 16 lbs. 10 oz.

2. What is the cost of 8,630 yards of cloth at £1 9*s.* 3*d.* per yard?

3. Divide 355 tons 13 cwt. 2 qrs. 13 lbs. 12 oz. by 132.

4. A person bought 11 cwt. 1 qr. of sugar for £48 19*s.* $0\frac{1}{2}$*d.*, and sold it at 10*d.* per lb.; did he gain or lose, and how much?

5. What is the pay of 548 labourers for 15 weeks at 2*s.* $9\frac{1}{2}$*d.* per day?

6. A bankrupt pays 14*s.* $7\frac{1}{2}$*d.* in the £ off a debt of £9,655; how much do the creditors lose?

7. If 4 oz. 15 dwts. of silver plate cost £1 11*s.* 6*d.*, what will five articles, each 3 oz. 12 dwts., cost?

8. If 5 yds. of linen cost 11*s.* 2*d.*, what must be given for 9 pieces, each containing 21 yds. 1 qr.?

9. (1) Extract the square root of 172,265,625.
(2) From $1\frac{1}{15}$ take $\frac{11}{30}$.
(3) Reduce $\frac{41123}{351386}$ to its lowest terms.
(4) Multiply $10\frac{1}{8}$ by $5\frac{1}{9}$.

10. (1) From 7·00001 take ·700001.
(2) Multiply ·049 by 3·417.
(3) Divide 1138·47 by 125,000.

11. A gardener has a piece of matting 73 yds. 1 ft. 8 in. long, and 3 ft. 9 in. wide, to cover a wall 94 ft. long and 10 ft. high; how many square feet of wall will remain uncovered?

12. Express as a vulgar fraction $7{\cdot}\dot{2}8371\dot{4}$.

13. Find the least fraction which, when added to the sum of $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{5}{12}$, will make the result a whole number.

VI. (*a.*)

1. Find the difference between 17 tons 14 cwt. 1 qr. 17 lbs. and 19 tons 12 cwt. 3 qrs., and multiply the result by 175.

2. Find the G.C.M. of 742, 795, and 901.

3. Convert £426 17*s.* 9*d.* into dollars, at 4*s.* $7\frac{1}{2}$*d.* per dollar.

4. Divide £814 among 3 persons, in the ratios $\frac{2}{5} : \frac{3}{7} : \frac{5}{8}$.

5. How many yards of cloth, at 9*s.* per yard, must be given in exchange for 3 cwt. 3 qrs. 12 lbs. of sugar at 7*d.* per pound?

6. If the expenses of a family of 10 persons for 3 weeks be £22 1*s.*, what will be the expenses of a family of 15 persons for 2 weeks 1 day?

7. Find the simple interest on £312 6*s.* 8*d.* for 1 year 120 days, at $4\frac{1}{2}$ per cent. per annum.

8. A person spending annually £240, saves £2 15*s.* of it

quarterly by ready payment: what is the rate of discount? If he by this means make an increase of $20\frac{5}{8}$ per cent. upon his annual saving, what is his annual income?

9. (1) Add together $\frac{2}{3}$, $\frac{1}{7}$ of $3\frac{1}{3}$, and $\dfrac{5\frac{3}{7}}{6\frac{2}{3}}$.

(2) Find the difference between $\frac{2}{3}$ of 12*s.* 6*d.* and $\frac{3}{8}$ of 15*s.* 4*d.*; and reduce the result to the decimal of 4*s.* 6*d.*

10. If $\frac{3}{5}$ of a ship cost £5,427, what will be the cost of $\frac{1}{6}$ of her?

11. Extract the square root of $3\frac{1}{16}$.

12. (1) Reduce ·0225 and $1{\cdot}03\dot{4}\dot{7}$ to vulgar fractions.

(2) Divide 17·28 by ·0144.

13. Find the sum of 2·024 of 3*s.* 8*d.*, ·128 of 12*s.* 9*d.*, and $1{\cdot}02\dot{3}$ of £1·28. (*Assistant Clerks'.*)

SANDHURST AND LINE PAPERS.

VII. (*a.*)

1. (1) Multiply £1,425 13*s.* $4\frac{1}{2}$*d.* by 29: after the multiplication has been performed, find the number of halfpence in the product.

(2) Find the price of 28 tons 11 cwt. at £3 17*s.* 6*d.* per ton.

2. If 3 steps of a soldier measure $2\frac{1}{4}$ yds., how many steps will he take in 3 miles?

3. If the pay and provision of 750 soldiers for 12 days cost £900, how many days will £3,000 suffice for 2,000 soldiers at the same rate?

4. (1) Reduce $\frac{1253}{6802}$ to its lowest terms.

(2) Add $\frac{7}{10}$, $\frac{9}{11}$, $\frac{13}{16}$, $\frac{589}{880}$, and reduce the result to its simplest form.

(3) Express $\frac{4{\cdot}173}{125}$ in the form of a decimal.

(4) Reduce 7*s.* $9\frac{3}{4}$*d.* to the decimal of £1.

5. If £562 10*s*. be the simple interest on £5,000 for 3 years, what is the rate per cent.?

6. Find $(23{\cdot}04)^2$ and $\sqrt{144{\cdot}24250201}$.

7. Find the value of $a^2-2ab+b^2$, when $a=2$, $b=\frac{1}{2}$; and of $(x-a)(x-b)(x-c)$ when $a=1$, $b=2$, $c=3$.

8. Perform the operations indicated in the following examples:—

(1) $\left(1+\frac{x}{2}-2x^2+\frac{x^3}{3}\right)-\left(1-\frac{x}{3}-2x^2+\frac{x^3}{2}\right)$.

(2) $(x^2+y^2+z^2-xy-xz-yz)\times(x+y+z)$.

(3) $(4x^6-9x^2-6x-1)\div(2x^3+3x+1)$.

9. Find the G.C.M. of $x^2-(a+b)x+ab$, and $x^2+(a-b)x-ab$; and prove that $1+x+x^2+\frac{x^3}{1-x}=\frac{1}{1-x}$.

10. Solve the equations:

(1) $(x-2)(x-3)=(x+4)(x-6)$.

(2) $\frac{2x-5}{3}-\frac{3x-2}{5}=x-5$.

(3) $\frac{x}{7}-\frac{y}{8}=17$, and $\frac{x}{8}+\frac{y}{7}=29$.

11. A farmer has 1,400 acres; for every 10 sheep that he keeps, he requires to plough an acre of land for turnips, and for every 4 sheep he requires an acre of land for pasture; how many sheep can he keep altogether?

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12. (1) Given a and l, the first and last terms of an arithmetical series of n terms; find the sum of the series.

(2) The sum of 3 nos. in A.P. is 21, and the sum of their squares 165; find them.

13. (1) If $\frac{2}{3}=\log_{10}x$, find the value of x to two places of decimals, without the aid of tables.

(2) Find by the tables a fourth proportional to $(41{\cdot}06)^{\frac{1}{3}}$, $({\cdot}00056)^2$, and $(120)^{\frac{2}{3}}$.

14. Find the whole surface and the content of a square *pyramid*: side of base, 10 ft.; slant edge, 20 ft.

VIII. (*a*.)

1. (1) Divide 425 tons 2 qrs. 21 lbs. 8 oz. by 90.

(2) If the annual cost of a cavalry-soldier be £52 11*s*. $3\frac{1}{2}$*d*., and that of an infantry-soldier £26 3*s*. $5\frac{1}{2}$*d*., what is the difference of the cost per annum of a cavalry-regiment of 400 men, and of an infantry-regiment of 800 men?

2. If 8 fires, kept burning 9 hrs. a day for 28 days, consume $3\frac{1}{8}$ tons of coal, how many tons will be consumed by 12 fires that are kept burning 7 hrs. a day for 15 days?

3. What is the cost of carpeting a room 25 ft. 9 in. long by 22 ft. 6 in. wide, at 5*s*. 4*d*. per square yard?

4. Find the simple interest on £396 13*s*. 4*d*. for 4 years at $3\frac{1}{4}$ per cent.

5. (1) Find the difference between the fractions $\frac{168}{169}$ and $\frac{142}{143}$; and multiply it by $\frac{1}{2}$ of $35\frac{3}{4}$.

(2) Express £1 3*s*. 8*d*. as the fraction and as the decimal of £5 18*s*. 4*d*.

(3) Find the value of ·1253125 of £100.

6. Two merchants form a partnership for conducting a business; one contributes a capital of £3,000, the other a capital of £5,000; at the end of the year the profit on the business is found to be £1,880; how much ought each to receive?

7. (1) Find the value of $(\cdot 05)^3 \times (1\cdot 04)^2$.

(2) Extract the square root of 15 to four decimal places.

8. (1) Find the value of $\left(\frac{a+b}{a-b}\right)^2$, when $a=2$, $b=\frac{3}{2}$, and of x^2-5x+6, $x=2$.

(2) Simplify $1-x-\frac{1-x^2}{x}$.

(3) Multiply $(x^2+ax-b^2)\,(x^2-ax+b^2)$.

(4) Divide $x^5+8x^2y^3+15xy^4-3y^5$ by $x^2+3xy+3y^2$.

(5) From $(x^{\frac{1}{2}}+y^{\frac{1}{2}})^2$ take $(x^{\frac{1}{2}}-y^{\frac{1}{2}})^2$.

9. (1) Find the G.C.M. of $x^2-50x+600$ and $x^2+10x-600$; and

(2) The L.C.M. of x^3-a^3 and $x^3+2ax^2+2a^2x+a^3$.

10. Solve the equations:

(1) $ax-b=1+b$.

(2) $\frac{x-2}{18}+10=x-\frac{x+4}{6}-\frac{2x-5}{7}$.

(3) $\left\{\begin{array}{l} 3x+7y+54=y \\ 6x-7y=13 \end{array}\right\}$

11. Two labourers working together can complete a piece of work in 12 days; working alone, one of them could have completed it in 20 days; in how many days could the other complete it, working alone?

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12. (1) Prove $(a+b-c)(b+c-a)+(b+c-a)(c+a-b)+(c+a-b)(a+b-c)+(a+b+c)^2=4(ab+ac+bc)$.

(2) Solve the equation $9x-4x^2+\sqrt{4x^2-9x+11}=5$.

13. (1) Find, without tables, $\log_{10}\sqrt{1000}$.

(2) Find, by the tables, $(32{\cdot}156)^{\frac{2}{7}}$;

(3) The amount of £1,365 10*s.* in 8 years, at $3\frac{1}{2}$ per cent. compound interest, due annually.

IX. (*a.*)

1. (1) Find the number of pounds and shillings in eighteen million and sixty thousand farthings.

(2) Divide £78 2*s.* 11*d.* by 62.

(3) Find the weight of 54 spoons, each weighing 5 oz. 5 dwt. 16 gr.

2. A case of liquid contains 12 gal. 3 qts. 1 pt.; what is its value at 5*s.* 6*d.* a gallon?

3. If the carriage of 3 cwt. for 150 miles is 4 guineas, what is the weight of a package which is carried 100 miles for £3 13*s.* 6*d.* at the same rate?

4. (1) Add together $\frac{3}{32}$ of 1 sq. mi., $\frac{9}{10}$ ac., and $\frac{8}{15}$ ro., giving the result in acres, roods and perches.

(2) Divide 51·731381 by ·00056047.

(3) From £·159375 subtract ·6875*s.*, giving the difference in pence.

5. Find the amount of £827 8*s.* 4*d.* in 4 yrs. at $3\frac{3}{4}$ per cent. per annum simple interest.

6. Extract the square root of ·00497025.

7. Find the value of $x^3-6x(x-2)+18$, when $x=-4$.

8. (1) Multiply $(x-3)^2$ by $x-4$.

(2) Divide $4x^4-12x^3+25x^2-24x+16$ by $2x^2-3x+4$.

(3) Reduce $\dfrac{x^3-9x^2+26x-24}{x^3-5x^2-2x+24}$ to its lowest terms.

9. Solve the equations;

(1) $\dfrac{5}{x-3}=\dfrac{6}{x-2}$.

(2) $(x-3)(x-5)-(x-4)(x-6)=3$.

(3) $\dfrac{3x-4}{2x+3}-\dfrac{2x-3}{3x+4}=\dfrac{5}{6}$.

(4) $\left.\begin{aligned}\frac{x}{4}+\frac{y}{3}&=30\\ \frac{3x}{8}-\frac{y}{10}&=12\end{aligned}\right\}$

10. (1) A sovereign was changed for half-crowns and sixpences; for 20 coins altogether. How many of each kind were there?

(2) Divide the number 42 into two parts, such that $\frac{2}{3}$ of the one part may be the same as $\frac{8}{9}$ of the other.

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11. Two lengths of cloth are bought for £5. One is 2 yds. longer than the other, and each costs as many shillings a yard as it is yards in length. What are their lengths?

12. In an A.P. in which, the common difference is 1, the third term from the beginning is 4, and the third from the end 9. How many terms are there ?

13. Employ logs to find—
   (1) The product of ·5684325 and 893 ;
   (2) The principal which amounts to £1,864 in 7 yrs. at 3 per cent. per annum compound interest.

X. (*a.*)

1. (1) How many times is £1 11*s.* 2*d.* contained in £162 1*s.* 4*d.* ?
   (2) A man steps 2 ft. 3 in.; how many steps will he take in 6 miles ?

2. If 180 men can make a road in 15 days, in what time would 270 men make a road twice as long ?

3. If 7 fires burning 10 hrs. a day consume 4 tons 10 cwt. of coal in 30 days, how much coal will be consumed in 12 days by 20 fires burning 14 hrs. a day ?

4. Find the simple interest on £5,656 5*s.* for 6 yrs. at $4\frac{1}{2}$ per cent. per annum.

5. (1) Add together $\frac{7}{8}$ of $3\frac{1}{3}$, $\frac{5}{7}$ of $1\frac{10}{11}$, $\frac{241}{363}$; divide the result obtained by $\frac{9}{11}$ of $\frac{10}{11}$.
   (2) If $5\frac{1}{2}$ yds. lineal measure = 1 pole, find the number of square yards in an acre.
   (3) What fraction of an acre is a field 55 yds. long by 40 yds. wide ?

6. (1) Multiply 21·56 by ·0035.
   (2) Divide ·25 by 31·25, and verify the result by vulgar fractions.
   (3) Find the values of ·125 of £88 16*s.* and $\cdot\dot{3}$ of 5 guineas.

7. (1) Determine by how much the square of 1·732 differs from 3.
   (2) Find the square root of 71 to three places of decimals.

8. (1) Show that $a-2b+3c=\frac{c^2}{3(b-a)}$ if $a=4$, $b=5$, $c=6$.

(2) Find the value of $(1+2x+3x^2)^2$ when $x=1$.

9. Perform the operations indicated in the following examples:

(1) $\left(a+\frac{2b}{3}\right)-\left(\frac{a}{2}-\frac{4b}{3}\right)+\left(\frac{a}{3}-2b\right)$.

(2) $(a^4-7a^3b+6a^2b^2-5ab^3+4b^4)+(a^2-ab+2b^2)$.

(3) $(a-x)^2(a+x)^2(a^2+x^2)^2$.

(4) $(x^4-81y^4)\div(x-3y)$.

10. (1) Find the G.C.M. of $3x^2-5xy-2y^2$ and $2x^2-3xy-2y^2$.

(2) Prove that $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-1)(x-3)}+\frac{1}{(x-2)(x-3)}=\frac{3}{(x-1)(x-3)}$.

11. Solve the equations:

(1) $\frac{2x-5}{3x-5}=\frac{1}{2}$.

(2) $\frac{3}{5}(x-1)+\frac{2x}{7}-\frac{x-7}{14}=\frac{x-1}{5}+13$.

(3) $3x+5y=154$ and $\frac{x}{5}-\frac{y}{6}=1$.

12. An engineer had a certain number of telegraph posts to extend the wires a certain distance; he found that if he set them 15 yards apart he should have too many by 150, but if he set them 5 yards apart he should have too few by 70; how many posts had he?

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13. (1) Prove that the number of combinations of n things taken 3 and 3 together is $\frac{n(n-1)(n-2)}{1.2.3.}$.

(2) The number of combinations of n things taken 3 at a time : n :: 55 : 3; find n.

14. (1) Given $\log_{10}44 = 1\cdot643453$; write down the logarithms of $44000\cdot$ and of $\cdot000044$.

(2) Given $\log_{10}2 = \cdot301030$; find from the above, without the tables, $\log_{10}11$.

(3) Find by the aid of tables, $(10)^{\frac{3}{5}}$.

(4) A third proportional to $(5\cdot14)^{\frac{1}{3}}$ and $(\cdot0072)^{\frac{1}{3}}$.

15. (1) Find the area of a circle when the radius of the inscribed hexagon is 10 ft.

(2) Compare the areas of two similar triangles, two of their homologous sides being 4 and 12 ft. respectively.

(3) Find $\sin 60°$ and $\tan 45°$.

XI. (*a.*)

1. (1) Find the value of 1*s.* $3\frac{1}{2}$*d.* × 365.

(2) A bag of sugar weighs 70 lbs.; how many of such bags will weigh 20 tons?

2. If 1,000 labourers can construct $1\frac{1}{2}$ mile of railroad in 3 months, what length of railroad would be constructed by 600 labourers in 18 months?

3. The simple interest on £3,000, in 7 years, is £236 5*s.*; what is the rate per cent.?

4. Express an English mile in terms of the French kilomètre, the kilomètre being 1,000 mètres, and 64 mètres =70 yards.

5. (1) Find the G.C.M. of 2,431, 3,861, 4,433.

(2) Reduce to a simple fraction $\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}+\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}-\frac{1}{3}}$.

(3) Express 3*s.* 6*d.* as the fraction, and as the decimal of £5.

6. Find the cost of matting for a floor 3 yds. 4 in. long by 17 ft. 9 in. wide, at 2*s.* 3*d.* a square yard.

7. (1) Find the square of $\cdot 00013$;

(2) The square root of $83\cdot 00849881$.

8. (1) Represent $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$ algebraically when $\frac{1}{2}=x$.

(2) Prove $(a-x)^2=(x-a)^2$.

9. Perform the operations indicated in the following examples:

(1) $\left(1+\frac{a}{2}-\frac{a}{3}+\frac{3a}{4}\right)\times\left(1+\frac{a+b}{2}+\frac{a-b}{2}\right)$.

(2) $(a^4-4a^3x+6a^2x^2-4ax^3+x^4)\times(a^2-2ax+x^2)$.

(3) $(4x^4-9a^2x^2-6a^3x-a^4)\div(2x^2+3ax+a^2)$.

(4) $(4x-6x^{\frac{1}{2}}y^{\frac{1}{2}}+9y)\times(2x^{\frac{1}{2}}+3y^{\frac{1}{2}})$.

10. (1) Reduce to its lowest terms $\frac{2x^3+x^2y-6xy^2-3y^3}{6x^3+3x^2y+2xy^2+y^3}$.

(2) Prove $(a^2+b^2)(x^2+y^2)=(ax-by)^2+(ay+bx)^2$.

11. Solve the equations:

(1) $\frac{x}{6}+\frac{x}{8}=\frac{x+1}{7}+\frac{x}{12}+3$.

(2) $\frac{2x-7}{2x-3}=\frac{x+7}{x+11}$.

(3) $\frac{a+x}{x}+\frac{b-y}{y}=\frac{13}{6}$ and $\frac{2a}{x}+\frac{6b}{y}=3$.

12. A number consisting of two digits when divided by 6 gives a quotient equal to the digit in the unit's place, and the number formed by inverting the digits is less by 6 than the double of the original number: find the digits.

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13. Solve the equations:

(1) $\frac{x+5}{5}-\frac{x}{x+5}=\frac{3}{2}$.

(2) $ab(x-y)=a^2-b^2$; $xy=1$.

14. (1) Find, without tables, the number of which $\cdot 5$ is the logarithm to a base 10.

(2) Find, by the aid of tables, a third proportional to $(51\cdot76)^{\frac{1}{2}}$ and $(\cdot007)^2$;

(3) The value of x^3-2x-5 when $x=2\cdot0946$.

15. The parallel sides of a trapezoid are 27·5 and 38·5 feet, and the other sides are 12·5 and 15·5 feet: find its area.

XII. (a.)

1. (1) Find the number of inches in 800 miles, and write the answer in words.

(2) Divide £41 18*s*. 2*d*. by 107.

(3) Find the cost of 12 tons 8 cwt. 76 lbs. at 7*s*. per cwt.

2. (1) Multiply together $7\frac{3}{5}$, $9\frac{7}{8}$, and $\frac{7}{19}$, and divide the product by $15\frac{4}{5}$.

(2) From $\frac{3}{8}$ of 14*s*. 4*d*. subtract $\frac{1}{24}$ of 1 guinea.

3. (1) Find the cost of 3·40625 quarters of corn at 1*s*. 8*d*. a peck.

(2) Express 161,172 square feet in acres and decimal parts of an acre.

(3) Divide 10356·48 by ·0384.

4. What is the principal on which the simple interest in 8 yrs. at £3 15*s*. per cent. per annum is £760 19*s*.?

5. Extract the square root of 49,984,900.

6. (1) Multiply together $y+z-x$, $x+z-y$, and $x+y-z$.

(2) Divide $48x^3-8x^2+31x-15$ by $5-12x$.

(3) Reduce to its lowest terms

$$\frac{3a^3-10a^2b+9ab^2-2b^3}{3a^3-4a^2b-5ab^2+2b^3}.$$

(4) If $x=-5$, find the value of $x^3+x^2-\dfrac{x-3}{x+7}$.

7. Solve the equations:

(1) $x-\dfrac{8-x}{5}-\dfrac{x-2}{20}=\dfrac{77}{10}$.

(2) $(x+14)(x-6)-(x-12)(x+7)=39$.

(3) $\sqrt{4+x}+\sqrt{x}=3$.

(4) $\dfrac{x-y}{x} = \frac{1}{2}$ and $7(x+y) = 32$.

8. If B give to A 10*s.*, A will have twice as much as B. If A give B 10*s.*, B will have thrice as much as A. What money has each?

9. Two travellers, starting at the same time, to meet one another, and proceeding uniformly, at different rates, meet at a point distant 3 and 4 miles from their respective starting places; if the faster traveller had been detained 20 minutes on his way, they would have met at the half-way between their starting places. What are their rates of travelling?

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10. Divide $8a^{\frac{3}{2}} - 12ab^{-\frac{1}{3}} + 4a^{\frac{1}{2}}b^{-\frac{2}{3}}$ by $2a^{\frac{1}{2}} - b^{-\frac{1}{3}}$.

11. Solve the equations:

(1) $2x + \sqrt{x} = 3$.

(2) $\left.\begin{aligned} y+z &= 2yz \\ x+z &= 3xz \\ x+y &= 4xy \end{aligned}\right\}$

12. A merchant bought, for £40, a number of articles at the same price. Reserving 5 of them, he sold the rest for £36, thus gaining 8*s.* on each. How many articles were there?

13. Employ logarithms:

(1) To find the 5th root of ·0463297;

(2) To find the rate per cent. of compound interest at which £1 will amount to £3 in 18 years.

14. On a base of 10 yards a right-angled triangle is formed with one side two yards longer than the other. Find its area.

15. (1) Find the area of a regular decagon of which each side is 1 yd.

(2) Two sides of a field, which are parallel, are 7 chains 33 links and 16 chains 19 links in length,

and the perpendicular distance between them is 27 chains 8 links. Find the area of the field in acres.

XIII. (*a.*)

1. (1) Find the number of seconds in six hundred and ninety-five days, expressing the answer in words.

(2) Find the cost of $87\frac{3}{4}$ yards of cloth at 8*s.* 6*d.* a yard.

(3) If 13 spoons weigh 2 lbs. 4 oz. 2 dwt. 19 grs., what is the average weight of each?

2. Three horses ate 12 bushels of corn in 16 days. How many horses can be fed, at the same rate, for 24 days on 150 quarters of corn?

3. (1) Find the difference between $\frac{7}{12}$ of a sovereign and $\frac{3}{5}$ of a guinea, giving the result in pence.

(2) Express 1 ton 3 cwt. 16 lbs. as the decimal of 1 ton 10 cwt. 3 qrs. 12 lbs.

(3) Divide 72,184,401 by 88·352.

(4) Find the number of perches and square yards in ·3375 of an acre.

4. Compute the simple interest on £1,730 in 15 months at the rate of $3\frac{1}{2}$ per cent. per annum.

5. Extract the square root of 321489.

6. (1) Multiply $(a+b)x-(a-b)y$ by $(a-b)x-(a+b)y$.

(2) Divide $x^3+(a+b+c)x^2+(bc+ac+ab)x+abc$ by $x^2+ax+bx+ab$.

(3) Reduce $\dfrac{x^3-6x-9}{x^4+3x^3-9x-9}$ to its lowest terms, and give the value of the result when $x=-3$.

7. Solve the equations:

(1) $\dfrac{2x-3}{4}-\dfrac{x-1}{8}=\dfrac{x}{6}$.

(2) $\dfrac{x+a}{x-b}=a-\dfrac{x}{x-b}$.

(3) $4x-3y=3$ and $7x+5y=36$.

8. A contractor begins a work with 21 bricklayers and 8 carpenters, and their weekly wages are £25 1*s*. As the work proceeds he takes off 6 bricklayers and adds 4 carpenters, thus lessening the weekly wages by £1 10*s*. Find the weekly wages of a bricklayer.

9. A gentleman is in the habit of walking from his house, to take a certain train, in 20 minutes. One day he is detained beyond his usual time of starting, and although he quickens his pace to $\frac{10}{7}$ of his accustomed speed, he is too late at the station by 2 minutes. How long was he detained?

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10. Prove that, $(b+c-a)a^{\frac{1}{2}}+(a+c-b)b^{\frac{1}{2}}+(a+b-c)c^{\frac{1}{2}}$ $=(a+b+c)(a^{\frac{1}{2}}+b^{\frac{1}{2}}+c^{\frac{1}{2}})-2(a^{\frac{3}{2}}+b^{\frac{3}{2}}+c^{\frac{3}{2}})$.

11. One of the two digits of a number is double the other, and if 27 be added to the number the digits become inverted. Prove that the number is double the product of its digits.

12. An earthwork has a breadth of 12 ft. at the top and 38 ft. at the bottom, its height being 12 ft. Find the number of cubic yards in a rod of its length.

13. Employ logarithms to find,

(1) $8{\cdot}64 \div {\cdot}03827654$;

(2) The number of years through which £100 must lie at compound interest of 4 per cent. per annum before it exceeds £160, the interest being paid yearly;

(3) To find the sum of 6 terms of the series 2, $\frac{2}{7}$, $\frac{2}{49}$, &c.

14. Find, by the aid of tables, to the nearest second, the angle whose cosine is $\frac{1}{3}$.

15. Find the diameter of a hemispherical cup which will hold half a pint, taking a gallon to be $277{\cdot}3$ cubic inches.

XIV. (*a*.)

1. (1) How many times is £17 14*s*. 5*d*. contained in £655 13*s*. 5*d*.?

   (2) Find the price of a mixture of 1 cwt. of black tea at 3*s*. 2*d*. a pound, with 20 lbs of green tea at 5*s*. 3*d*. a pound.

2. If 20 lamp-posts are required to light a road 1,600 yards long, how many will be required for a road 4 miles long?

3. Find the cost of carpeting a room 34 ft. 6 in. long, by 18 ft. 4 in. wide, at 3*s*. 9*d*. a square yard.

4. Find the simple interest on £9,062 10*s*. for six years, at $3\frac{3}{4}$ per cent. per annum.

5. (1) Subtract $2\frac{13}{216}$ from $3\frac{1}{4}$.

   (2) Divide $\frac{23}{535}$ by $\frac{77}{107}$, expressing the result in its lowest terms.

   (3) Multiply 24·35 by ·074.

   (4) Divide 1·8019 by 243·5.

6. (1) What fraction of £1 2*s*. 6*d*. is $\frac{4}{3}$ of 2*s*. 6*d*.?

   (2) Find the value of ·075 of £3 5*s*.

7. (1) Find $(\cdot 03)^3$.

   (2) Extract the square root of 484·176016.

8. (1) Find the value of $1+2x^2+3x^3$, when $x=2$.

   (2) If $x=7$ show that $\dfrac{6}{7(x-1)}=\dfrac{2}{x+7}$.

9. Reduce to their simplest forms :—

   (1) $7a+5b-c-(4a-3b-2c)$;

   (2) $\dfrac{1}{a+b}+\dfrac{1}{a-b}$; (3) $\dfrac{(x^2-4)(x+3)}{(x^2-9)(x+2)}$.

10. (1) Multiply $a^3-3a^2b+3ab^2-b^3$ by $a^2-2ab+b^2$.

    (2) Divide $x^4-4x^2y^2+12xy^3-9y^4$ by $x^2-2xy+3y^2$.

    (3) Find the L.C.M. of x^2-y^2, and x^3-y^3.

11. Solve the equations:

(1) $\frac{1}{x}-\frac{1}{3}=\frac{1}{6}$.

(2) $\frac{2x-1}{3}-\frac{3x-2}{4}=\frac{5x-4}{6}-\frac{30+x}{12}$.

(3) $\left.\begin{array}{l}10x-y=40\\ 2x+3y=40\end{array}\right\}$

12. Two farmers have each a flock of sheep containing the same number; one farmer loses 390 sheep by a disease, and the other 930; it is then found that one farmer has just twice as many sheep remaining as the other: how many sheep had each at first?

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13. (1) Find x and y from the equations $\left.\begin{array}{l}ax^2+by^2=c\\ a_1x^2+b_1y^2=c_1\end{array}\right\}$ and explain the result if $ab_1-a_1b=0$.

(2) Solve the equation: $\frac{x+2}{x-1}+\frac{x-1}{x+2}=\frac{65}{28}$.

14. (1) Find, without the tables, to two decimal places, the number of which ·25 is the logarithm to the base 10.

(2) Find, by the aid of tables, $(\cdot 00645)^{\frac{2}{3}}$;

(3) The value of x to two decimal places when $10^{\frac{1}{x}}=2\cdot 45$.

15. Find the area of an isosceles triangle, having each of the angles at the base double of the third angle, one of the equal sides being 12 feet.

XV. (*a.*)

1. How many times is £2 15*s.* 6*d.* contained in £152 12*s.* 6*d.*?

2. A commissariat officer bought 27 bullocks at £13 14*s.* each, and 140 sheep at 36*s.* each. This supply served 4,146 men with meat rations for 9 days. What was the daily cost of the meat rations for each man?

3. If 144 lbs. avoirdupois be equivalent to 175 lbs. troy; express, in troy weight, 7 cwt. 2 qrs. 24 lbs. avoirdupois.

4. Find the cost of carpeting a room 26 ft. 8 in. long by 13 ft. 6 in. wide, with carpet at 4*s.* 6*d.* a square yard.

5. Find the simple interest on £9,687 10*s.* for 6 yrs. at $4\frac{3}{4}$ per cent. per annum.

6. (1) Explain why the values of fractions are not changed by reducing them to a common denominator.
   (2) Give the sum of the fractions: $\frac{7}{10}$, $\frac{11}{12}$, $\frac{13}{15}$, $\frac{17}{20}$, $\frac{19}{24}$ in its lowest terms.
   (3) Add together $\frac{3}{5}$ of 15*s.* 10*d.*, ·75 of £4 16*s.* 8*d.*, and $\dot{6}$ of £3 12*s.*

7. (1) Show from the notation of decimal fractions that ·25 is ten times as great as ·025.
   (2) Multiply 2·345 by ·0024.
   (3) Divide 56·28 by ·2345.
   (4) Divide $(\cdot 01)^3$ by $(\cdot 05)^2$: extract the square root of 100·020001.

8. (1) Find the value of $(x-3)(x-2)+(x-3)(x-1)+(x-2)(x-1)$, when $x=3$.
   (2) Find the difference between $\dfrac{a+b}{2}$ and $\sqrt{ab}$ when $a=4$, $b=9$.

9. Reduce the following expressions to their simplest forms:
   (1) $\dfrac{2a+3b-c}{2}-\dfrac{3a+2b-4c}{3}$.
   (2) $c^2(a+b)(a-b)+b^2(c+a)(c-a)+a^2(b+c)(b-c)$.
   (3) $\dfrac{(6x^2-5xy-6y^2)(20x^2+9xy-20y^2)}{10x^2-23xy+12y^2}$.
   (4) $\dfrac{(a^{\frac{1}{3}}+b^{\frac{1}{3}})^2+(a^{\frac{1}{3}}-b^{\frac{1}{3}})^2}{a^{\frac{2}{3}}+b^{\frac{2}{3}}}$.

10. Find the L.C.M. x^2-5x+6, $2x^2-3x-2$, and $2x^2-7x+3$.

11. Solve the equations:

(1) $\frac{x}{a}+\frac{x}{b}-\frac{x}{c}=\frac{ab+bc-ac}{abc}$.

(2) $\frac{x}{6}+\frac{3x-6}{10}-\frac{x-7}{7}=\frac{3x}{14}+\frac{x-2}{5}-3$.

(3) $\frac{3x+y}{\cdot 5}-\frac{5y-3x}{4}=\frac{39}{4}$, $2(x+y)=5y$.

12. A sum of prize-money has to be divided between a serjeant and a corporal, so that the serjeant's share shall be to that of the corporal as 5 to 3; and the serjeant's share is to exceed $\frac{5}{9}$ of the whole sum by £50. What is the share of each?

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13. (1) The number 4 is the logarithm of 625 to a certain base; find that base, without the tables.

(2) Find, by the aid of the tables, $(\cdot 03457)^{\frac{3}{5}}$.

(3) The value of x for which $(\frac{5}{3})^x=17\cdot 4$.

14. Distinguish between *Permutations* and *Combinations*, and find an expression for the number of permutations of n things taken 3 at a time.

The number of permutations of n things taken 5 together is 20 times the number taken 3 together; find n.

15. If 30 cubic inches of gunpowder weigh 1 lb., what is the radius of the base of a cone whose altitude is 6 feet and which contains 350 lbs. of powder?

XVI. (*a*.)

1. (1) Find the number of inches in two hundred thousand yards, and write the answer in words.

(2) If 53 articles cost £10 7*s*. 7*d*. what is the price of each?

(3) Find the value of 17 qrs. 3 bu. 1 gal. of corn at 1*s*. 4*d*. the peck.

2. In a piece of plate weighing 3 lbs. 9 oz. 7 grs. there is alloy weighing 10 oz. 8 dwts. 19 grs. What is the weight of the silver?

3. If the carriage of 8 cwt. for 120 miles be 24*s*., what weight can be carried 32 miles, at the same rate, for 18*s*.?

4. (1) Reduce $\frac{1}{1001}$ to a decimal.
   (2) Find the value of ·3625 of £4.
   (3) Express 1 ac. 3 ro. 26 po. as the decimal of a square mile.
   (4) Extract the square root of 9042049.

5. Find the simple interest on £830 for 15 months at $3\frac{1}{4}$ per cent. per annum.

6. (1) Multiply $a^9-a^2x^3+ax^6-x^9$ by $a+x^3$.
   (2) Add together $(ax^2-by^2)^2$ and $(ay^2+bx^2)^2$, and divide the result by a^2+b^2.
   (3) Prove $(a^2+2a-3)(a+2)=(a^2+5a+6)(a-1)$.
   (4) Prove $\frac{1}{z}\left\{\frac{x}{x+z}-\frac{y}{y+z}\right\}=\frac{1}{y+z}-\frac{1}{x+z}$.

7. Solve the equations:
   (1) $\frac{3}{5x}+\frac{5}{3x}=68$.
   (2) $\frac{x-1}{2}-\frac{x-3}{5}=\frac{2x-3}{3}$.
   (3) $$\left.\begin{array}{l}\frac{x}{2}-\frac{y}{2}+\frac{z}{3}=4\\ \frac{x}{5}+\frac{y}{3}-\frac{z}{5}=3\\ x+y-z=7\end{array}\right\}$$
   (4) $(x^2+x-1)^2-(x^2+x-2)^2=2x^2$.

8. (1) Divide £7 15*s*. into three portions, such that the second may be three times the first, and the third double the second.
   (2) Two men start together from the same point and run round a ring with different uniform speed. If they run in opposite directions they meet at 36 yards from the starting-point. If they run in the same direction, they are at the starting-

point together after one has been seven times round, and the other six times. Find the length of the ring.

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9. Solve the equations:

(1) $6x^2+x=15$, (2) $\left\{\begin{array}{l}\sqrt{x+y}+\sqrt{x-y}=3x \\ \sqrt{x+y}-\sqrt{x-y}=3y\end{array}\right\}$

10. The hind and front wheels of a carriage have circumferences 14 and 16 ft. respectively. How far has the carriage advanced when the smaller wheel has made 51 revolutions more than the larger one ?

11. Find the sum of 13 terms of the progression 12, 10, 8, &c.

12. Employ logarithms:

(1) To multiply $\sqrt[4]{\cdot 086}$ by 39·86427 ;

(2) To find to 8 places the decimal corresponding to the fraction $\frac{1}{21979}$.

13. (1) The diagonal of a square is 3362 feet. Find the length of a side of the square.

(2) The sides of an equilateral triangle are each 17 chains 4 links. Find the area in acres.

14. Two angles of a triangle are 47° 18′ 39″ and 98° 7′, and the side between them is 864 ft. Find the longest side of the triangle.

15. Find the content of a circular right cone, when the diameter of the base is 174 in., and the length of the slope from the vertex to the base is 145 inches.

XVII. (*a.*)

1. (1) Multiply £15 17*s.* 6½*d.* by 271.

(2) In the coinage of India 12 pice make 1 anna, and 16 annas make 1 rupee; if the value of a rupee be 2*s.*, how many pice make a farthing ?

(3) A tradesman consumes in a certain time 63,500 cubic feet of gas, at a cost of £11 18*s.* $1\frac{1}{2}$*d.*; what is the charge for 1,000 cu. ft. of gas?

2. If a quantity of flour serve 7,500 men for 12 weeks at the rate of 20 oz. a day for each man, how many ounces a day will there be for each man when the same quantity of flour lasts 11,250 men for 20 weeks?

3. The area of a rectangular field contains 474 sq. yds., its length is 25 yds. 2 ft.; find its width.

4. A banker allows interest on deposits at the rate of $5\frac{1}{3}$ per cent. per annum; what interest should be received on a deposit of £3,650 for 90 days, at this rate?

5. (1) Reduce to a simple fraction in its lowest terms:

$$\frac{\frac{2}{3}+\frac{1}{2}}{\frac{2}{3}-\frac{1}{2}}+\frac{\frac{2}{3}-\frac{1}{2}}{\frac{2}{3}+\frac{1}{2}}-\frac{8}{9}.$$

(2) Reduce to a finite decimal $\frac{113}{125}$.

(3) Multiply £333 6*s.* 8*d.* by ·015.

(4) Find the value of $(\cdot 0012)^2 \div (\cdot 02)^3$.

(5) Extract the square root of 19895·1025.

6. (1) Express by a single number $2+2^2+2^3$.

(2) Find the value of $(1+\frac{x}{2}+\frac{x}{3})^2$ when $x=2$.

7. Perform the operations indicated in the following examples:

(1) $a+2b-3c-\{a-2b-(a+b-c)\}$.

(2) $\frac{x+y}{2}-\frac{x-y}{3}-\frac{x-5y}{6}$.

(3) $(x^3-4x^2y+3xy^2-4y^3)\times(x^2+4xy-3y^2)$.

(4) $(a^5-16a^3b^2+20a^2b^3-25ab^4+12b^5)\div(a^2+4ab-3b^2)$.

8. Reduce to their simplest forms:

(1) $\frac{(x^2-y^2)(x^3+y^3)}{(x^2-xy+y^2)(x-y)}$.

(2) $\frac{12x^4+x^2y^2-6y^4}{6x^3+9x^2y-4xy^2-6y^3}$.

9. Solve the equations :

(1) $(x+4)^2-(x-4)^2=120+x$.

(2) $\frac{4x+3}{9}-\frac{7x-29}{5x-12}=\frac{8x+19}{18}$.

(3) $\frac{x}{2}+\frac{y}{3}-\frac{5x}{6}=2; \frac{2x}{9}-\frac{y}{8}=1$.

10. Two regiments went into action, of the same strength; 50 men of one regiment were killed, and 450 of the other, and it was found after the action that the number of men remaining of one regiment was twice as great as the number remaining of the other. Of how many men did each regiment consist ?

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11. (1) Insert 5 arithmetic means between $a-b$ and $a+b$.

(2) Find the sum of 12 terms of the series 7, 12, 17, &c.

(3) Four numbers are in geometrical progression: the sum of the first and second numbers is 15, and the sum of the third and fourth is 60; find the numbers.

12. (1) Find, without tables, to three decimal places the number of which ·5 is the logarithm to the base 10.

(2) Find, by the aid of tables, $(22{\cdot}075)^{\frac{3}{5}}$.

(3) $(1{\cdot}56)^{1{\cdot}5}\times(1{\cdot}56)^2\times(1{\cdot}56)^{2{\cdot}5}$. . . . to 6 factors.

13. Find the area of the circle described about an equilateral triangle of which the side is 6 in.

14. (1) Given $\sin 30°=\frac{1}{2}$, $\sin 45°=\frac{1}{\sqrt{2}}$: find $\sin 75°$.

(2) If $2 \sin A=\tan A$, find the angle A.

(3) Prove $\frac{\cos 2A-\cos 4A}{\cos 2A-\cos 4A}=\tan 3A \tan A$.

15. (1) In a right-angled triangle ABC, C the right angle: given $AB=43{\cdot}2$, $AC=41{\cdot}031$; find B.

(2) Two sides of a triangle are 850 yds. and 750 yds. respectively, and the included angle is 75°; find the remaining angles of the triangle.

XVIII. (*a.*)

1. (1) How many times is £1 11*s.* 2*d.* contained in £81 0*s.* 8*d.*?

   (2) A chest of tea weighing 4 cwt. 3 qrs. 14 lbs. is to be made up into parcels containing 8 lbs. and 6 lbs. each, there being the same number of parcels of each sort. How many parcels will there be?

2. A gentleman, after paying an income tax of 7*d.* in the £, has £497 1*s.* 4*d.* left; what was his gross annual income?

3. (1) A banker allows interest at the rate of 5 per cent. per annum on deposits. A customer deposited £7,300 for 90 days; what interest did he receive?

   (2) A person buys £500 consols at $90\frac{1}{3}$, and sells out at 93; what sum of money does he gain?

4. (1) Reduce $\frac{8280}{11385}$ to its lowest terms.

   (2) Find the value $10+\frac{11}{10}+\frac{111}{100}+\frac{1111}{1000}+\frac{11111}{10000}$, and reduce the result to a decimal.

   (3) Express £$\frac{7}{10}$ as the fraction of a guinea.

   (4) What decimal of £2 is 11*s.* $9\frac{3}{4}$*d.*?

5. (1) Divide 15·73 by $(\cdot 0011)^2$.

   (2) Extract the square roots of $\frac{526}{2401}$ and ·081.

6. Show that $x(x^2+11)=6(x^2+1)$ whether $x=1$ or 2. If $2^x=8$, what is the value of x?

7. Simplify the expressions:

(1) $\left(1+\frac{x}{2}\right)^2-\left(1-\frac{x}{2}\right)^2$. (2) $\dfrac{\frac{a}{a+}+\frac{b}{a-b}}{\frac{a}{a+b}-\frac{b}{a-b}}$.

(3) $\dfrac{2x^3+3x^2-2x-3}{6x^3+9x^2+2x+3}$.

(4) Expand $(a^{\frac{1}{3}}-x^{\frac{1}{3}})^3$.

8. Solve the equations :

(1) $\dfrac{7x+5}{3}-\dfrac{16+4x}{5}=\dfrac{3(x-1)}{2}$.

(2) $\dfrac{6x+7}{9}-\dfrac{2x+4}{3}=\dfrac{13-7x}{6x+3}$.

(3) $\dfrac{4}{y}-\dfrac{4}{x}=1, \dfrac{5}{x}+\dfrac{4}{y}=\dfrac{7}{x}+\dfrac{3}{2}$. (4) $\dfrac{10}{x}-\dfrac{14-2x}{x^2}=\dfrac{22}{9}$.

(5) $\dfrac{1}{x}+\dfrac{1}{y}=1$ and $\dfrac{1}{x^2}+\dfrac{1}{y^2}=5$.

(6) Two towns are 374 mi. apart. Couriers start from each town on the same day to meet; one travels 9 mi. a day, the other 8; in how many days will they meet?

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9. (1) Given $\log_{10} 2 = \cdot 3010300$: find, without tables, $\log_{10} 3\frac{1}{8}$.

(2) Find, by the aid of the tables, $\dfrac{(24\cdot 76)^{\frac{2}{7}}}{(\cdot 0045)^{\frac{3}{2}}}$;

(3) The time in which a sum of money will double itself at $6\frac{1}{2}$ per cent. compound interest ?

10. The radius of a circle is 8 ft. ; find the area of the sector of the circle, the angle of which is 36°.

11. (1) Prove $\tan A = \dfrac{2 \tan \frac{1}{2} A}{1-\tan^2 \frac{1}{2} A}$; if $\tan A = \sqrt{3}$, find $\tan \frac{1}{2} A$.

(2) Prove $\dfrac{\sin A + \sin B - \sin C}{\sin A + \sin B + \sin C} = \dfrac{\cos \frac{A-B}{2} - \cos \frac{A+B}{2}}{\cos \frac{A-B}{2} + \cos \frac{A+B}{2}}$

when $A+B+C=180°$.

12. In a triangle ABC, if $A=41° 10'$, $BC=145\cdot 3$, $AC=$

178·3 ft.; show that the solution is ambiguous, and determine the angle contained by A C and B C in the greater of the triangles to which the solution belongs.

13. If two forces are in equilibrium on a straight lever round a fulcrum, show that the forces are to each other inversely as the perpendiculars drawn from the fulcrum upon the direction of the forces: find the pressure on the fulcrum.

At one extremity of a horizontal lever 16 in. long a weight of 10 lbs. is suspended; at the other end a force of 12 lbs. acts at an angle of 150° with the lever: find the place of the fulcrum in the case of equilibrium, and the pressure upon it.

14. How is *accelerating force* measured? Investigate the relation between the measure of gravity and the space through which a body falls from rest in one second.

15. A body in falling from the top of a steeple described one-third of its path in the last second: determine the time of its fall.

WOOLWICH PAPERS.

XIX. (*a.*)

1. (1) Find the cost of 183 articles at £7 16*s.* 7*d.* each.

(2) Divide 14*s.* 4$\frac{3}{4}$*d.* by ·034, giving the result to the nearest penny.

2. A gentleman at his death leaves one-third of his property to his widow, and the remainder to be equally divided amongst his children. The widow's legacy proves to be three times that of one of the children. How many children were there? (*Arithmetic.*)

3. A franc being taken to be worth 9$\frac{3}{4}$*d.*, find the sum of money which can be paid by an exact number of either

shillings or francs, the number of francs exceeding the number of shillings by 27. (*Arithmetic.*)

4. Solve the equations:

(1) $\dfrac{\sqrt{3x+1}+\sqrt{3x}}{\sqrt{3x+1}-\sqrt{3x}}=4.$ (2) $\left.\begin{aligned} x+y+z &= 6 \\ 4x+y &= 2z \\ x^2+y^2+z^2 &= 14 \end{aligned}\right\}$

(3) $x^4+x^2+2x\sqrt{2}=1.$ (4) $4x^3-15x-63=0,$ by Cardan's method; find all three roots.

5. On a sum of money borrowed, interest is to be paid at the rate of 5 per cent. per annum. After a time £200 of the loan is paid off, and the interest on the remainder is now reduced to 4 per cent., and the yearly interest is now lessened by one-third. What was the sum borrowed?

6. (1) Extract, by means of logarithms, the fifth root of ·0034629.

(2) Find a third proportional to ·00046 and 83725.

7. Find the present value of £1 due 12 years hence, when money bears $4\frac{1}{2}$ per cent. compound interest.

8. A quadrilateral is inscribed in a circle; one side, being the diameter of the circle, is 10 ft. long, and the opposite one, which is parallel to it, is 6 ft. What is the area of the quadrilateral?

9. Write down all the angles, between $-800°$ and $800°$, which have ·5 for the value of their sine.

10. Assuming the expression for sin (A+B) in sines and cosines of A and B, prove that

$$2\sin A = \pm\sqrt{1+\sin 2A}\pm\sqrt{1-\sin 2A}:$$

Ex. $2A=315°$, so that $\sin 2A=-\dfrac{1}{\sqrt{2}}$; find sin 157° 30′.

11. Find the radius of a circle, inscribed in a triangle, in terms of the area and perimeter of the triangle.

In a right-angled isosceles triangle the radius of the inscribed circle is 1 ft.; find the lengths of the sides of the triangle.

12. Find the equation to a circle referred to two perpendicular tangents as axes of coordinates.

13. Straight lines being drawn at right angles to one another through the fixed points (a, b) and $(-a, -b)$, find the locus of their intersection.

14. Prove that the straight line $y-x = \sqrt{a^2+b^2}$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and find the corodinates of the point of contact.

15. If $xy = x \sin x + \cos x$, prove that $\frac{d^2y}{dx^2}+y = \frac{2y}{x^2}$.

16. Find the asymptotes of the curve $y^4+(a^2+x^2)y^2 = a^2x^2$, and the angle between the two tangents at the origin.

17. Integrate the following expressions with respect to x:

(1) $\frac{x\,dx}{(a+x)^{\frac{3}{2}}}$. (2) $\frac{xdx}{x^4+2x^2-3}$.

(3) $\frac{dx}{(\cos x)^4}$. (4) $\frac{\cos x\,dx}{1+\cos x}$.

18. A parabolic area bounded by the *latus rectum* revolves about the tangent at the vertex; find the volume of the solid generated.

XX. (*a*.)

1. If 25 gas-burners, which are lighted 5 hours every evening for 20 days, consume a quantity of gas which costs £2 2*s*. 6*d*., how many burners may be lighted 4 hours every evening for 30 days at a cost of £7 13*s*.?

2. A merchant lost a cargo at sea, which he had insured; the broker offered him a sum of money for the loss, which the merchant refused, as 10 per cent. below the estimated value of his loss; the broker then offered £379 15*s*. more than at first, and the amount of the second offer was $5\frac{1}{2}$ per cent. in excess of the estimated value. What was that value, and what sum did the broker first offer?

3. (1) Of what number is -5 the logarithm to the base 10?

(2) Prove $\log_{10}5 \times \log_5 10 = 1$.

(3) What value is the log 0? does it vary with the base?

(4) Find, by the aid of tables, a mean proportional between 2^{11} and $\cdot000256$.

(5) The amount of £1,420 10*s.* in 14 years at 5 per cent. compound interest payable quarterly.

4. If $\frac{A}{x} = \frac{B}{y} = \frac{C}{z}$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$; prove that

$$\frac{A^2}{a^2} + \frac{B^2}{b^2} + \frac{C^2}{c^2} = \frac{A^2+B^2+C^2}{x^2+y^2+z^2}.$$

5. Solve the equations:

(1) $c^2b(x^2-1) = x(b^2-c^4)$.

(2) $$\left.\begin{aligned} x^3+y^3+z^3 &= 153 \\ x+y-z &= -1 \\ (x+z)\{y(x+z)+y^2+xz\} &= 192 \end{aligned}\right\}$$

(3) $x^4-14x^3+60x^2-50x-125 = 0$ which has equal roots.

6. When is an infinite series said to be *convergent* or *divergent*? Show that the series $1+\frac{1}{2x}+\frac{1}{3x}+\frac{1}{4x}$ is convergent if x be greater than unity, and divergent if $x = 1$.

7. ABC is an equilateral triangle; in AB take AD BE each equal to one-third of AB; through D draw DF parallel to BC meeting AC in F, and through E draw EG parallel to AC meeting BC in G. Show that a circle may be inscribed in the pentagon FDEGC.

8. In any triangle, A B C, let B E be drawn perpendicular to A C, then B O being a diameter of the circumscribing circle, if from B, B D be drawn parallel to the tangent at A and cutting A C in D; the rectangle B O . B E is equal to the rectangle A C . B D.

9. What is the *circular measure* of an angle? When this measure is referred to, examine the length of the arc subtending the *angular unit* at the centre of a given circle.

10. The radius of a circle is 10 ft.; express in degrees the angle subtended at the centre by an arc 3 ft. in length.

11. (1) Prove $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$; verify this when $A=30°$.

(2) Prove $\frac{\cos 3A+\sin 3A}{\cos A-\sin A}=1+2\sin 2A$.

(3) In any plane triangle, A an obtuse angle, prove $\cos A=\frac{b^2+c^2-a^2}{2bc}$.

12. (1) When an object is viewed from an elevation above it, what is the angle of depression?

(2) An observer in a balloon observes the angle of depression of an object on the ground, due south, to be 35° 30′; the balloon drifts due east, at the same elevation, for $2\frac{1}{2}$ miles, when the angle of depression of the same object is observed to be 23° 14′. Find the height of the balloon.

13. In a right-angled spherical triangle, given the hypothenuse 74° 20′, and one angle 34° 15′. Find the side opposite to the given angle, proving the formula used in the solution.

14. The equation to a straight line referred to rectangular coordinates is $\frac{x}{3}-\frac{y}{4}=1$. Find the distance of the origin from the line.

15. If $y^2=4m(x-m)$ be the equation to a parabola, where is the origin? Find the equation to the normal drawn at the extremity of the *latus rectum* of this parabola.

16. (1) If $u=f(x)$, examine the condition, that u may have a minimum value when $\frac{du}{dx}=0$, and $\frac{d^2u}{dx^2}$ remains finite.

(2) Through a given point between two lines inclined at a given angle draw a straight line, forming with these lines the least possible triangle.

(3) If $\frac{dy}{dx}=\sqrt{\frac{2a-y}{y}}$ is the equation to the cycloid

from an extremity of the base, find the radius of curvature at any point, and show that it is equal to twice the normal to the curve at the same point.

17. Integrate the functions:

(i) $\int \frac{dx}{\sqrt{a^2+x^2}}$. (ii) $\int \frac{x^2 dx}{\sqrt{1-x^2}}$.

(iii) $\int \frac{dx}{\sin^3 x}$.

18. (1) If A be the area of a plane curve referred to rectangular coordinates, prove $\frac{d\text{A}}{dx}=y$.

(2) Find the equation to a curve such that its area is equal to twice the rectangle contained by its coordinates.

XXI. (*a.*)

1. (1) Find the cost of 131 articles at £3 9*s.* 6*d.*

(2) Divide £1 16*s.* 5*d.* by $5\frac{3}{4}$.

2. A traveller in Russia finds that a length denoted as 3 versts 84 sachines is 11,088 English feet; and that another length denoted as 2 versts 21 sachines is 7,147 feet. How many sachines make a verst? (*Arithmetic.*)

3. There are two mixtures of wine and water, the quantities of wine in which are respectively ·34 and ·46 of the whole. If a gallon of the first is mixed with two gallons of the second, what decimal part will the wine be in the compound? (*Arithmetic.*)

4. (1) Prove that two circulating decimals multiplied together may produce a terminating decimal.

(2) Multiply $\cdot\dot{4}2857\dot{1}$ by $1\cdot1\dot{6}$.

5. Solve the equations:

(1) $\frac{2x-8}{6}-\frac{7-x}{4}=\frac{5}{12}$. (2) $\frac{5-x}{x-1}-\frac{x-3}{x+1}=1$.

(3) $\left\{\begin{array}{c} 4y-22x=7 \\ 2-x=23y \end{array}\right\}$ (4) $4x^3+6x^2+x=1$.

6. The population of a place, in which the numbers of males and females are as 14 to 11, is reduced, by an emigration in which twice as many males as females depart, and the numbers of males and females are afterwards as 12 to 13. What fraction of the population emigrated ?

7. The sum of either 5 or 15 terms of an A.P. is 75: find the 10th term.

8. Five ladies and three gentlemen are going to play at croquet. In how many ways can they divide themselves into two sides of 4 each, so that the gentlemen may not be all on one side ?

9. Employ logarithms to find :

(1) The fifth root of ·0378642;

(2) The amount of £1 in 37 yrs. at $4\frac{1}{3}$ per cent. compound interest.

10. If a semicircle be inscribed in a triangle with the diameter in the base of the triangle, prove that its centre divides the base into segments which are as the sides of the triangle.

11. (1) Trace the change in sign and magnitude of the secant of an angle, as the angle increases from 0 to 360°.

(2) From tables extending from 0 to 90° how do we find the secants of angles above 90° ?

12. Prove that $\sin(A-B)=\sin A \cos B-\cos A \sin B$; and that $\sin(A+30°)+\cos(A+60°)=\cos A$.

13. If the sides of a triangle, 27·6 and 37·54 yds. long, include the angle 126° 18′ 54″, find the area of the triangle.

14. When an observer is at the base of a tower, an object has an elevation of 36° 18′, and when he is at the summit of the tower, 30 ft. vertically above his former position, the elevation of the object is lessened by 5° 37′ 9″. Find the height of the object.

15. A triangle, whose sides are 3, 4, and 5 inches long, revolves about its longest side. Find the volume of the *solid thus* generated.

16. (1) If $y=x\tan\alpha+c$, $y=x\tan\alpha'+c'$ be the equations to two straight lines, find the equation to a straight line passing through their intersection and bisecting the angle between them.

(2) From the definition of a parabola find its polar equation, the focus being the pole. Hence find the inclination to the axis of a chord of the parabola, through the focus, whose length is double the *latus rectum.*

17. (1) Find the maximum and minimum values of the expression $x^3-6x^2+9x+12$.

(2) Obtain the equation to the cycloid, and find the radius of curvature at the vertex.

18. Integrate with regard to x the following expressions:

(1) $\dfrac{dx}{x(1-x)}$. (2) $\dfrac{dx}{\sqrt{x}-x^{\frac{3}{2}}}$.

(3) $\dfrac{\cos x\,dx}{1-\sin^2 x}$. (4) $x^2a^x dx$.

XXII. (*a.*)

1. A rectangular tank is 14 ft. long and 10 ft. wide: what must be its depth that it may just contain $31\frac{1}{4}$ tons of water, a cubic foot of water weighing 1,000 ounces?

2. (1) Find the cube root of 27·270901.

(2) Reduce to its simplest form $\dfrac{4}{12\sqrt{\cdot0625}+1}$.

3. Three merchants invest respectively in a cargo the sums £5,200, £3,600, £2,000; when sold, there is a clear profit on the whole cargo of £1,647: how much ought each to receive, and at what rate per cent. is the profit?

4. (1) Find, without tables, to two decimal places, the number of which ·75 is the logarithm to the base 10.

(2) Find, by the tables, $(1\cdot013)^{\frac{3}{20}}-1$.

5. Reduce to their simplest forms:

(1) $\dfrac{x^4-a^2x^2-2ab^2x-b^4}{x^4+2ax^3+a^2x^2-b^4}$.

(2) $\dfrac{(a+b\sqrt{-1})^3+(a-b\sqrt{-1})^3}{(a+b\sqrt{-1})^2+(a-b\sqrt{-1})^2}$.

(3) If a, b, c, d are unequal and positive numbers, prove $abcd$ less than $\left(\dfrac{a+b+c+d}{4}\right)^4$.

6. Solve the equations:

(1) $x^3-7x^{\frac{3}{2}}=8$.

(2) $\left.\begin{aligned} 2\sqrt{x^2-y^2}+xy&=26 \\ \frac{x}{y}-\frac{y}{x}&=\frac{9}{20} \end{aligned}\right\}$.

(3) $6x^4-35x^3+62x^2-35x+6=0$.

(4) $x^3-2x^2-x=-2$.

7. (1) Find the sum of n terms of the series
a, $(a+b)\,r$, $(a+2b)\,r^2$, $(a+3b)\,r^3$ &c.

(2) Show how to find the coefficient of an assigned power of x in the expansion of $(a_0+a_1x+a_2x^2+\&c.)^n$, when n is fractional or negative, and find the coefficient of x^4 in the expansion of $(1+x+x^2)^{-5}$.

8. (1) ABC is an isosceles triangle, vertex A; D and E are the points of bisection of AB, AC respectively; BE and CD intersect in F; prove the triangle ADE equal to three times the triangle DEF.

(2) O is a point in the circumference of a given circle; with centre O and any radius describe a circle cutting the given circle in A and B; place BD equal to OB in the circle whose centre is O; join AD, cutting the given circle in P; prove PD and PB each equal to the radius of the given circle.

(3) ACB is a right-angled triangle, CD bisects the right angle C, cutting the base AB in D; prove AD^2, CD^2, DB^2 in harmonical progression.

9. (1) Find, without the aid of the integral calculus, the convex surface of a right cone with a circular base.

(2) What length of canvas $\frac{3}{4}$ yd. wide will be required to make a conical tent 10 ft. high and 12 ft. in diameter at the base?

10. (1) The cosine of an angle is $\sqrt{\frac{2}{3}}$; find the tangent of the angle and the tangent of its complement.

(2) Prove $(\cos \text{A} + \cos \text{B})^2 + (\sin \text{A} + \sin \text{B})^2 = 4 \left(\cos \frac{\text{A}-\text{B}}{2}\right)^2$.

(3) If θ be the circular measure of an acute angle, prove $\theta > \sin \theta$ and $< \tan \theta$.

(4) Given $\pi = 3{\cdot}1415927$; find sin 1′.

11. (1) Assuming the formula for the cosine of an angle of a plane triangle in terms of its sides, find the sine of an angle of the triangle and its area. If A, B, C be the angles, a, b, c the sides opposite to them, and s half the perimeter of the triangle, prove $\sin \text{A} + \sin \text{B} + \sin \text{C} = \frac{4s}{abc} \times \text{area}$.

(2) In a triangle ABC, given A$=42°\ 35'$, B$=47°\ 25'$, BC$=2963{\cdot}5$ ft.; solve the triangle.

12. A is the top of a vertical column, AB, standing on a hill, BC is perpendicular to the horizontal plane on which a base DE of 380 ft. is measured; find the height AB, the following angles having been observed: ADC$=59°$, ADB$=26°$, ADE$=70°\ 20'$, AED$=65°\ 30'$.

13. What length of a gun of 6 inches bore will be filled with 20 lbs. of powder of which 30 cubic inches weigh 1 lb.?

14. (1) Show that any two sides of a spherical triangle are together greater than the third side, and the sum of the three sides less than the circumference of a great circle.

(2) Find the area of a spherical triangle in terms of the spherical excess.

15. (1) Construct the circle whose equation is $x^2+y^2-8y-6x=0$, and find the equation to the straight line which is a tangent to the circle at the origin.

(2) Prove that the rectangle contained by the perpendiculars from the foci of an ellipse on the tangent at any point, is equal to the square of half the minor axis.

16. (1) If $u=\frac{f(x)}{\phi(x)}$, where $f(x)$ and $\phi(x)$ are functions of the same independent variable x, show how to obtain $\frac{du}{dx}$.

(2) If $u=\frac{ax}{\sqrt{a^2-x^2}}$, find $\frac{du}{dx}$ and $\frac{d^2u}{dx^2}$.

(3) Inscribe the greatest right cone in a given sphere.

17. Find the radius of curvature at any point of an ellipse. If ρ be the radius of curvature at the extremity of the major axis, and ρ_1 that at the extremity of the minor axis, prove $\rho\rho_1=ab$, a and b being the semi-axes.

18. The equation to a cycloid, the base being the axis of x, is $\frac{dy}{dx}=\sqrt{\frac{2a-y}{y}}$; find the volume of the solid generated by the revolution of the cycloid round its base.

XXIII. (*a.*)

1. (1) If land has been sold in the city of London at the rate of a million pounds an acre, find the value of a square inch, to the nearest halfpenny.

(2) Find, in grains, the least weight which can be expressed by an exact number of ounces in both troy and avoirdupois weights.

2. What is the amount of £1,384 15*s.* 8*d.* in 7 yrs. 6 mo. at £6 13*s.* 4*d.* per cent. per annum, simple interest?

3. The officers of a regiment are ·042 of its strength, but,

after 50 privates have been added, the officers are ·04 of the whole. What is the number of officers? (*Arithmetic.*)

4. Solve the equations:

(1) $(x-3)(x+2)-(x-2)(x+3)=6$.

(2) $7y-2x=1$, $2w-x=15$, $2y+z=7$, $10y+3x=19$.

(3) $6(x+4)^2+(x-4)^2=5(x^2-16)$.

(4) $x^2+y^2+1=3xy$, $2(xy+4)=3y^2$.

5. In the election to an office the numbers of the supporters of the three candidates, A, B, C, are in arithmetical progression. C, who has the fewest supporters, withdraws before the day of election, and of his supporters 9 decline to vote, and the rest so divide themselves that 4 times as many of them vote for A as for B. On the whole, 125 votes are given for A and 82 votes for B. Find the number of supporters of each candidate.

6. In the expansion of $\left(\frac{3}{5}+\frac{5x}{3}\right)^{-7}$ in ascending powers of x by the binomial theorem, give the first three and the tenth terms. Show whether the expansion has or has not a greatest coefficient as to numerical value.

7. Employ logarithms to find:—

(1) A fourth proportional to ·546837, 7·6298 and 762·98.

(2) To two places of decimals the real root of the equation $7^x-6\times 7^{-x}=1$.

8. If the straight line which bisects an angle of a triangle also bisects the opposite side, prove that the triangle is isosceles.

9. (1) Prove that an angle has only one sine, but that a decimal given as a sine has an unlimited number of angles corresponding to it. What is the relation between the two angles between 0 and 180° which have the same sine?

(2) Given that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$; from $\tan 225°$ find $\tan 112° 30'$.

10. In the ambiguous case of the solution of a triangle from the given parts A, a, b, prove that the two values of the angle C differ by twice the angle whose cosine is $\frac{b \sin A}{a}$. Compute the value of this difference when $a=3{,}946$, $b=6{,}984$, $A=34° \; 19' \; 56''$.

11. A circle touches the hypothenuse of a right-angled isosceles triangle, and the two sides produced. Find the distance between the centre of this escribed circle and the right angle, the area of the triangle being 392 square yards.

12. A regular pyramid has a square base and equilateral triangles for sides. Its volume being 13,122 cubic inches, find its height.

13. Find the polar equation to a circle when the pole is a given exterior point, and the spiral angle is measured from a line passing through the centre of the circle. Hence prove that the two tangents drawn to a circle from an exterior point are equal.

14. Given the equation to the tangent to an ellipse at the point (h, k) $\frac{xh}{a^2}+\frac{yk}{b^2}=1$; prove that the tangent is equally inclined to the focal distances of the point (h, k).

15. Find the radius of curvature of a parabola.

16. Integrate with respect to x—

(1) $\sqrt{\frac{a+x}{a-x}}\,dx$. (2) $\frac{xdx}{a^4-x^4}$.

(3) $(\cot x)^2\,dx$.

17. Evaluate $\int_0^{\pi}(\sin x)^4\,dx$.

18. A semi-ellipse bounded by the minor axis revolves about the tangent at the vertex of the major axis. Find the solid generated.

XXIV. (*a.*)

1. (1) If $1\frac{1}{2}d.$ be the interest on a £100 exchequer bill for 1 day in the year 1865, at what rate per cent. per annum is the interest paid?

(2) Extract the square root of 250080·0064.

(3) Extract the cube root of 8·741816.

2. At an examination a candidate was required to obtain ertain decimal of the whole number of marks allowed the papers, in order to pass. A obtained for his mark 5, which was not enough, and B obtained ·55, which was re than was necessary. If, however, 360 marks had en added to A's total, and 120 marks taken from B's al, A and B would each just have obtained marks enough pass. Required the number of marks allotted to the pers, and the decimal required for a pass.

3. (1) Explain why negative numbers have no logarithms.

(2) If N be any whole number, prove $\frac{\log_{10}N}{\log_5 N}=\log_{10}5.$

(3) Find by logarithms $\frac{\left(\frac{125}{123}\right)^{10}}{(\cdot0043)^{\frac{2}{5}}}.$

(4) Find the present value of an annuity of £1,575 10*s.* at $4\frac{1}{2}$ per cent. compound interest, to be paid for 14 years.

4. (1) If $\frac{x}{y}+\frac{y}{x}=a$, $\frac{y}{z}+\frac{z}{y}=b$, $\frac{z}{x}+\frac{x}{z}=c$, prove $a^2+b^2+c^2-abc=4.$

(2) If $\alpha=\frac{-1+\sqrt{-3}}{2}$, $\beta=\frac{-1-\sqrt{-3}}{2}$, prove $\alpha^2+\alpha\beta+\beta^2=0.$

(3) Solve the equation $x^4-x^3-3x^2+5x-2=0$, which has equal roots.

5. Find the number of shot in a complete pyramidal pile ith a triangular base, the number of shot in a side of the ase being given.

In an engagement a number of shot were fired away from the top of a pyramidal square pile, equal to 20 times the number of shot in one side of the base; and it was found that the shot remaining would form a triangular pile, the number of shot in a side of the base course being the same for both the triangular and the square piles. Find the number of shot in a side of the base course.

6. (1) Assuming the general form for the expansion of a binomial, write down the coefficient of the $(r+1)$th term of $(1+x)^{-n}$, and show that it is a whole number when n is a whole number.

(2) Prove that $1+n+\frac{n(n+1)}{1\,.\,2}+\ldots$

$$+\frac{n(n+1)(n+2)\ldots(n+r-1)}{\lfloor r}=\frac{(n+1)(n+2)\ldots(n+r)}{\lfloor r}.$$

7. (1) How is probability represented mathematically? If $\frac{a}{a+b}$ represent the probability of an event happening, what is the probability of its failing? If $\frac{a}{a+b}, \frac{a_1}{a_1+b_1}$ represent the separate probabilities of the happening of two independent events, find the probability that *both* events (1) happen, (2) fail.

(2) The odds are 7 to 1 against A's solving a problem, and 3 to 1 in favour of B's solving it: what is the probability that the problem will be solved, (1) by either A *or* B, (2) by both A *and* B?

8. (1) ACB is a right-angled triangle, C the right angle; if AB be bisected in E, and AE and BE in G and H, respectively, prove the sum of the squares described on the three sides of the triangle CGH is to the square described on AB as 7 to 8.

(2) Inscribe an equilateral and equiangular hexagon in a given equilateral triangle, and compare the hexagon so inscribed with the area of the triangle.

(3) If the diagonals of a quadrilateral inscribed in a circle intersect at right angles, show that the sum of the squares of either of the two opposite sides of the quadrilateral is equal to the square of the diameter of the circle.

9. (1) Trace all the changes of sign in $\sin \frac{1}{2} A + \cos \frac{1}{2} A$ as A varies from 0 to 360°.

(2) Assuming $\sin 3A = 3 \sin A - 4 \sin^3 A$, deduce from this equation $\cos 3A$ in terms of $\cos A$, and hence prove $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$.

10. (1) Account for the values of the characteristics of the logarithms of the sines of angles, as registered in the tables: show that, in general, the change of the tabular logarithmic sine of an angle is approximately proportional to the change of the angle. Is this true when the angle is nearly equal to 90°?

(2) Given log sin 31° 15′ = 9·714978, log sin 31° 16′ = 9·715186: find log sin 31° 15′ 20″.

(3) Given log tan 35° 20′ = 9·850593: find without tables log cot 35° 20′.

11. Given the three sides of a triangle, 101·5, 80·5, 59·4 feet: find the greatest angle of the triangle.

12. A lighthouse is 12 miles distant from a ship, and has a bearing north-east from the ship. The ship sails in a direction due east at the rate of 4 miles an hour: after *what times* will the distance of the ship from the lighthouse be 9 miles?

13. The axis of a hollow cone is 2·3106 feet; find the radius of its circular base, so that it shall contain 8·48232 gallons. (1 gal. = 277·272 cub. in.; $\pi = 3·1416$.)

14. (1) If A, B, C be the angles, a, b, c the corresponding sides of a spherical triangle, assuming the expression for cos A in terms of the sides, obtain from it the expression for $\cos\frac{a}{2}$ in terms of the angles of the triangle.

(2) Given $A = 102° 25'$, $B = 57° 43'$, $C = 90°$; find a.

15. (1) Find the straight lines represented by the equation $x^2-6y^2+xy-2x-y+1=0$.

(2) If S be the focus of a parabola, and A the vertex, SY the perpendicular on the tangent drawn from the point P, prove $SY^2 = SP \cdot SA$.

(3) If the normal at any point P intersect the axis in G, prove $PG = 2SY$.

16. (1) If $u = \log_e x$, find from definition $\frac{du}{dx}$.

(2) If $u = \log_e\left(\frac{1-\cos mx}{1+\cos mx}\right)^{\frac{1}{2}}$, find $\frac{du}{dx}$.

(3) Find the point, in the line joining the centres of two given spheres, from which the greatest portion of spherical surface is visible. (Radii of spheres r_1, r_2; distance of centres, d.)

17. Integrate the functions:

(1) $\frac{du}{dx} = \frac{a}{1+x+x^2}$. (2) $\frac{du}{dx} = e^{ax}\cos bx$.

(3) $\frac{du}{dx} = \frac{x^2}{\sqrt{2ax-x^2}}$.

18. Find an expression for finding the area of a plane curve referred to polar coordinates: show that the curve $r = a\sin 3\theta$ has loops, and find the area of one of its loops.

XXV. (*a.*)

1. (1) Compute the cost of 225 articles at £8 7*s.* 8*d.* each.

(2) If a deduction of $12\frac{1}{2}$ per cent. be made from £896 5s., what sum remains?

2. A gallon of water weighs 10 lbs. If new weights and asures be adopted, in which a litre is 1·761 pints and a ogram 2·205 lbs., express in kilograms the weight of a re of water, to four places of decimals.

3. If $\frac{1}{3}$ of a sum of money is invested at the interest of 3 r cent. per annum, and the remainder at 4 per cent., and the whole yearly interest is £3,709 11*s.* 4*d.*, find, *without 'gebra*, the sum invested.

4. Solve the equations:

(1) $\dfrac{x+a}{x+b}+\dfrac{x-a}{x-b}=\dfrac{a}{b}$.

(2) $\frac{1}{4}(2y-7)-\frac{3}{4}(x-6)=\frac{3}{5}(x+2)-\frac{2}{3}(y-1)=1$.

(3) $x^3-2x=\frac{7}{8}$.

(4) $\sqrt{ay(a-x)}=(a-b)\,x,\ \sqrt{ayx}=(a+b)(a-x)$.

5. 46 lbs. of tea are worth 2*s.* 10*d.* more than 73 lbs. of ffee; 9 lbs. of tea are worth 5*s.* 4*d.* less than 17 lbs. of ffee: find the value of 1 lb. of tea.

6. (1) Find, by logarithms, the quotient of ·08642639 divided by $\sqrt[3]{3{\cdot}4276}$.

(2) A sum of £500 is borrowed at 5 per cent. interest, and £30 is paid at the end of every year to discharge the interest and reduce the principal; what debt will remain at the end of 20 years?

7. In how many ways can the letters of the word *Chelsea* arranged, under the restriction that the two vowels *e* all never stand together?

8. (1) A square is the greatest rectangle of a given perimeter.

(2) Bisect the sides of a triangle, and join the points of bisection. Prove that the triangle thus formed is similar to the original triangle, and a quarter of it in size.

9. Prove sin (A—B) = sin A cos B— cos A sin B, in the case which A is an obtuse angle less than 180°, and A—B is an ute angle.

10. The angle between the equal sides of an isosceles triangle is $50° 18' 54''$, and the area of the triangle is 10 sq. ft.; find the length of the base.

11. A ship observes a certain landmark due north of her, and after sailing three miles in the north-east course, she observes the same mark to be 23° to the west of north; what is the distance of the mark for her latter position?

12. Place a string a yard long to form a semicircle and its diameter; what is the area enclosed?

13. A circular tube an inch in diameter and 8 in. long contains liquid to the height of 5 in. when it is held vertically: find the greatest slope at which the tube may be held before any of the liquid runs out.

14. (1) Find the angle between the straight lines

$$3x+4y=1 \text{ and } 4x+3y=1.$$

(2) The equation to the parabola being $y=3x-4x^2$, find the length of the *latus rectum*.

15. Find the greatest rectangle which can be inscribed in a given ellipse, with its sides parallel to the axes.

16. Investigate an expression for the length of the radius of curvature of a plane curve. Find the radius of curvature at any point of the curve $xy=a^2$.

17. Integrate the expressions:

(1) $\dfrac{x\,dx}{x^4+3x^2+2}$. (2) $\dfrac{dx}{x\sqrt{1+x^2}}$.

(3) $\dfrac{d\theta}{\cos\theta-\sin\theta}$. (4) $xe^{4x}dx$.

18. Find, by integration, the surface of a hemisphere.

XXVI. (*a*.)

1. (1) Extract the square root of ·009, and write down the true value of the remainder in the operation when four decimal places have been obtained in the root.

(2) Find the cube root of 1006·012008.

2. The value of 1 oz. troy of standard gold is £3 17*s.* 10½*d.*; find the least exact number of ounces that can be coined into an exact number of sovereigns.

3. A railway train travels 30 miles an hour including stoppages, and 35 miles an hour when it does not stop: in what distance will the train lose 2 hrs. by stoppages?

4. (1) Given log 3 = ·477121; find the number of digits in 3^{15}, and the number of ciphers between the decimal point and the first significant digit of the decimal equal to $\frac{1}{3^{15}}$.

(2) Find, by the aid of the tables, the sum of the series $19^{\frac{1}{3}}$, $19^{\frac{1}{3}}$, $19^{\frac{1}{3}}$ to six terms;

(3) The rate per cent. at which £1,320 will amount to £1,778 8*s.* in 6 yrs. compound interest payable quarterly.

5. Prove $$\frac{x^2y^2}{(z^2-x^2)(z^2-y^2)}+\frac{x^2z^2}{(y^2-x^2)(y^2-z^2)}+\frac{y^2z^2}{(x^2-y^2)(x^2-z^2)}=1.$$

Solve the equations:

(1) $\dfrac{(x-a)(x-b)}{(x-c)(x-d)}=\dfrac{x-a-b}{x-c-d}$.

(2) $x^4-2x^3-3x^2+2x+2=0$, one root being $1+\sqrt{3}$.

(3) $2x^3+3x^2=1$.

(4) $x^3-3x^2=-4$.

6. (1) Find the factor which will rationalise $a^{\frac{1}{2}}+b^{\frac{1}{3}}$.

(2) Find the square root of $9-2\sqrt{3}-2\sqrt{5}+2\sqrt{15}$.

7. (1) In the scale of notation of which the radix is r, show that any whole number when divided by $r+1$ will leave the same remainder as the difference between the sum of the digits in the odd places and the sum of the digits in the even places, when divided by $r+1$.

(2) Express 240·25 in the scale whose radix is 8, and extract the square root of it in that scale.

8. Expand $\left(1+\frac{1}{n}\right)^{nx}$, and show that when n is indefinitely great the expansion is the same as the known expansion for e^x, where e is the base of Napier's system of logarithms.

In the expansion of $(e^x-1)^6$, show that the coefficient of x^3 vanishes.

9. (1) If two equal straight lines bisect each other, the figure formed by joining their extremities is a rectangle.

(2) Two circles intersect, AB is their common chord, a point O is taken in AB produced, and any straight line OCD is drawn cutting one of the circles in C and D, and a straight line OEF is drawn cutting the circumference of the other circle in E and F; show that the circle circumscribing the triangle CDE will pass through F.

(3) A pyramid has four faces which are equilateral triangles, and from one of the angles a perpendicular is drawn to the opposite face intersecting it in O; show that this perpendicular is equal to three times the perpendicular dropped from O upon either of the other faces of the pyramid.

10. (1) Define the sine of an angle, and show when the sine of an angle is 0. Obtain a relation between the sine and cosine of the same angle from which we may infer the extreme limits, both *positive* and *negative*, of the sine and cosine.

(2) Given $\tan\frac{A}{2}=\frac{1}{\sqrt{3}}$; find $\sin A$.

(3) Prove $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$.

(4) Eliminate θ from the equations $\frac{\sin(\phi-\theta)}{\sin\theta}=\frac{a}{b}$, $\frac{\cos(\phi-\theta)}{\cos\theta}=\frac{c}{d}$; and find $\sin\theta$ or $\cos\phi$.

11. If A, B, C, are the angles of any plane triangle, and a, b, c, the corresponding sides, prove:

(1) $a = b \cos C + c \cos B$;

(2) $\dfrac{a^2 - b^2}{c^2} = \dfrac{\sin (A - B)}{\sin (A + B)}$.

(3) In a plane triangle ABC, given AB = 200 ft., BC = 160 ft., B = 36° 52′ 11″; find the remaining angles of the triangle.

12. From a ship at sea a rock and a headland are observed to be in the same line, which line has a bearing 18° E. of N. of the ship; after the ship has sailed 6 miles in its own course, which is NW., the rock is observed from the ship to be due E. and the headland NE.; find the distance of the rock from the headland.

13. A regular pyramid with a hexagonal base has the perimeter of its base 30 ft., and its altitude 30 ft.; find its content, and the inclination of one of its plane faces to its base.

14. (1) If one angle of a spherical triangle be greater than another, the side opposite to the greater angle is greater than the side opposite to the other.

(2) In a right-angled spherical triangle, given one side and the adjacent oblique angle; show how to determine a formula for finding the hypothenuse of the triangle.

15. (1) Find the equation to the circle referred to rectangular co-ordinates, the radius of which is 5, and the centre of which is on the axis of y, at a distance 8 from the origin. Find also the equation to the line passing through the centre of the above circle, and the centre of the circle whose equation is $x^2 + y^2 = 24x$.

(2) If S and H be the foci of an ellipse, PG a normal to the curve at P, cutting the axis major AM in G, prove $SG = SP \dfrac{SH}{AM}$.

16. (1) State the theorem known as that of Maclaurin, and, by means of it, expand $\sin^{-1}x$ as far as the term involving x^5.

(2) Define the asymptote of a curve, and show how the asymptotes of a curve may be found. Draw all the asymptotes of the curve $y^2=\frac{x^3-8}{x-4}$.

17. Integrate the function $\int\frac{dx}{(x^2+1)(x-2)^2}$.

18. Obtain the differential expression for finding the volume of a solid of revolution: apply the expression to determine the volume of a given frustum of a right cone.

XXVII. (*a.*)

1. (1) What is the value in English money of 83,628 francs, when a franc is worth $9\frac{7}{8}d.$?

(2) Find the rent of 3 ac. 3 ro. 29 po., at the rate of £1 3*s.* 4*d.* the rood.

(3) Express 136 lbs. as the decimal of 1 cwt. 1 qr.

2. An income first pays parish rates, and, on its amount thus reduced, an income-tax of 6*d.* in the £ is levied. These deductions are together 35 per cent. of the original income. How much in the £ are the parish rates? (To be solved without algebra.)

3. Prove the rule for pointing the figures of a number of which the square root is to be extracted.

Compute to two places of decimals the value of—

$$\frac{1}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{5}-\sqrt{3}}.$$

4. (1) Having given $\log 1{\cdot}28=k$, find $\log {\cdot}016$.

(2) By the aid of tables compute the value of—

$$\frac{1{\cdot}9864\times{\cdot}1188269}{{\cdot}504567}.$$

5. Solve the equations:

(1) $\frac{x-}{2}-\frac{3x-5}{8}=1.$

(2) $x+\sqrt{6x+x^2}=3.$

(3) $\frac{1}{x+5}=\frac{1}{x}+\frac{5}{6}.$

(4) $\frac{x+y}{x-y}+10\frac{x-y}{x+y}=7$ and $xy^3=3.$

6. Find two numbers whereof the greater added to a quarter of the less exceeds the less by 6, and the less added to a third of the greater falls short of the greater by 2.

7. Assuming the binomial theorem, obtain the series $a^x=1+kx+\frac{k^2x^2}{1.2}+\frac{k^3x^3}{1.2.3}+\ldots$, where k is the logarithm of a to the base e, and e is the series $1+1+\frac{1}{1.2}+\frac{1}{1.2.3}+\ldots$

8. (1) The complements of the parallelograms about the diameter of a parallelogram are greatest when those parallelograms are equal to one another.

(2) Three equal circles touch one another; compare the area of one of them with the area of the circle which circumscribes them all.

9. (1) Assuming that $\sin(A+B)=\sin A \cos B+\cos A \sin B$, express $\cos A$ in terms of $\sin 2A$; account for the appearance of four values.

(2) Find the values of θ between 0 and 90° which satisfy the equation $\tan\theta+\cot\theta=5$.

(3) Compute the value of $\cos^3\frac{A}{3}$, when $A=69° 15' 34''$.

10. The diagonals of a parallelogram are 6 and 8 ft. long, and include an angle of 67°. Find the length of the shorter sides of the parallelogram.

11. An object is observed at two points on the horizontal plane on which it stands, one of them being due north, and the other due east of it, and the distance between them

468 yds. At both of these points the summit of the object has the same elevation, 32° 47′. Find its height from the horizontal plane. By what instrument may such angular elevations be measured?

12. The diameter of the base of a circular right cone is 1 ft., and the distance of the vertex from the edge of the base is 8 in.; find the whole surface of the solid.

13. Find the radius of the circle whose equation is $x^2+y^2=4x+2y+4$.

14. Investigate the equation to the tangent to the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, at a given point. Hence show that from any external point two tangents and no more can be drawn to the ellipse.

15. From the definition of a differential coefficient, find directly $\frac{d(\tan\theta)}{d\theta}$.

16. Find the minimum value of $\frac{x^3}{(x-1)^2}$. Is this the least value of which the expression admits?

17. Find the volume of the solid produced by a quadrant of a circle revolving about one of the radii which bound it.

18. Integrate the expressions:

(1) $\left(x^2-\frac{a}{x}\right)dx$. (2) $\frac{x+1}{x^2-1}$.

(3) $(x+1)\,a^x\,dx$. (4) $\text{Sin}^2 x\,dx$.

XXVIII. (*a.*)

1. (1) Reduce $\dfrac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\text{ of }\frac{3}{11}}$.

(2) Prove $\sqrt{\cdot\dot{1}}+\sqrt[3]{\cdot 29\dot{6}}=1$.

2. A lump of metal, worth £190, is formed by a fusion of gold and silver, the values of which are £3 17*s.* 6*d.* and 5*s.* an ounce respectively; the weight of the gold is ·75 of the whole weight: find the weight of each metal contained in the mixture.

3. A farmer purchased 749 sheep, and sold 700 of them

for the price he paid for the whole; he afterwards sold the remaining sheep at the same price per head as the others; find his gain per cent.

4. (1) What is meant by the modulus of a system of logarithms?

(2) Given the logarithm of a number to any base, show how to find the logarithm of the number to any other base. If e be the base of Napier's system, $\log_e 5 = 1{\cdot}60944$, $\frac{1}{\log_e 10} = {\cdot}4343$, find $\log_{10} 5$.

(3) Find by the aid of tables $\frac{(4{\cdot}37)^{\frac{2}{5}}-1}{(4{\cdot}37)^{\frac{2}{5}}+1}$.

(4) $\frac{101}{100}+\left(\frac{101}{100}\right)^2+\left(\frac{101}{100}\right)^3+$&c. to 10 terms.

5. (1) Prove $(x+y+z)^3-(x^3+y^3+z^3)=3(x+y)(x+z)(y+z)$.

(2) Eliminate y and z from the equations

$$\left.\begin{array}{r} e^x+e^y+e^z=p \\ e^{x+y}+e^{x+z}+e^{y+z}=q \\ e^{x+y+z}=r \end{array}\right\}$$

6. Solve the equations:

(1) $(x-3)(x-2)+(x-3)(x-1)+(x-2)(x-1)=2$.

(2) $(ax+by)^2+(ay-bx)^2=2\left\{\frac{a}{b}+\frac{b}{a}\right\}^2$

$$\frac{x}{y}+\frac{y}{x}=2\,\frac{a^2+b^2}{a^2-b^2}.$$

(3) Take away the second term from the equation $y^3-9y^2+15y+25=0$, and solve it by Cardan's method.

7. (1) Show how to find the number of homogeneous products of r dimensions that can be formed out of n letters a, b, c, and their powers.

(2) Find the number of terms in the expansion of $(a+b+c+d)^5$.

8. (1) Show how to convert any given fraction into a continued fraction, and prove the law of formation of the successive convergent fractions.

(2) Find by this method the first three convergents to 3·1416.

9. (1) Prove that the area of any right-angled triangle is equal to the rectangle contained by the semi-perimeter, and the excess of the semi-perimeter above the hypothenuse.

(2) Show that the base of an isosceles triangle, whose vertex is at the centre of a circle, and the angles at the base of which are each equal to three-fourths of the angle at the vertex, is the side of a regular pentagon inscribed in the circle; and that the base of the isosceles triangle, whose angles at the base are each equal to double the angle at the vertex, is the side of a regular decagon inscribed in the same circle.

(3) ABCD is a trapezium, BD, AC the diagonals, E, F, G, H the points of bisection of the four sides AB, BC, CD, DA respectively; join these points of bisection by straight lines cutting the diagonals in K, L, M, N; prove the figure KLMN to be a trapezium similar to ABCD, and equal in area to one-fourth of ABCD.

10. (1) Show that the angle at the centre of a circle which is subtended by an arc equal to the radius, is invariable. If this angle be taken as the unit of measurement, any other angle will be properly expressed by the value $\frac{arc}{rad}$.

(2) Find the circular measure of an angle of a regular hexagon.

(3) Prove

$\sin 3\theta = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$, without assuming the formula for $\sin(A+B)$.

11. (1) If A, B, C be the angles of a triangle, prove

$$\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{A}{2}\tan\frac{C}{2} + \tan\frac{B}{2}\tan\frac{C}{2} = 1.$$

(2) In any plane triangle, given a, b, A, show that the third side may be found by the formula $c=b \cos \text{A} \pm \sqrt{a^2-b^2\sin^2\text{A}}$: if the data be $\text{A}=75°34'$, $b=85$, $a=75$; explain the result.

12. A person walking along a straight road observes a tall tree standing in front of a tower, both being in the road before him. The elevation of the top of the tower is 34° 15′, and of the top of the tree 25° 10′; on advancing 400 ft. he finds the tower and the tree to have the same elevation, 60° 15′; find the height of the tower *or* tree.

13. If a hemisphere be bisected by a plane parallel to its base, show how to find the height of the spherical segment. If the radius of the sphere be 5, show that the height of the segment will be between 3·25 and 3·3.

14. (1) If A, B, C be the angles of a spherical triangle, and a, b, c the sides subtending them, prove $\tan \frac{1}{2}(\text{A}+\text{B}) = \dfrac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \frac{1}{2} \text{C}$. To what general description does this formula belong? State the other forms that are included under the same name.

(2) In a spherical triangle C is a right angle; prove $\tan \frac{1}{2}(c+a) \tan \frac{1}{2}(c-a)=\tan^2 \frac{1}{2}b$.

15. (1) The axes being rectangular, find the angle between two intersecting straight lines whose equations are given.

(2) At what angle do the lines $2y+x+3=0$ and $3y-x+2=0$ intersect?

(3) Find the polar equation to the parabola, the focus being the pole. If PSp be any chord of a parabola drawn through the focus S, and if L be the *latus rectum* of the parabola, prove $4\,\text{SP}.\text{S}p=\dfrac{\text{L}}{2}(\text{SP}+\text{S}p)$.

16. (1) Define a differential coefficient; if $y=\sin^{-1}x$, find $\dfrac{dy}{dx}$.

(2) If $y=b\sin 2x+a\cos 2x$, prove $\frac{dy^2}{dx^2}+4y=0$.

(3) Find the coordinates of the centre of the circle of curvature, drawn at any point of a plane curve, to rectangular coordinates. Find these for any point of a parabola, and hence determine the evolute of the parabola.

17. Integrate the functions :

(1) $\int \frac{x^2dx}{(1+x^2)^{\frac{1}{2}}}$;

(2) $\int \frac{dx}{(\cos x)^4}$; and find the value of the latter integral between the limits $x=0$ and $x=\frac{\pi}{4}$.

18. Find the whole length of the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.

XXIX. (*a.*)

1. (1) Divide £71 2*s.* 2½*d.* by 107.

(2) Find the value of 3·458333 . . . shillings.

(3) Express £6·944 in rupees and decimal parts of a rupee, taking a rupee to be worth 2*s.* 4*d.*

2. Five brothers join in paying a sum of money. The eldest pays $\frac{1}{3}$ of it, and the others pay the remainder of it in equal shares, and thereby each of them pays £84 less than the eldest brother. What is the sum of money? (*Arithmetic.*)

3. A deduction is made on a debt of £1,373 6*s.* 8*d.*, and £1,308 2*s.* is accepted in discharge of it. At what rate per cent. is the deduction made ?

4. Solve the equations :

(1) $2\sqrt{x(x-4)}=1-4x$.

(2) $\left.\begin{aligned}(3x+y)^2-(3y+x)^2&=24\\ x^2+y^2&=5\end{aligned}\right\}$ (3) $\left.\begin{aligned}4x^2-6xy+y^2&=11\\ 3y^2-2xy&=14\end{aligned}\right\}$

5. A rectangular area of 8 ac. 480 yds. has a perimeter of 840 yds. Find its length and breadth.

6. Skilled workmen and labourers are employed on a

work, and a skilled workman receives 1*s.* 6*d.* a day more than a labourer. The average of their daily wages exceeds by $1\frac{1}{2}d.$ the sum which would be the average if skilled workmen and labourers were employed in equal numbers. If 6 men of each kind are discharged, the average of the daily wages is raised by $1\frac{1}{2}d.$ Find the number of men of each kind.

7. Employ logarithms to compute $\sqrt[3]{\cdot 038 \times (1\cdot 03847)^4}$ to 6 places of decimals.

8. If a, b, c are the roots of the equation $x^3+3x^2+1=0$, find the value of $a^2\left(\frac{1}{b}+\frac{1}{c}\right)+b^2\left(\frac{1}{a}+\frac{1}{c}\right)\times c^2\left(\frac{1}{a}+\frac{1}{b}\right)$

9. (1) Define a square.

(2) Prove that the parallelograms about the diameter of a square are squares.

(3) If the diagonal of a square is 1 ft. longer than the side, find the area of the square.

10. If A B, C D are parallel chords in a circle, and if their opposite extremities are joined by the straight lines A D, B C intersecting in O; prove that A O = B O and C O = D O. If C D = 2 A B, prove that A D and B C are trisected in O.

11. (1) From the definition of a cosine prove that $\cos 60^\circ = \frac{1}{2}$. Account for the tables, giving $9\cdot 69897$ as the logarithm of this cosine.

(2) Compute, by the tables, to 3 places of decimals the value of $\frac{\sin^3 A}{(1-\cos A)^2}$ when A is $18^\circ\ 40'$.

12. (1) Find to the nearest inch the length of the longest side of a triangle in which one side is 49 yds. and the angles at the end of this side $123^\circ\ 14'\ 28''$ and $40^\circ\ 27'$.

(2) Compute the area of a regular pentagon of which each side is 250 yds. long.

13. If C be the centre of a sector C A B of a circle, of which the radii C A, C B, are each 23 in. long, and the straight line A B is 14 in., find the area of the sector.

14. The rectangular coordinates of two points being (3, 5)

and (4,4) respectively, find the equation to the straight line which bisects at right angles the straight line joining these points.

15. Define a parabola, and thence deduce its polar equation with the focus for a pole. Deduce the length of a chord of the parabola which passes through the focus, and is inclined at 60° to the axis of the curve.

16. Find the maximum and minimum values which the expression $\sin^3 \theta \cos \theta$ admits, whilst θ ranges from 0° to 180°.

17. Find the equation to the normal of the curve $y^2 = \dfrac{x^3}{2a-x}$ at the point $x=a$.

18. (1) Integrate the expressions :

$$\text{(i)}\ \frac{dx}{x^2-3x+2}. \quad \text{(ii)}\ \frac{dx}{1+\sqrt{x}}. \quad \text{(iii)}\ xa^{3x}dx.$$

(2) Evaluate $\int_0^\pi x \sin x \, dx$.

(3) Find the area of the curve $r=a \sin 4\theta$.

XXX. (*a.*)

1. (1) Reduce to a single fraction $\dfrac{\frac{4}{5}-\frac{16}{25}+\frac{24}{125}}{\frac{2}{3}-\frac{2}{9}+\frac{52}{81}}$, and express the result as a decimal.

(2) If 200 men can dig a trench in 12 hrs., what number of men would be required to dig a trench 3 times as long in $\frac{1}{5}$ of the time?

2. A contractor, after buying a flock of 500 sheep, found that by selling each sheep at £3 13*s.* 6*d.* he could realise a profit of $22\frac{1}{2}$ per cent. on his purchase. What did his flock cost him?

3. (1) Explain why, if $\log_{10} N$ be given, the logs of all numbers, to the same base, that result from multiplying or dividing N by powers of 10, may be found by changing the characteristics of the logs only.

(2) Find, by the aid of tables, $\dfrac{(12{\cdot}45)^{\frac{2}{3}}}{({\cdot}00746)^{\frac{1}{6}}}$;

(3) The value of x^3-2x-4 when $x=2{\cdot}0946$.

4. (1) Find the L. C. M. of $x^4-(a^2+b^2)\,x^2+a^2b^2$, $x^4-(b^2+c^2)\,x^2+b^2c^2$, $x^4-(a^2+c^2)\,x^2+a^2c^2$.

(2) Prove $\dfrac{x^2-(y-z)^2}{(x+z)^2-y^2}+\dfrac{y^2-(z-x)^2}{(x+y)^2-z^2}+\dfrac{z^2-(x-y)^2}{(y+z)^2-x^2}=1$.

5. Solve the equations:

(1) $\dfrac{1+x^3}{(1+x)^3}=\dfrac{13}{25}$.

(2) $\dfrac{1}{\sqrt{a-x}+\sqrt{a}}+\dfrac{1}{\sqrt{a+x}-\sqrt{a}}=\dfrac{\sqrt{a}}{x}$.

(3) $x^3-10x^2+31x-30=0$.

6. (1) Sum the series a, $(a-b)\,r$, $(a-2b)\,r^2$, $(a-3b)$ to n terms.

(2) If a be the first term, r the common ratio, n the number of terms, S the sum, S_2 the sum of the squares of the terms of a geometrical progression, prove $S_2\,(r+1)-S^2\,(r-1)=2aS$.

7. (1) In the expansion of $(a+x)^n$, n, a positive integer, show that the coefficient of the second term is n, of the third term $\dfrac{n(n-1)}{1\,.\,2}$, and the number of terms $n+1$.

(2) Find the coefficient of $x^2y^2z^3$ in $(x+y+z)^7$.

8. (1) If in any quadrilateral figure ABCD the opposite sides be equal, viz. AB=CD and BC=AD, show that the figure is a parallelogram; and if the diagonals be equal, show that the parallelogram is rectangular.

(2) Prove that the sum of the angles in the four segments of a circle exterior to the sides of an inscribed quadrilateral are together equal to six right angles.

(3) If the side of a regular hexagon be 6 in., what will

be the length of the side of the equilateral triangle having the same area?

(4) A pyramid has an equilateral triangle for its base; each of three plane angles at its vertex is a right angle; if h be the altitude of the pyramid, a a side of the base, prove $h^2 = \dfrac{a^2}{6}$.

9. (1) Prove $\cos A + \cos (180° - A) = 0$.

(2) Find $\sin 18°$.

(3) Prove $\cos^2 36° + \sin^2 18° = \frac{3}{4}$.

(4) Assuming $\tan 2A = \dfrac{\sin 2A}{\cos 2A}$ prove $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$.

(5) Prove $\tan\left(45° + \dfrac{A}{2}\right) + \cot\left(45° + \dfrac{A}{2}\right) = 2 \sec A$: if $A = 225°$, what are the proper signs of $\tan\left(45° + \dfrac{A}{2}\right)$, $\cot\left(45° + \dfrac{A}{2}\right)$, and $\sec A$?

10. (1) In any triangle, if a, b, c are the sides subtending the angles A, B, C respectively, show that $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$; and thence prove that

$$c = (a - b) \cos \frac{C}{2} \sqrt{1 + \left(\frac{a + b}{a - b} \tan \frac{C}{2}\right)^2}.$$

(2) If $a = 161{\cdot}25$ ft., $b = 100$ ft., $C = 48° 24'$, determine by the aid of this formula the value of c.

11. During the war, a steam privateer was watching a harbour from a distance of $13\frac{1}{2}$ miles, and in a position SW. of the harbour. A merchantman sails from the harbour in a direction SE., at the rate of 9 mi. an hour: in what direction and at what rate must the privateer steam to come up with the merchantman in 2 hrs.?

12. (1) A field is in the form of a sector of a circle, its angle is 125° 15′, and radius 100 ft.; find the length of the arc of the circle, and its area.

(2) If a pony be tethered by a rope fastened to the

angle, determine the length of the rope that he may graze over exactly half the field.

3. (1) In spherical trigonometry, show that the sides of the polar triangle are the supplements of the angles of the polar triangle.

(2) In a right-angled spherical triangle ABC, C being the right angle, given the angle A and the side BC, proceed to solve the triangle: if A and BC are both acute, what condition must hold to make the triangle possible? Show, in this case, how there may be an ambiguity in the solution.

4. (1) If $x^2+y^2=r^2$ be the equation to a circle, find the equation to its tangent line at any point (x_1, y_1), and show that if the straight line $\frac{x}{a}+\frac{y}{b}=1$ touches the circle, then $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{r^2}$.

(2) Find the polar equation to an ellipse, one of the foci being the pole. If P S p be a focal chord of an ellipse, S the focus, SL half the *latus rectum*, prove $\frac{1}{\text{SP}}+\frac{1}{\text{S}p}=\frac{2}{\text{SL}}$.

5. (1) If $a=f(x)$, can $f(x+h)$ be always expanded in a series ascending by regular and integral powers of x when x retains its general value? Point out when Taylor's Theorem is said to fail for a particular value of x, and give an example.

(2) Expand $\sin^{-1}(x+h)$ to three terms by Taylor's Theorem.

6. Find the length of the normal drawn at any point of lane curve. If PG be the normal at any point P of an ›se, SP, HP the focal distances, a, b the semi-axes, major minor respectively, prove that

$$\text{PG}^2 : \text{SP.HP} :: a^2 : b^2.$$

17. Find $\int \frac{ax}{(x^2-a^2)^2}$.

18. Find the surface of a sphere included between two parallel planes drawn on different sides of the centre and at distances from the centre equal to half the radius.

ROYAL MARINES PAPERS.

1. Multiply 3 cwt. 2 qrs. 17 lbs. by 14, and divide the result by 35.

2. If 10 oz. of silk can be spun into a thread 5 furlongs long, what weight of silk would supply a thread sufficient to reach to the moon, a distance of 240,000 miles?

3. What must be the breadth of a room whose length is $31\frac{1}{2}$ ft., that the area may contain 46 sq. yds.?

4. If a person's estate be worth £692 8*s.* a year, and the land tax be assessed at 2*s.* $9\frac{1}{2}$*d.* in the pound, what is his net annual income?

5. If I lay out £4,000 in the 3 per Cent. Consols when they are at $88\frac{1}{2}$, what income shall I thence derive?

6. If $6\frac{5}{8}$ oz. of tea cost $12\frac{3}{8}$*s.*, what will $30\frac{3}{7}$ lbs. cost?

7. Give definitions of the following terms: '*common multiple*,' '*common measure*,' '*vulgar fraction*,' '*decimal fraction*.'

8. Divide 647·2352 by 12·5, and 3·1475 by ·00125.

9. Find the value of $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}$.

10. Extract the square root of 12345, and also of 234·135.

11. Find the value of $x-\{yz-[x^2y^2-4y^3z-(2xyz+3xy)-y]\}$ when $x=1$, $y=2$, $z=3$.

12. (*a*) Multiply together $3b+4a$, $4b+3a$, $3b-4a$, and $4b-3a$.

(*b*) Divide $a^4-4a^2b^2+4ab^3-b^4$ by $a^2-2ab+b^2$.

(*c*) Find elementary factors of $a^2+b^2-c^2-2ab$;

(*d*) Also of $a^2+b^2+c^2-2ab+2bc-2ac$.

3. (*a*) Simplify $\frac{21y^2-12x^2+9xy}{7ay-4ax+14by-8bx}$;

(*b*) Also $\dfrac{\dfrac{x-y}{x+y}+\dfrac{x+y}{x-y}}{\dfrac{x-y}{x+y}-\dfrac{x+y}{x-y}}$.

4. Solve the following equations:

(*a*) $\frac{x}{9}-\frac{2x}{4}+\frac{3x}{6}=\frac{5x-10}{8}-8.$

(*b*) $amx-\frac{nx}{b}=bx-c.$

(*c*) $\left\{\begin{array}{c}\frac{4y-3x}{9}-\frac{3y-5\frac{1}{3}x}{11}=\frac{x}{6}-\frac{47}{9}\\ 4x-5y=4.\end{array}\right\}$

5. A and B have equal incomes; A's annual expenditure o B's as 6 : 5; at the end of two years A has saved ?0, which is $\frac{3}{5}$ of B's savings: what is the income of h?

6. The diagonals of a parallelogram bisect each other; parallelogram whose diagonals are equal is rectangular.

7. The square of the sum of two lines is equal to four es the rectangle contained by them, together with the are of their difference.

8. The centres of the inscribed and circumscribed circles ın equilateral triangle coincide, and the diameter of one ouble that of the other.

9. If two straight lines are at right angles to the same ne, they shall be parallel to one another.

:0. Solve the following equations:

(*a*) $x-\frac{x-9}{\sqrt{x}+3}=15.$

(*b*) $x^2-2x+6\sqrt{x^2-2x+5}=11.$

!1. Given log $3=\cdot4771213$; find log $\cdot000027$ without ɜrence to the tables.

Find, by the aid of the tables,

(*a*) The fifth root of 14·52 ;

(*b*) The cube root of ·000048.

22. What is the expression for the volume of an oblate spheroid?

Find the volume of a prolate spheroid whose diameters are 40 and 50 inches respectively.

23. Give diagrams and explanations of positive and negative straight lines and angles; and find the value of cot (-270°).

24. (*a*) Prove $\frac{\cot x+\tan x}{\cot x-\tan x}=\sec 2x$.

(*b*) Prove $\sin^2 x-\sin^2 y=\sin(x+y)\sin(x-y)$.

25. In any plane triangle ABC, find cos A in terms of the sides.

26. Prove that the sines of the sides of a spherical triangle are proportional to the sines of the opposite angles.

27. Define '*weight*,' '*mass*,' '*accelerating force*,' '*moving force*;' show how to compare the effects of projectiles of different weights, moving with different velocities, on an immoveable object.

28. A heavy beam (weight$=w$, length $=l$) rests on a peg, with its lower end against a smooth vertical wall; find the angle made by the beam with the horizon when there is equilibrium.

29. A person jumped off the monument horizontally with a velocity due to a height of 2 ft.; he fell 40 ft. from its base: find the height of the monument.

30. Find the common velocity of two inelastic bodies after direct impact.

STAFF COLLEGE, ENTRANCE EXAMINATION.

1. What is the cost of 3 cwt. 2 qrs. 10 lbs. at 2*s*. $4\frac{3}{4}$*d*. per pound?

2. A fraudulent tradesman sells a piece of cloth for $31\frac{1}{2}$ yds., at 19*s*. 6*d*. a yd.; what is his profit from having used a yard measure one inch too short?

3. If 5 compositors set up 12 sheets of 32 pages, each page containing 45 lines of 60 letters, in 21 days, how many compositors will set up 20 sheets of 30 pages, each containing 54 lines of 64 letters, in 35 days?

4. Distinguish between simple and compound interest. Find the simple interest on £440 for 7 months, at the rate of $3\frac{3}{4}$ per cent. per annum.

5. Explain the rule for the multiplication of two vulgar fractions; and show that if each of the fractions be less than unity their product must be less than either of them.

   Multiply $\frac{3}{5}+\frac{20}{39}-\frac{7}{117}$ by $\frac{3}{7}+\frac{5}{8}-\frac{8}{77}$.

6. State the rule for the division of decimal fractions, when the number of decimal places in the divisor exceeds the number in the dividend.

   Divide ·34935 by ·000137, and verify the result by vulgar fractions.

7. A rectangular tank is 13 ft. 6 in. long by 9 ft. 9 in. wide; how many cubic yards of water must be drawn off to make the surface sink a foot?

8. Reduce ·025 of 25*s*. + ·063 of 100 guas. − $3\frac{7}{8}$ of 13*s*. 4*d*.

9. Extract the square roots of—
   (1) 3249·456016;
   (2) 17 to 4 decimal places. What is the true numerical value of the remainder in the latter example?

10. Perform the operations indicated in the following examples:

(1) $\left(\frac{2a}{3}-\frac{3b}{4}-c\right)-\left(\frac{a}{6}-\frac{2b}{3}+2c\right)$.

(2) $(1+x-2x^2-3x^3)\,(1+x+2x^2+3x^3)$.

(3) $(a^2+2ab+b^2-c^2+2cd-d^2)\div(a+b-c+d)$.

11. Prove—

(1) $\frac{(ac+bd)^2+(bc-ad)^2}{(a^2+b^2)\,(c^2+d^2)}=1$.

(2) $\frac{6a^2+2ab-20b^2}{21a^2-29ab-10b^2}=\frac{2a+4b}{7a+2b}$.

12. Solve the equations:

(1) $\frac{3x+5}{8}-\frac{21+x}{3}=30-5x$.

$7x+3y=11$.

(2) $3xy=(x+1)\,(3y+1)$.

13. A cistern into which water is supplied by two pipes, A and B, will be filled by both of them running together in 24 hrs., and by A alone in 40 hrs.; in what time will it be filled by B alone?

14. State and examine a rule for determining the sum of the interior angles of any rectangular figure.

15. The semicircle described upon the hypothenuse of a right-angled triangle is equal to the sum of the semicircles described upon the sides.

16. If AB, CD be the opposite sides of a quadrilateral figure circumscribing a circle whose centre is O, prove the angle AOB equal to the two angles ODC, OCD, taken together.

17. Prove—

(1) $\frac{1}{(a^2-b^2)(x^2+b^2)}+\frac{1}{(b^2-a^2)(x^2+a^2)}$

$=\frac{1}{(x^2+a^2)(x^2+b^2)}$.

(2) If $\sqrt{ax}+\sqrt{by}+\sqrt{cz}=0$, $a^2x^2+b^2y^2+c^2z^2$ $=2\,(abxy+acxz+bcyz)$.

18. Solve the equations:

(1) $x^{\frac{7}{3}}=56x^{-\frac{2}{3}}+x^{\frac{5}{6}}$.

(2) $xyz=48,\ \frac{x}{yz}=\frac{1}{12},\ \frac{xy}{z}=\frac{4}{3}.$

19. Write down the first five terms of the expansion of $(1+x)^{2x}$, and show, when n is a whole number, that the sum of the coefficients of the expansion $=(2^n)^2$. Also assuming the $(p+1)$th coefficient of the expansion equal to the $(p+3)$th coefficient, prove $p=n-1$.

20. Find—

(1) The base of the system of logarithms in which $\cdot\dot{6}$ is the logarithm of 25;

(2) By the tables, a mean proportional between $\sqrt[3]{\cdot 04}$ and $(\cdot 0004267)^4$;

(3) The sum of the series 1, $\frac{19}{20}$, $(\frac{19}{20})^2$, $(\frac{19}{20})^3$, &c. to 100 terms.

21. In any plane triangle ABC:

(1) $(a+b)\cos C+(a+c)\cos B+(b+c)\cos A= a+b+c.$

(2) Radius of circumscribed circle

$$=\frac{1}{2}\left(\frac{abc}{\sin A \sin B \sin C}\right)^{\frac{1}{3}}.$$

22. Wishing to know the distance of an inaccessible object P, and having no instrument for measuring angles, I measured a horizontal base AB, 400 ft., and from A to B, in a line directly away from P, I measured AC, 300 ft., BD, 450 ft.; I also measured BC, 500 ft., and AD, 750 ft.; find PA or PB.

23. The elevation of a tower standing on a horizontal plane is observed; on advancing 240 ft. nearer to it, the angle of elevation is found to be the complement of the first observed angle, and on advancing 100 ft. nearer still to the tower, its angle of elevation is found to be double the first. Find the height of the tower.

If the given distances be a and b, between what limits must the first angle of elevation lie, that the angular data, as above, may be possible.

24. It being given that the ratio of the circumference of

a circle to its diameter is 3·1416 to 1, and that 30 c. in. of gunpowder weigh 1 lb., find the internal diameter of a spherical shell that will be just filled by 37·6992 lbs. of powder.

25. A string without weight having its extremities fastened to the ends of a uniform bar weighing 10 lbs. passes over 4 tacks so as to form with the bar a regular hexagon, the bar being horizontal; find the tension of the string, and the vertical pressure on each tack.

26. If α be the angle of inclination of a rough inclined plane, β the angle which the direction of the force P supporting a weight W makes with the plane, θ the limiting angle of friction, show that when P is just on the point of moving W up the plane $P = \frac{W \sin(\alpha+\theta)}{\cos(\beta-\theta)}$.

27. If a body be projected perpendicularly downwards with a velocity V, obtain an equation for determining the space it will describe in the time t.

With what velocity must a body be projected downwards that in n'' it may overtake another body that has already fallen through a feet?

28. In a wheel and axle, P hanging vertically at the wheel draws up Q hanging vertically at the axle; find the accelerating force on P, the inertia of the machine being considered, but not the friction.

29. A weight of 12 lbs. is suspended at the circumference of a wheel whose radius is 2 ft., and a weight of 20 lbs. at the axle whose radius is $\frac{1}{2}$ ft.; find the accelerating force on the descending weight, if the weight of the wheel and axle be 4 lbs.

30. If a fluid issue into a vacuum through an extremely small orifice in the bottom of a vessel kept constantly full, find the velocity of the issuing fluid.

A cylindrical vessel filled with fluid rests with its base on a horizontal plane; find the position of an orifice so that the range of the issuing fluid on the plane may be the greatest possible.

STAFF COLLEGE, FINAL EXAMINATION.

1. Explain the terms: '*vertical line*,' '*vertical plane*,' '*horizontal plane*,' '*angle of elevation*,' '*angle of depression*,' '*dip of the horizon*.'

2. In a right-angled triangle ABC, C the right angle, given BC=36·25 ft., AC=42·15 ft.; solve the triangle.

3. In a triangle in which each angle at the base is double of the third angle, given the base 100 ft.; solve the triangle.

4. Given the sides of a triangle 50·25, 40·5, 29·75 ft.; find the greatest angle.

Is there any formula for determining the angle in terms of the sides which it would be unadvisable to employ?

5. The angle of elevation of a tower 400 ft. high, when due N. of the observer, was 55° 14′; what will the elevation be when the observer has walked 720 ft. due E. on the horizontal plane?

6. A railroad AB runs N., and a railroad AC, NE. from A. A train starts from A along AB with a velocity of 25 mi. an hr.; after half-an-hour a train starts from A along AC with a velocity of 30 mi. an hr.; find the distance of the trains from each other, three hours after the first train has started.

7. From a boat on a river, the angle of elevation of the top of a column on the bank was observed to be 32° 15′, and the angle subtended by the top of the column and a boat down the river was 48° 12′; after sailing past the column towards the barge for a distance of 480 yards, the observer in the boat found that the angle subtended by the top of the column and the first position of the boat was 20° 20′: find the height of the column.

8. A person wishing to determine the length of an

inaccessible wall, places himself due S. of one end and then due W. of the other, at such distances that the angle which the length of the wall subtends at each station is 30° ; find the length of the wall, the distance between the stations being 120 yds.

9. Four points A, B, C, D are in the same horizontal plane; if the distance CD be known, show how to determine AB by means of angles measured at A and B.

10. Examine and illustrate the following general principle: 'The number of units of work yielded by any agent in a given time is the true measure of its efficiency or working power.'

A weight of 3 tons is to be raised from a depth of 50 fathoms in one minute ; determine the horse-power of the engine capable of doing the work.

11. If P, Q, R are three pressures in equilibrium about a point A in one plane, establish the relation

$$R^2 = P^2 + Q^2 + 2\,PQ \cos PAQ.$$

Two forces which are to each other as 2 : $\sqrt{3}$ when compounded give a resultant 1; find the angle at which the directions of the forces are inclined to one another.

12. Show how the moment of a force with respect to a point may be represented by an area; prove that the moment of the resultant of any two pressures whose directions are not parallel, is equal to the algebraical sum of the moments of those pressures with respect to that point, taking the case in which all the moments are not positive.

13. Find the centre of parallel pressures of any system, all the pressures acting in a plane.

Hence show that every system of heavy particles has one and only one centre of gravity.

Squares are described on the three sides of a right-angled isosceles triangle; prove that the distance of the centre of gravity of the whole figure including the triangle, from the hypothenuse c, is equal to $\frac{c}{54}$.

14. If a surface be described by the revolution of a plane curve round a fixed axis, show how to determine the area of the surface described.

It being given that the surface of a sphere is equal to the area of 4 great circles of the sphere, find the centre of gravity of the area of a semicircle.

15. When one rough surface rests upon another, explain what is meant by the angle of resistance; and show how it may be determined for two substances.

A mass of wrought iron weighing 500 lbs. rests on a plank of oak inclined at an angle of 20° to the horizon; a pressure acts at an angle of 12° to the plane to pull the weight up the plane, and it is in a state bordering on motion; find the pressure, it being given that the limiting angle of resistance for wrought iron upon oak is 31° 50′.

16. A wheel and axle weigh 1 cwt., the radius of the wheel is 2 ft., of the axle 6 in., of the diameter of the axis 1 in., and the angle of resistance for the axis and its bearing 5° 40′; find the magnitude of the weight which hanging from the circumference of the wheel will just support a weight of 1,000 lbs. hanging from the circumference of the axle.

17. Define a '*parabola*,' '*its axis*,' '*directrix*,' '*latus rectum*.' Show that the tangent at a given point of a parabola bisects the angle between the focal distance and the perpendicular drawn from the point to the directrix.

18. How is accelerating force estimated in dynamics? If a body be projected vertically downwards with a given velocity, find the space it will describe in a given time under the action of gravity.

If the initial velocity be 64 ft. a second, find the space which the body will describe *in vacuo* in the 5th second, the force of gravity being taken at 32 ft.

19. If a heavy body is dragged up an inclined plane, show that the units of work expended will equal the num-

ber that would be expended in dragging it along the base, supposed equally rough, and in lifting it vertically through the perpendicular height.

A train weighs 40 tons, the friction is 7 lbs. a ton; determine the horse-power of the engine that will draw the train 10 miles in 25 minutes with a uniform velocity up an inclined plane that rises 1 ft. in 200.

20. Explain generally how to determine the amount of pressure produced on the summit of a wall that supports an isosceles roof.

A roof weighs 25 lbs. a sq. ft., the pitch is 60°, the distance between the walls 30 ft.; determine the magnitude of the pressure on the wall at the extremity of a rafter, the rafters being 5 ft. apart.

21. Find the time of flight of a projectile on an inclined plane passing through the point of projection.

22. If there are two inclined planes, and the angle between them is bisected by the horizontal plane, and if the times of flight on the two inclined planes are t_1 and t_2, and on the horizontal plane t, prove that $t_1+t_2=2t$, the velocity and angle of projection being the same in each case.

23. Explain the construction of the cycloid; and show how to cause a point suspended by a perfectly flexible string to oscillate in a cycloid.

24. Show how to find the number of oscillations which a seconds pendulum will lose in a day when lengthened by a given small quantity: apply the result when the pendulum is increased by $\frac{1}{100}$ of its length.

25. How is the work accumulated in a moving body estimated?

A train is moving at the rate of 40 miles an hour when the steam is turned off; how far will it ascend an incline of 1 in 100, taking the friction at 8 lbs. a ton?

26. How is the fluid pressure at any point of a rigid body, *in* contact with it, estimated—

(1) When the pressure is constant ?

(2) When it is variable ?

27. A piston has a radius of 4 in. and is in contact with eam on its under surface, exerting a pressure of 30 lbs. on .e sq. in.; find the pressure downwards on the piston :cessary to keep it at rest.

28. What is the centre of pressure on any plane surface ımmersed in a heavy fluid? Show how to find it when the rface is a rectangle, one side of which is horizontal, and incident with the surface of the fluid.

29. What distinctive property is there between liquids d gases ?

If v be the volume of gas at a temperature t, v_1 the volume at a temperature t_1, investigate the formula $v_1 = \frac{460+t_1}{460+t} \cdot v$.

If 200 c. in. of gas, whose temperature is 60° and pressure 30 in., be raised to 280°, and its pressure reduced to 20 in., calculate its volume.

30. Explain the action of the common suction-pump ; d show what is the theoretical limit of the height to which e water is raised by it.

If the diameter of the piston be 2 in. and the height of the water in the head of the pump be 24 ft. above the well, what pressure does the piston bear?

ANSWERS.

ABBREVIATIONS EMPLOYED IN REFERENCES.

AR. . . .	Colenso's Arithmetic, 1863.
ALG. I. . .	Colenso's Algebra, Part I., 1862.
ALG. II. . .	Colenso's Algebra, Part II., 1863.
PL. TRIG. .	Colenso's Plane Trigonometry, 1863.
(III. 20) . .	Euclid, Book III., Proposition 20.
CON. SEC. .	Salmon's Conic Sections, 1864.
CALC. .	Hall's Differential and Integral Calculus, 1864.
PRAC. MEC.	Twisden's Practical Mechanics, 1860.
EL. MEC. .	Parkinson's Elementary Mechanics, 1861.
EL. HYD. .	Besant's Elementary Hydrostatics, 1863.
MEC. PHIL.	Tate's Mechanical Philosophy, 1853.

ANSWERS.

I.

(3) 1. £7652194 0*s.* 9*d.* (4) 1. $2\frac{31}{36}$; 2. $\frac{64}{105}$; 3. 14645·61; 4. 7·8125.
(5) £144 11*s.* 3*d.* (6) £767. (7) £184 14*s.* 8*d.*
(8) £4502 1*s.* $9\frac{1}{4}$*d.* (9) $2\frac{1}{4}$. (10) £36 13*s.* 1·44*d.*
(11) 8064. (12) $36159\frac{3}{4}$ hrs.

II.

(2) 1. £9897 5*s.* 10*d.*; 2. £1238 3*s.* $11\frac{3}{4}$*d.*
(3) 1. 506722; 2. 1358880. (4) 1. $\frac{20}{21}$; 2. $1\frac{17}{60}$.
(5) 1. 7782·82521; 2. 3011·5026; 3. 5424·8324832; 4. 397543.
(6) £51 1*s.* $8\frac{1}{2}$*d.* (7) £7144 7*s.* 6*d.*
(8) 1. £179 17*s.* $10\frac{3}{4}$*d.*; 2. £570 8*s.* $5\frac{3}{4}$*d.*
(9) £38 2*s.* (10) £214 3*s.* 4*d.* (11) £4125. (12) $106\frac{2}{3}$ yds.

III.

(2) 1. £1053462 17*s.* $7\frac{1}{2}$*d.*; 2. £2328 6*s.* $4\frac{1}{4}$*d.*
(3) 1. $37\frac{1}{2}$ lbs.; 2. 3055709″. (4) $\frac{267}{385}$.
(5) 2. 2·6991408. (6) 2*s.* $2\frac{1}{2}$*d.* (7) £7.
(8) £343 12*s.* $4\frac{3}{4}$*d.* (9) £17 10*s.* (10) 45 mi. an hr.
(11) 3*s.* 6*d.* (12) 20 hrs.

IV.

(2) 2. 340674. (3) £2282 11*s.* $6\frac{3}{4}$*d.*
(4) 1. £180892 15*s.* $0\frac{3}{4}$*d.*; 2. £862287 5*s.* $7\frac{1}{2}$*d.*
(5) 1. £12204 14*s.* $2\frac{1}{2}$*d.*; 2. £78 4*s.* $11\frac{3}{4}$*d.*
(6) £414 13*s.* $0\frac{1}{2}$*d.* (7) 1. 673328; 2. 192669.
(8) 2 cwt. 3 qrs. 14 lbs. (9) $4\frac{27}{32}$ oz.
(10) 1. £1060 6*s.* $7\frac{1}{4}$*d.*; 2. £419 3*s.* $4\frac{1}{2}$*d.*
(11) £293 8*s.* 5·4*d.* (12) 4.

V.

(2) 2. £558308 19*s.* $0\frac{3}{4}$*d.* (3) £87 13*s.* $8\frac{3}{4}$*d.*
(4) 1. 3 yds. 20 ft. 1365 in.; 2. 46 yds. 5 ft. $127\frac{1}{2}$ in.
(5) 1. $16\frac{4}{5}$; 2. 26. (6) $\frac{16}{27}$.
(7) 32. (8) 1. £343 12*s.* $4\frac{3}{4}$*d.*; 2. £1281 19*s.* $0\frac{3}{4}$*d.*
(9) 1. ·015625; 2. ·4255952380. (10) £·46553.
(11) £94 3*s.* 4*d.* (12) £750.

VI.

(1) 1. $10\frac{1}{25}$; 2. £3 10*s*. (2) 8203·6; 3·12834; 13400; ·00134
(3) 1. ·1041$\dot{6}$; 2. $2\frac{89}{2000}$; 3. $\frac{1}{1600}$; 4. $\frac{13}{90}$; 5. $\frac{8546}{24975}$; 6. ·09375; 1·$\dot{2}\dot{1}$.
(4) $45\frac{1}{2}$. (5) 10. (6) 18 lbs. 8 oz.
(7) £269 19*s*. $8\frac{1}{4}$*d*. (8) £1411 9*s*. 2*d*. (9) $4\frac{1}{2}$ yrs.
(10) 10. (11) £40 8*s*. (12) £16206 15*s*.

VII.

(1) $\frac{8}{9}$. (2) 1. $\frac{11}{2400}$; 2. $22\frac{2}{9}$; 3. £5 2*s*. 5*d*.
(3) 19801·12156. (4) 1. ·28182; 2. 10556·4673.
(5) 17526. (6) 24 ft. 9 in. (7) 30 oz.
(8) $36\frac{2}{5}$ lbs. (9) £50 2*s*. 3*d*. (10) £260.
(11) 8 yrs. (12) £173 17*s*. 6*d*.

VIII.

(1) 2894 f. 3 c. (2) £8 11*s*. (3) £223 11*s*. 3*d*.
(4) £19127. (5) £188 10*s*. (6) 9 hrs.
(7) 12800. (8) £100. (9) 2340 lbs.
(10) £15727 16*s*. 8*d*. (11) 1. $\frac{7}{4000}$; 2. $\frac{36}{275}$. (12) $46\frac{29}{40}$.

IX.

(1) 1. 78256; 2. 29 mi. 1 fur. 63 yds. 10 in.; 3. 2 yds. 26 ft. 57·in.
(2) £36 10*s*. $2\frac{1}{4}$*d*. (3) 1. $1\frac{7}{8}$; 2. £3 12*s*. 2*d*.
(4) ·078125; ·$\dot{2}$14285$\dot{7}$; $23\frac{61}{1800}$; $\frac{11}{16000}$. (5) ·148809523···; ·[illegible].
(6) $\frac{67}{495}$; $\frac{17}{1375}$. (7) 17*s*. 6*d*. (8) 45 days.
(9) £36 18*s*. 1*d*. (10) $4\frac{1}{4}$. (11) 140 yds.
(12) 1. 729 ; 2. $5\frac{2}{3}$.

X.

(1) 1. $\frac{7}{8}$; 2. $\frac{46}{285}$; 3. $11\frac{4}{5}$*d*. (2) 1. ·7625; ·07392; 2. 32·9415; 500
(3) 101. (4) 2 hr. $3\frac{3}{7}$ mi. (5) $22\frac{6}{7}$.
(6) 1. £370703 2*s*. 6*d*.; 2. £31907 9*s*. 11*d*. (7) £438 8*s*. ·165*d*.
(8) £12. (9) £270 2*s*. 5·28*d*. (10) 10*s* 6*d*.; $8\frac{24}{127}$.
(11) 327; 28·349. (12) Ar. p. 1.

XI.

(1) 2. 36430031911. (2) 85001997; 885768804.
(4) 2. £351154 5*s*. $10\frac{1}{2}$*d*. (5) 60. (6) 112640.
(7) £74 7*s*. 6*d*. (8) 1. £132 2*s*. $3\frac{171}{320}$*d*.; 2. £625 19*s*. $0\frac{3}{8}$*d*.
(9) £7 15*s*. $2\frac{2}{5}$*d*. (10) $3\frac{113}{229}$.
(11) 1. £4 2*s*. $1\frac{33}{35}$*d*.; 2. 970·57005897. (12) 40000.

XII.

(1) 1. £1199679 6*s*. 3*d*.; 2. 45 mi. 7 f. 25 p. 2 yds.
(2) $47\frac{2}{3}$ ft. (3) 14 mo. (4) £12 14*s* $9\frac{11}{12}$*d*.
(5) $\frac{1017}{1955}$. (6) £2338 3*s*. (7) ·0$\dot{6}$.
(8) 6*s*. 11·04*d*.; 21 da. 4 hr. 25 m. 40·8 sec.
(9) 1. ·107151; 2. 23·456. (10) £500.
(11) £35049 12*s*.; £31782 8*s*.; £105148 16*s*.; £128515 4*s*.
(12) 2256 dol. 96 c.

XIII.

(1) £6 5*s*. $8\frac{1}{4}$*d*. (2) 141 tons 16 cwt. 8 oz.
(3) £10. (4) $13\frac{1}{5}$ mi. (5) Ar. p. p. 38, 45.
(6) 1. $3\frac{13}{30}$; 2. $3\frac{4}{57}$; 3. $33\frac{1}{4}$. (7) 21 times. (9) 15 days.
(10) $3\frac{3}{4}$ wks. (11) £601 11*s*. $10\frac{1}{2}$*d*. (12) 232 yds.

XIV.

(1) 2. 125550; Remainder = 79. (2) £10656 5*s*. (3) 23.
(4) 2 ro. 27 po. (5) 1. $12\frac{11}{20}$; 2. 2; 3. $\frac{79}{82}$; 4. $\frac{13}{77} > \frac{14\frac{1}{2}}{88}$ by $\frac{5}{1232}$.
(6) 1. $\frac{1}{2}$; 2. $\frac{3}{10}$. (7) 1. £1156 0*s*. $7\frac{1}{2}$*d*.; 2. £30 19*s*. 8*d*.
(8) £513. (9) 282 cwt. 2 qrs. (10) 115 ft.
(11) £40 10*s*.; £46 10*s*.; £67 10*s*. (12) 2907.

XV.

(1) 5112 tons 18 cwt. 2 qrs. (2) 14 da. 1 h. 4 m. 19 sec.
(3) 4200 cr. = 10500 fl. = 84000 3*d*. pieces.
(4) £1 5*s*. 3*d*.; and £309 6*s*. 3*d*. (5) 960.
(6) Ar. p. p. 38, 39, 40. (7) 1. $11\frac{865}{432}$; 2. $\frac{1}{6}$; 3. 18; 4. $\frac{3}{[illegible]}$.
(8) £46 3*s*. 10*d*. (9) £543 18*s*; £207 11*s*. $10\frac{1}{2}$
(10) 4*s*. $0\frac{3}{4}$*d*. (11) 150 ft. (12) 96.

XVI.

(1) 1907920; 105536. (2) £548 3*s*. $4\frac{1}{2}$*d*. (3) £5 14*s*. 5*d*.
(4) £1 12*s*. $9\frac{1}{2}$*d*. (5) 10710. (6) $\frac{16}{17}$; $\frac{1}{3}$.
(7) 1. $21\frac{31}{36}$; 2. $2\frac{53}{180}$. (8) $48\frac{15}{44}$. (9) £34 12*s*. $10\frac{2}{15}$*d*.
(10) £12 2*s*. $3\frac{193}{320}$*d*. (11) 90 miles. (12) $47\frac{237}{829}$.

XVII.

(1) 67168. (2) 51964; 1643. (3) 75 tons 5 cwt. 1 qr. 25 lbs.
(4) £2 0*s*. 10*d*. (5) 2331; 19 m. 3 fur. 193 yds. 1 ft. 7 in.
(6) 36. (7) 1. $\frac{3}{11}$; 2. $3\frac{13}{36}$; 3. 5; 4. $\frac{29}{76}$.
(8) $\frac{94}{30129}$; 14*s*. $9\frac{3}{4}$*d*. (9) £119 0*s*. $7\frac{7}{20}$*d*.
(10) £6 8*s*. 4*d*. (11) 27 days. (12) 10800 feet.

XVIII.

(1) 154096 oz.; 396 ells. (2) £3 3*s*. $3\frac{3}{4}$*d*.
(3) £8 6*s*. 8*d*.; £6 6*s*. 8*d*.; £5 6*s*. 8*d*. (4) 119; 18480.
(5) Ar. p. 41.; $\frac{13}{11}$. (6) 1. $10\frac{97}{120}$; 2. 80; 3. $31\frac{26}{35}$.
(7) 10*s*. $1\frac{1}{4}$*d*. = £$\frac{97}{193}$. (8) 1. £1246 16*s*. $6\frac{3}{4}$*d*.; 2. £15 5*s*. $4\frac{1}{2}$*d*.
(9) £4 7*s*. 6*d*.; 7*d*. per lb. (10) 7*s*. 6*d*.
(11) 27 days. (12) 6 ac. 293 yds. 3 ft.

XIX.

(1) $238\frac{3}{21}$; 3 f. 24 p. 3 yds. 0 ft. 4 in. (2) 128.
(3) 1. $15\frac{6}{13}$; 2. $1\frac{1}{23}$; 3. 45 : 48 : 40. (4) $\frac{1}{6}$; $\frac{4}{5}$.
(5) £499 19*s*. 3*d*.; £32 19*s*. 4*d*. (6) £595 6*s*. $11\frac{63}{80}$*d*.
(7) 2·8; 420; 1000. (8) 10*s*. 6*d*.; ·1696428751.
(9) £16 8*s*. $8\frac{1}{2}$*d*. (10) 8. (11) 1 yd. 2 ft. (12) $3\frac{1}{2}$.

XX.

(1) 6 ac. 1 ro. 26 po. 23 yds. 4 ft. 110 in. (2) 1. $\frac{8}{105}$; 2. $3\frac{553}{561}$.
(3) £21 17*s*. $2\frac{36}{137}$*d*.; $\frac{12}{55}$. (4) £100. (5) 23·544 oz.
(6) ·001875; 2040000; $76\cdot\dot{9}2307\dot{6}$. (7) ·7317; $\cdot 84\dot{2}5\dot{9}$.
(8) £27. (9) $6\frac{2}{3}$. (10) £494; $3\frac{21}{73}$.
(11) Ar. p. 99. £245; £1012 7*s*. 3·264*d*.
(12) 15 ft. 7 in.; £18 5*s*. $2\frac{13}{16}$*d*.

XX*a*.

(1) 1. 757 tons 3 qrs.; 2. £884818 6*s*. $7\frac{1}{2}$*d*.; 3. 9 m. 4 f. 39 p. $2\frac{26}{39}$ ft.
(2) 1. 31622400; 2. £2 5*s*. 7*d*.; £4 11*s*. 2*d*.; 3. $56224\frac{32}{37}$.
(3) 1. £25 11*s*. 4*d*.; 2. £47 16*s*. 3*d*.; 3. £21 11*s*. 3*d*.; 8*d*.
(4) $\frac{1}{3}$. (5) $\frac{23}{38}$. (6) $\frac{63}{160}$. (7) ·025592. (8) 2040000.
(9) ·36875. (10) 80; $10\frac{10}{11}$. (11) $9\frac{79}{84}$. (12) 64.

XXI.

(1) 1. 363825; 2. 17325. (2) £3237 7*s*. $3\frac{1}{2}$*d*.
(3) $1\frac{2}{3}$; Ar. p. 41. (4) 1. ·00875; 2. ·01.
(5) $149\frac{2}{9}$ yds.; £33 11*s*. 6*d*.; $\frac{79}{600}$; $\cdot 131\dot{6}$.
(6) 1. 8·869743; 2. 5·724; 3. $\frac{5}{6}$. (7) 10. (8) £2874 13*s*. 1·8*d*.
(9) 1. $1+2x+8x^2$; 2. $\dfrac{2(a^2+9x^2)}{a^2-9x^2}$.
(10) $3x^6-142x^4y^2+71x^3y^3-49x^2y^4+13xy^5-y^6$.
(11) 1. $2x-7y$; 2. $\left(\dfrac{a^3+b^3}{a+b}\right)^2$.
(12) 1. $x=3$; 2. $x=7$; 3. $x=6$, $y=12$. (13) $\frac{5}{18}$.

XXII.

(1) £103 4*s*. $3\frac{1}{2}$*d*.; 24. (2) £65 9*s*.; 9*d*. (3) 7200.
(4) $\frac{1}{791}$; $3\frac{5}{21}$. (5) Ar. p. 41. (6) $\frac{18}{103}$; $51\frac{1}{2}$ days.
(7) 1. 2·6; 2. ·203125; 3. ·0001287; 4. 12·34. (8) $\frac{121}{144}$; 23·07.
(9) $a^3-b^3+3a^2b$; 4. (10) 1. $3xy-\dfrac{5y^2}{12}$; 2. $\dfrac{28x}{1-49x^2}$;
3. $1-3x^2+3x^4-x^6$; 4. $7a^2+3ab-9b^2$; 5. $4b(a+c)$.
(12) 1. $x=20$; 2. $x=68$; 3. $x=12$, $y=4$. (13) 25 yrs. and 35 yrs.

XXIII.

(1) £454 6*s*. 8*d*. (2) £152 2*s*. $3\frac{6}{7}$*d*. (3) £3100. (4) $\frac{7}{16}$; $\frac{2}{3}$.
(5) ·6875; $1\cdot\dot{4}\dot{5}$; 16·864; 2. 125000. (6) 2. 7·625; £3 10*s*.
(7) 1. £128 1*s*. $7\frac{1}{2}$*d*.; 2. 4. (8) £10000000; £16000000.
(9) Alg. i. p. p. 1, 2. (10) 1. $\dfrac{a}{3}+b$; 2. $\dfrac{a^3+a^2x-x^3}{x(a^2-x^2)}$;
3. $a^3+b^3+c^3-3abc$; 4. $-7x^{\frac{1}{3}}y^{\frac{1}{3}}$; 5. $3x^2-2ax+5a^2$. (11) $\dfrac{x+1}{2x+5}$.
(12) Al. i. p. 26. 1. $x=\frac{1}{2}$; 2. $x=45$; 3. $x=15$, $y=5$.
(13) $\frac{1}{2}$ pint at 5*s*., and $1\frac{1}{2}$ pt. at 3*s*. a quart.

XXIV.

(1) £16830 7*s.* $7\frac{3}{4}$*d.* (2) £425 10*s.* $7\frac{1}{2}$*d.*; 9*s.* $10\frac{3}{4}$*d.* (3) 329280.
(4) £8 11*s.* $10\frac{1}{4}$*d.* (5) £9 18*s.* 2*d.* (6) ·1875.
(7) 6*s.* $8\frac{1}{4}$*d.*; ·675 ac. (8) 62573; $6\frac{7}{15}$. (9) 31. (10) $3x^2$.

(11) 1. $x^8+2x^6+3x^4+2x^2+1$; 2. $3a^2+4ab+b^2$; 3. $\dfrac{x^2+4}{x^2+x+1}$.

(12) 1. $x=\frac{6}{7}$; 2. $x=2\frac{3}{13}$; 3. $x=18, y=6$. (13) £3 10*s.*

XXV.

(1) £1 11*s.* $7\frac{1}{2}$*d.* (2) 7040. (3) 11 mo. (4) £2450 8*s.*
(5) £670 15*s.* $7\frac{1}{2}$*d.* (6) 1. $10\frac{287}{360}$; 2. ·628125. (7) 3 ft. 9 in.
(8) 1. 12·416; 2. 3·6055. (9) 4. (10) 1. $2b(3a^2+b^2)$; $y(x-y)$.

(11) 1. $4x^4-x^2y^2+6xy^3-9y^4$; 2. $2x^2+xy-3y^2$; 3. $\dfrac{24xy}{4x^2-9y^2}$.

(12) 1. $x=20$; 2. $x=10$; 3. $x=18, y=12$. (13) 9000.

XXVI.

(1) 1. £28214 18*s.* $8\frac{1}{4}$*d.*; 2. £3479 9*s.* $1\frac{1}{2}$*d.*
(2) 1. 4 tons 3 cwt. 2 qrs. 11 lbs.; 2. £12 6*s.* 6*d.* (3) £4 17*s.* 6*d.*
(4) 1. 4·36625; 2. ·004; 3. 213·2. (5) 1*s.* $3\frac{3}{4}$*d.* = £·065625.
(6) £3 3*s.* 4*d.* (7) ·437962. (8) 194. (9) $a+x$.

(10) $\dfrac{3x+2}{x+1}$. (11) 0. (12) 1. $x=4$; 2. $x=4, y=1$. (13) $\frac{4}{15}$.

XXVII.

(1) 1. £13999 10*s.* $2\frac{1}{2}$*d.*; 2. £2 15*s.* $7\frac{1}{2}$*d.*
(2) 1. 2595 tons 5 cwt. 3 qrs. 12 lbs.; 2. 28726. (3) £428.
(4) £1716 18*s.* $6\frac{1}{2}$*d.* (5) 1. $\frac{53}{122}$; 2. ·155; 3. 8·4 (6) £481 5*s.*
(7) £322 8*s.* $1\frac{1}{2}$*d.* (8) 115·23. (9) 109.

(10) 1. $x^6-5x^4+3x^3+6x^2-7x+2$; 2. x^3+3x^2+x-2; 3. $\dfrac{x-3}{x+1}$.

(11) $\dfrac{a^2b^2}{a^4-b^4}$; $\frac{9}{80}$. (12) 1. $x=4$; 2. $x=12, y=6$; 3. $x=\dfrac{ab(a+3b)}{a^2+ab+2b^2}$.

(13) 27.

XXVIII.

(1) 1. £4666 10*s.* $0\frac{1}{2}$*d.*; 2. £425 10*s.* $7\frac{1}{2}$*d.* (2) 28246.
(3) £839 14*s.* $4\frac{1}{2}$*d.* (4) Gain, £5 10*s.* 6*d.* (5) ·189.
(6) 1. ·011214; 2. ·314; 3. $\frac{25}{168}$ gua. = £·15625. (7) $314\frac{1}{2}$.
(8) £1682. (9) 1. $3x^2$; 2. $2b(x+y)$.

(10) 1. $(x^2-xy+y^2)+(x+y+1)$; 2. $\dfrac{x-1}{x+1}$. (11) $\dfrac{1}{x+2}$.

(12) 1. $x=13$; 2. $x=3$; 3. $x=\dfrac{ab}{a+b}$. (13) 24 yds. at 5*s.* a yard.

XXIX.

(1) 1. 2 tons 4 cwt. 9 lbs.; 2. £237829 5s. $11\frac{1}{2}d.$ (2) 901.
(3) 485 miles. (4) 84. (5) £1 2s. 6d. (6) £251 14s. $4\frac{1}{2}d.$
(7) 1. 103; 2. 12·34; 3. ·2. (8) ·0125. (9) 27.
(10) 1. $\frac{2a}{3}-\frac{b}{6}$; 2. $1+3x-10x^2-24x^3$; 3. $5xyz$.
(11) 1. $(a-x)(a+x)^2$; 2. $2x^2+ax-a^2$.
(12) 1. $x=63$; 2. $x=15$; 3. $x=12$, $y=15$. (13) 100 miles.

XXX.

(1) 1. £4332 3s. 2d.; 2. £2656 19s. $4\frac{1}{2}d.$
(2) 1 629579; 2. 3 mi. 5 f. 39 p. 1 yd. 1 ft. 8 in. (3) £17 2s. $2\frac{1}{4}d.$
(4) £8 11s. $10\frac{1}{2}d.$ (5) 1. $15\frac{5}{6}$; 2. 6·25; 3. ·03003. (4) £·09765625.
(7) £299 4s. ·03d. (8) ·125. (9) 63.
(10) 1. $x^6-2ax^5-a^2x^4+4a^3x^3-a^4x^2-2a^5x+a^6$; 2. x^2-a^2;
3. $\frac{2x^3-4x^2+2x-3}{4x^3+3x^2-18x+27}$. (11) 4. (12) 1. $x=3$; 2. $x=7$, $y=4$.
(13) Cost of 1 lb. of tea = 2s. 6d.; of sugar = 8d.

XXXI.

(1) £16477 8s. 11d. (2) £7 8s. $7\frac{1}{2}d.$ (3) 22862.
(4) £1844 15s. 10d. (5) £2000. (6) £4484 13s. 9d.
(7) 1. $1\frac{4}{7}$; 2. $\frac{2}{75}$; 3. 3·54; 4. $\frac{1}{2}$; 5. ·5625. (8) 2·0501.
(9) 25. (10) 1. x^2-a^2; 2. $a(a+b)$; 3. $\frac{x-5}{2x+3}$.
(11) 1. $4x^4-4x^3-23x^2+12x+36$;
2. $x^5y^5+2x^4y^4+4x^3y^3+8x^2y^2+16xy+32$.
(12) 1. $x=19$; 2. $x=15$; 3. $x=13$. $y=17$. (13) 1000.

XXXII.

(1) £28540 17s. 11d. (2) 300500. (3) £1717 12s. 6d.
(4) £28; £1 8s.; 7s.; 3d. (5) £4831 18s. $2\frac{1}{2}d.$ (6) £900
(7) 1. £172 0s. $11\frac{1}{4}d.$; 2 £5 10s. 8d.
(8) 1. 1·09375; 2. $\frac{9}{40}$; 3. 7006; $9\frac{1}{2}$.
(9) 1. $7a+3b-10c-2d$; 2. $x^3-x^2y+7xy^2-5y^3$; 3. $\frac{a}{6}+\frac{9b}{10}+\frac{c}{14}$.
(10) 1. $(x-y)^7$; 2. $(a+b)^3$. (11) 1. $\frac{2x}{x^2-16}$; 2 $\frac{x-3}{3x+2}$.
(12) 1. $x=20$; 2. $x=13$; 3. $x=125$, $y=81$. (13) $\frac{4}{5}$.

XXXIII.

(1) 3200 days. (2) £405. (3) $592\frac{7}{8}$.
(4) 1. $\frac{11}{54}$; 2. $2\frac{5}{9}$. (5) 1. ·155; 2. 28·4.
(6) 1. 5 lbs. 3 oz. 13·1328 dr.; 2. ·375; 3. ·692820.
(7) £14 16s. $10\frac{1}{2}d.$ (8) £5 17s. $11\frac{5}{48}d.$

(9) 1. $2\frac{1}{4}$, 2. $x^5-10ax^4+40a^2x^3-80a^3x^2+80a^4x-32a^5$.

(10) $a-b-c+d$. (11) 1. $2x^2+2xy-3y^2$; 2. $\frac{x-y}{x+y}$.

(12) 1. $x=\frac{b}{a-1}$; 2. $x=7$; 3. $x=2y=60$.

(13) Distance $=60$ mi.; rate 6 mi. an hour.

XXXIV.

(2) 9*s.* 1*d.* (3) 2*s.* 6*d.* (4) 5 dwt. 10 grs.
(5) 1. $1\frac{31}{42}$; 2. 7*s.* $4\frac{1}{2}$*d.*; 3. 736. (6) 64.
(7) 8. (8) 2·828...; ·014....

(9) 1. $(\cdot 01)^2$; 2. $x^4+2x^3-24x^2+62x-105$; 3. $\frac{9a^2}{4}-3a+1$.

(10) $x+c$. (11) 1. $4bc$; 2. $\frac{2x-y}{x^2-1}$; 3. $\frac{x^2+a^2}{x}$.

(12) $x=6$. (13) 1. 21 yrs.; 18 yrs.; 2. 371; 283.

XXXV.

(1) 1. 2*s.* 6*d.*; 2. 175. (2) 1. 12; 2. 29. (3) 50 miles.
(4) £2 8*s.* 2*d.* (5) $3\frac{3}{4}$. (6) 2. 2.
(7) 1. 37·56; 2. ·00875. (8) 2. 3·60555.

(10) 1. $\frac{2a^2+3c^2}{12}$; 2. $a^6-10a^5b+29a^4b^2-24a^3b^3-14a^2b^4+22ab^5-4b^6$;

3. $a^2-b^2+c^2$; 4. $(x+y)^2$.

(12) 1. $x=-1$; 2. $x=56$; 3. $x=7, y=9$. (13) £5000.

XXXVI.

(2) £1809 9*s.* 9*d.* (3) £9 8*s.* (4) £9 9*s.* (5) 47553.
(6) 1. £4 9*s.* 10*d.*; 2. ·0053125; 3. 3490000. (7) £537 16*s.* 8*d.*

(8) 709. (9) 1. $\frac{x^2}{6}-\frac{y^2}{6}-\frac{z^2}{16}+\frac{5xy}{36}-\frac{xz}{24}+\frac{5yz}{24}$;

2. $a^2+5ab+3b^2$; 3. $\frac{(x^2+2xy-y^2)(x^2+y^2)}{(x+y)(x-y)^2}$.

(10) 1. 1; 2. $\frac{x^2+2x+1}{x^3+2x^2+3x+5}$; 3. $\frac{x+4a}{x+2a}$.

(11) 1. $x=\frac{40}{67}$; 2. $x=\frac{\pm\sqrt{7}}{2}$; 3. $x=50, y=20$.

(12) £1 13*s.* (13) 209 boys, 191 girls, 200 infants.

XXXVII.

(1) 1. 2462 yds. 7 in.; 2. 117. (2) £267 1*s.* $0\frac{3}{8}$*d.*
(3) 2848 miles. (4) 7 yrs.
(5) A, £1228; B, £2140; C, £1790 16*s.* 8*d.*

(6) AR. P. P. 38, 39. (7) 1. 1·271; 2. 37·25; 4. ·825.
(8) 1. £1464 2*s.*; 2. ·05643. (9) 1. 189; 2. 12.

(10) 1. $\frac{a^2+7ab-5b^2}{10}$; 2. x^5+y^5; 3. $2a^2+5ab-b^2$. (11) 1. $2x+5$.

(12) 1. $x=10$; 2. $x=5$; 3. $x=\frac{ac+bd}{a^2+b^2}$, $y=\frac{bc-ad}{a^2+b^2}$.

(13) 28 miles an hour.

XXXVIII.

(1) 562069. (2) 10000. (3) 147. (4) 200.
(5) 1. £322 10*s.*; 2. 4 yrs. (6) 1. $\frac{9}{560}$; 2. 1·05.
(7) 1. ·06640625; 2. 1·74. (8) 1. 70·205; 2. $\frac{13}{15}$; 3. 1.
(10) 1. $x^7-9x^5y^2+7x^4y^3+13x^3y^4-19x^2y^5+8xy^6-y^7$; 2. $a+2b-c+3d$.

(11) 1. $\frac{4a^2x^2}{x^4-a^4}$; 2. $\frac{x-a}{x+a}$.

(12) 1. $x=75$; 2. $x=\frac{ac(b-d)}{bc-ad}$; $y=\frac{bd(a-c)}{bc-ad}$.

(13) 600 E.; 400 I.; 200 S.

XXXIX.

(1) £24506 15*s.* 8*d.* (2) 1. 1470407; 2. 3 P.M. Jan. 15.
(3) £35. (4) 6. (5) £3300; £148 10*s.*
(6) 1. $1\frac{601}{690}$; 2. $\frac{1}{3}$. (7) 1. ·0017283; 2. $\frac{257}{1110}$; 3. ·008; 4. 30·1111.
(8) 22 cwt. 2 qrs. N.; 4 cwt. 2 qrs. C.; 3 cwt. S. (9) 2. -1.
(10) 1. $c+14b-a$; 2. $x^3+y^3+z^3-3xyz$; 3. $x-2x^2+3x^3$;

4. $x^{\frac{2}{3}}y^{\frac{2}{3}}+a^{\frac{1}{3}}x^{\frac{1}{3}}y^{\frac{1}{3}}+a^{\frac{2}{3}}$.

(12) 1. $x=7$; 2. $x=4$, $y=5$. (13) 12 at 12*s.*, 24 at 9*s.*

XL.

(1) 138 tons 16 cwt. 1 qr. 27 lbs.; 2. £2 17*s.* 6*d.* (2) £24 19*s.*
(3) 336. (4) £1458 6*s.* 8*d.* (5) £405.
(6) 1. $\frac{4}{5}$; £168; 2. 002; 3. 1·0625. (7) 4 yds. by 20 yds.

(8) 59·899. (9) 19. (10) 1. $\frac{x^2}{12}$; 2. $2b^3$; 3. $x^2+2xy+4y^2$.

(12) 1. $x=21$; 2. $x=14$; 3. $x=2\frac{2}{3}$; $y=3\frac{2}{3}$. (13) £1250.

XL *a.*

(1) £83347 18*s.* 4*d.* (2) 1. £1 12*s.* $4\frac{1}{2}$*d.*; 2. 13 wks. 5 da.
(3) 1. £3 7*s.* $11\frac{1}{4}$*d.*; 3. 27060.
(4) 1. 6 mi. 373 yds. 1 ft.; 2. $\frac{3}{32}$, ·09375.
(5) 1. 161172; 2. ·02604. (6) £4400.
(7) 1. $abc+a^2c-b^2c+a^2b-ab^2+bc^2+ac^2$; 2. $\frac{4x^2}{y^2}+\frac{12x}{y}+11+\frac{3y}{x}+\frac{y^2}{4x^2}$.

(9) $x^3+2x^2y+8xy^2+16y^3$.
(10) 1. $x=3$; 2. $x=5$; 3. $x=3$, $y=4$; 4. $x=5$, $2{\cdot}4$.
(11) 20. (12) 974. (13) 73.

XLI.

(1) £6264 11*s.* 10*d.* (2) 1. £2840 10*s.* $8\frac{3}{4}$*d.*; 2. 2 lbs. 9 oz.
(3) 44891 yds. (4) 60.
(5) 62. (6) 1. 8*s.* 6*d.*; 2. $\frac{1}{25600}$; 3. ·186.
(7) 1. $a^3-8b^3+27c^3-6a^2b+9a^2c+12ab^2-36abc+27ac^2+36b^2c-54bc^2$; 2. $x^6-2x^5+3x^4-4x^3+3x^2-2x+1$.
(8) $\dfrac{2x^3}{(a-x)(a^2+x^2)}$. (9) 1. $x=\frac{1}{8}$; 2. $x=1\frac{1}{2}$; 3. $x=-54$; $y=44$.
(10) 1. 9; 2. $\dfrac{1}{3}\cdot\dfrac{3^n-2^n}{2^{n-2}}$. (11) 11*s.* (12) $\frac{5}{8}$. (13) 120; 24.

XLII.

(1) £5 9*s.* 3*d.* (2) 1. £18 6*s.* 11*d.*; £34 1*s.* 5*d.*
(3) 39. (4) 1. ·3; 2. 1070.
(5) £183 12*s.* 4*d.* (6) 1000 ft. (7) £47 10*s.*
(8) ·2317. (9) 1. 12; 2. -36; 3. 10.
(10) 1. $2a^2b+7ab^2+3b^3$; 2. $\dfrac{x^3}{6}+\dfrac{x^2y-xy^2}{3}+\dfrac{5y^3}{6}$.
(11) 1. $-12x^{2(p-1)}y^5$; 2. $14x^5-6x^4y+2x^3y^2-7x^3+10x^2y-4xy^2+y^3$; 3. $x^3-2x^2y+xy^2+ax^2-2axy+ay^2$.
(12) 1. $\dfrac{x^2-3}{x^3-2}$; 2. $\dfrac{4x^2-18x-9}{x^2-9}$; 3. $\dfrac{x^2+xy+y^2}{x^2-xy+y^2}$.
(13) 1. $x=\dfrac{q+r}{c-b}$; 2. $x=5$, $y=4$; 3. $x=y=8$; 4. 56 and 49.

XLIII.

(1) Ar. p. 40. (2) 1. $\frac{85}{1386}$; 2. $\frac{3}{11}$.
(3) 1. ·003; 2. 7500; 3. $\cdot 1\dot{3}$; 4. $\frac{3}{40}$; 5. $2\frac{23}{66}$.
(4) 1. 2; 2. ·06896; 3. 2*s.* $3\frac{1}{2}$*d.* (5) £23 0*s.* $4\frac{11}{36}$*d.*
(6) 205. (7) 15. (8) 12*s.* $4\frac{1}{4}$*d.*
(9) 1. Alg. I. p.p. 13, 14; 2. $3a^5-7a^4x+8a^3x^2-8a^2x^3+\dfrac{10ax^4}{3}-x^5$.
(10) $a+bx$. (11) 1. $\dfrac{x+4}{x^2+2x-2}$; 2. $\dfrac{a+x}{ax}$; 3. $1-x$.
(12) 1. $x=6$; 2. $x=\dfrac{a^2}{b-a}$; 3. $x=9$, $y=7$.
(13) A.'s rate 75 yds., B.'s, 50 yds., a minute; length of course, 150 yds.

XLIV.

(1) 1. $\frac{1}{154}$; 2. 2; 3. 12. (2) AR. P. 58.
(3) 1. 130; 2. 19880; 3. ·03. (4) AR. P. P. 1, 34, 35.
(5) 1. £1 16s.; 2. £1 16s. 10d. (6) 1. ·02$\dot{6}$; 2. £551 16s. 0$\frac{1}{4}$d.
(7) 1. $1+2\sqrt{2}$; 2. 1. (8) $2(x-y)$.
(9) 1. $1-2(x+y)+x^2+y^2-2xy(x+y-2)$; 2. x^2-7x+5.
(10) $84\,x^3y^2z^3$. (11) $\frac{2a}{b}$. (12) 1. $x=7$; 2. $x=10$. (13) 4000.

XLV.

(2) £842 19s. 4$\frac{1}{2}$d. (3) 2s. 9d. (4) £40 8s. 9d.
(5) £337 19s. 4$\frac{1}{2}$d. (6) £119.
(7) $\frac{8a^2}{5}-\frac{5b^2}{4}+c^2-\frac{27d^2}{7}=-\frac{85}{4}$.
(8) 1. $\frac{1}{8}-\frac{3x}{4}+\frac{3x^2}{2}-x^3$; 2. $4x^4+6x^3-\frac{71x^2}{4}-15x+25$.
(9) $x^3-\frac{3x^2}{5}-2x+\frac{1}{3}$.
(10) 1. $\frac{a}{b(a-c)}$; 2. $\frac{x^2-y^2}{2(x^2+y^2)}$; 3. $\frac{x^2-4}{x^3+64}$; 4. $\frac{a(yz+n)}{xyz+nx+mz}$.
(11) $\frac{x^2-2x+3}{x^2+2x-3}$. (12) 1. $x=3\frac{1}{2}$; 2. $x=\frac{a^2-b^2}{ap-bq}$; $y=\frac{a^2-b^2}{aq-bp}$.
(13) 4 days.

XLVI.

(1) 1. 1 mi. 1 fur. 202 yds. 1 ft. 3 in.; 2. $\frac{1}{84}$.
(2) 1. 87$\frac{1}{2}$ yds.; 2. £9 16s. 10$\frac{1}{2}$d. (3) 1. $\frac{23}{40}$. (4) ·904.
(5) 18s. (6) 5·07; ALG. I. P. 41. (7) 1. 49; 2. 19.
(8) 1. $9x^4+12x^3y+4x^2y^2-y^4$; 2. $\frac{1}{4}(a^2-4b^2-12bc-9c^2)$.
(9) 1. $7a^2-3ab+2b^2$; 2. $\sqrt{x}+2\sqrt{y}$.
(10) 1. $x=13$; 2. $x=6$, $y=10$. (11) $1-2x+3x^2$.
(12) 840. (13) ALG. I. P. P. 87, 88.

XLVII.

(1) £149 10s. 7$\frac{1}{2}$d. (2) 7$\frac{1}{2}$ hrs. (3) £4 9s. 0$\frac{3}{4}$d.
(4) 1. $\frac{411}{256}$; 2. ·28125; 3. 26 lbs. 4 oz.
(5) 1. 3·7356; 2. ·0117; 3. 76·923; 4. 1·2854.
(6) £35 12s. 6d. (7) 1. 5·1984; 2. 2·2803; 3. 2·0615.
(8) 1. $a-b+c-d$; 2. $4x^8-16x^6-16x^5+12x^4+32x^3+24x^2+8x+1$;
3. $x^2-xy+y^2+x+y+1$. (9) $\frac{9x^2-x-3}{x+5}$.
(11) 1. $x=7$; 2. $x=18$, $y=12$. (12) £540; 17d. in the £.
(13) 1. $x^2-ax+2a^2$; 2. x^2-x+2; 3. 36.

XLVIII.

025312. (2) 11 lbs. (3) 332.
l. P. 49. 1. $2\frac{5}{13}$; 2. $6\frac{521}{540}$. (5) 1·001; ·1484375; ·0009; $\frac{1}{64}$; $\frac{1}{101}$.
. (7) 1. $-3(b+c)$; 2. $\frac{59y}{40}-\frac{x}{12}$.
$x^3-41x-120$; 2. $2(a^2b^2+a^2c^2+b^2c^2)-(a^4+b^4+c^4)$.
x^4-5x^2+4; 2. x^4-8x^2+16. (10) $x-3$.
$\frac{x^2-2x+3}{x^2+2x+3}$; 2. $\frac{4a}{a+x}$. (12) 1. $x=\frac{1}{3}$; 2. $x=19$; 3. $x=4$.
and 2.

XLIX.

$\frac{13}{36}$ mi. (2) $1562\frac{1}{2}$. (3) £335 8*s.* 4*d.*
$16\frac{16}{35}$; 2. $1\frac{10}{13}$. (5) 15.
G. I. P. 18. $15x^4-23x^3+27x^2+9x-28$. (7) $3(ax-by)$.
G. I. P. 6; $x+2$. (9). $\frac{5x^2-4x-8}{3x^2+4x+24}$. (10) $60a^2b^2c^2$.
$x=2$; $x=8$, $-\frac{295}{1134}$. 3. $x=4$, $y=9$. (13) 12 and 9.

L.

(2) 1. £1 17*s.* $1\frac{23}{32}d.$; 2. 7 hr. 13 mi.
9140625. (4) 83 cwt. 2 qrs. 22 lbs. 4 oz.
h. 50·4 m. (6) b. (7) 36. (8) x^4+4x^2+16.
$+3ab-4b^2$. (10) $(a-2)(a-4)(a-8)$. (11) ALG. I. P. 25; -3.
$x=\frac{2}{7}$; 2. $x=3$; 3. $x=12$; 4. $x=10$; 5. $x=4$, $y=3$.
4.

LI.

(2) £203 18*s.* 0·69*d.* (3) £30.
$\frac{2}{3}$. (5) 4. (6) 0.
$a^3+b^3+c^3-3abc$. 2. $-(x^4-13x^2+36)$.
$(x-1)^3$; 2. $1+2x+3x^2+4x^3+5x^4+$.
$-3y$. (10) 1. 0; 2. $\frac{1-x+x^2}{1-x^2}$.
$x=7$; 2. $x=-2\frac{2}{5}$; 3. $\sqrt{x}=-2$; 4. $x=2$, $y=5$.
at 3*s.*; 5 at 5*s.* (13) 48.

LII.

gal.; $14\frac{2}{7}$ p.c. (2) The latter; £83 6*s.* 6*d.*
$1\frac{13}{24}$; 2. 12 tons 13 cwt. 3 qrs. 17 lbs. 8 oz.
2; 2. 10. (8) 1. $x^4+4x^3-19x^2+64x-35$.
7. (10) $\frac{16x^3-28x^2+4x+7}{16x^4-1}$. (11) $3x^2-x$.
$x=10$; 2. $x=2$; 3. $x=3\frac{1}{2}$; 4. $x=7$, $y=4$. (13) 7 in. and 12 in.

LIII.

(1) 1. 8·74894; 2. 43·2457152.
(2) 1. $\cdot 3\dot{4}6153\dot{8}$; 2. $2\cdot\dot{9}1\dot{8}$; 3. $\frac{85}{198}$; 4. $7\frac{281}{666}$.
(3) 1. ·740625; 2. ·0390625. (4) £58 6*s.* $4\frac{43}{56}$*d.*
(5) 5 h. $27\frac{3}{11}$ m. (6) 123·904.
(7) $1+x+x^2+x^3+x^4+\ldots$ (8) $a^2+ab+2b^2$.
(9) $(x^2-7x+10)(x^2-3x-70)$. (10) $\frac{2x^3}{x^6-1}$.
(11) 1. $x=1\frac{5}{7}$, $y=2\frac{2}{5}$, $z=-12$; 2. $x=4, 3$; $y=3, 4$.
(12) Alg. I. p. 65; ·61419. (13) III. 17; I. 30; III. 16.

LIV.

(1) 1. $\frac{(1760)^2}{4840}=640$; 2. 24 tons 19 cwt. 2 qrs. 23 lbs. 13 oz.
(2) 4950. (3) Ar. p. 45; $2\frac{19}{30}$.
(4) ·00015344. (5) $\frac{a}{6}-\frac{7b}{6}+4c$.
(6) 1. $1+4x^{\frac{1}{2}}+4x+6x^{\frac{3}{2}}+12x^2+9x^3$; 2. $x^2-3x^3+4x^4$.
(8) 1. $x=4$; 2. $x=10$, $y=24$. (9) 1000.
(10) Alg. I. p. 121; $\frac{a^2-b^2}{a}$. (12) I. 4; Def. A. (13) VI. 4.

LV.

(1) 1. $\frac{1}{800}$, $\frac{3}{4}$; 2. ·04, ·25; 3. ·00064, 12000. (2) $\cdot 14\dot{6}\dot{9}$.
(3) 45. (4) £36 18*s.* 4·74*d.* (5) £5400.
(6) 1008 c. in. (7) Alg. II. p. 63.
(8) 1. ·009170328; 2. 1·078774.
(9) 1. $x=4, y=5, z=6$; 2. $x=\frac{1}{4}, \frac{1}{9}$. (10) $x=2, 16$; $y=2, \frac{1}{2}$.
(11) 26059. (12) I. 35. (13) I. 12, 30, 6; III. 12, 16.

LVI.

(1) 1. 2 mi. 3 fur. 35 po. 3 yds.; 2. 63; 3. 1200. (2) 3 lbs. 6 oz.
(3) 6 ac. 3 ro. 30 po. $22\frac{1}{2}$ yds.; 278 yards nearly.
(4) 1. 2; 2. 1. (5) $1\frac{7}{20}$. (6) 7*s.* 9*d.*
(7) $10\sqrt{10}$. (8) 1. ·08502884; 2. 21.
(9) $x=2\frac{1}{4}$, $y=3\frac{3}{8}$. (10) 50 ft. (11) $25\sqrt{6}$.
(12) 7200. (13) III. 31, 3.

LVII.

(1) 69. (2) 6 yrs.; £500. (3) ·0000056332611.
(4) £2 0*s.* 9·24*d.*; 13·728125. (5) £1 13*s.*
(6) 30. (7) Ar. p. p. 99, 105; £2050; £$487\frac{33}{41}$.
(8) 1. x^2+2x+3; 2. $(x+2)(x-1)(x-3)$.
(9) $\frac{x^2}{a^2(a^4-x^4)}$. (10) 1. $a-b$; 2. $\frac{5\sqrt{2}}{8}+\frac{1}{2}$.

1. $x=5$; 2. $x=4$; 3. $x=\frac{1}{4}, -1$;

4. $x=1, \dfrac{-1\pm\sqrt{-3}}{2}, 2, 1\pm\sqrt{-3}$; $y=2, 1$, &c.

1. $\dfrac{4a+b}{5}, \dfrac{3a+2b}{5}, \dfrac{2a+3b}{5}, \dfrac{a+4b}{5}$; 2. 81; 3. $14\frac{27}{32}$;

4. $\frac{3}{2}(\sqrt{2}+\sqrt{6})$.

B will pass A at end of 36th mile; A will pass B at end of 120th mile.

LVIII.

$11(a+b+c+d)$. (2) $\frac{18}{23}$.

1. $x^7-13x^5+10x^4+12x^3-2x-1$;

2. $x^6+9x^5+30x^4+45x^3+30x^2+9x+1$.

1. ALG. I. P. 8; 2. $-(a+7b)$.

1. $x^6+x^5-x^3+x+1$; 2. $(15x+4y+4)(7x+6y+10)$.

1. $x=5$; 2. $x=1\frac{1}{4}$. (8) 37 and 21.

$x-4$. (10) 48. (11) 1. $\dfrac{a^2-b^2}{2}$; 2. $1-16x^2$.

1. $x=7, y=-3$; 2. $x=0$; 3. $x=\frac{7}{18}$ or $\frac{1}{4}$.

$19\frac{3}{4}$ miles from Cambridge.

LIX.

1. $1+3x+4x^2+7x^3+11x^4+\ldots$; 2. $\dfrac{x^2}{2}+ax+\dfrac{a^2}{3}$.

2. x^2-y^2; 2. $\dfrac{x-1}{x+2}$; 3. $\dfrac{\sqrt[3]{12}}{7}$; $7a\sqrt{2x}$.

1. $x=2$; 2. $x=120$; 3. $x=5$; 4. $x=\dfrac{\sqrt{a}}{\sqrt{a}+2}$; 5. $x=3, y=15$.

3 sheep and 6 lambs. (6) 9 hrs.

2 hrs. 21′ 40″. (8) 1. $\dfrac{2a+b}{3}, \dfrac{a+2b}{3}$; 2. $r=\left(\dfrac{b}{a}\right)^{\frac{1}{4}}$

1. -102; 2. $6\frac{1}{3}$; 3. 2. (10) 7160.

$x=\dfrac{\log\frac{1}{2}(cd^{-1}+d)}{\log a}$; $y=\dfrac{\log\frac{1}{2}(cd^{-1}-d)}{\log a}$.

ALG. II. P. 76. (13) 11*s.* $9\frac{1}{2}d$.

LX.

x^2-3. (2) $\dfrac{x^3+2x^2+2x-2}{3}=6$.

$6x^4-7x^3+x$. (4) 1. $\dfrac{1}{1-x}$; 2. $\dfrac{x}{a-x}$.

1. $8a^6x^6-60a^5x^7+150a^4x^8-125a^3x^9$; 2. $\dfrac{-2x^2y^3}{3}$.

$p=25, q=-24$. (7) $2(a+\sqrt{a^2+b^2})$; $6-\sqrt{-5}$.

(8) 1. $x=6$; 2. $x=7,\ y=4,\ z=3$;
3. $x=\dfrac{am-2(b^2+c^2)}{a+4c},\ y=\dfrac{2cm+b^2+c^2}{a+4c}$; 4. $x=9,\ \dfrac{16}{9}$;
5. $x=7,\ 3,\ -\frac{1}{3}(31\mp\sqrt{205})$; $y=3,\ 7,\ -\frac{1}{3}(31\pm\sqrt{205})$;
6. $x=y=\frac{1}{2}$.

(9) 1. $x=89,\ 76$, &c.; $y=2,\ 13,\ 24$, &c.; 2. $x=2,\ y=1$.

(10) 1. 252; 2. $\dfrac{2x}{y}$; 3. $39\frac{3}{8}$.

(11) $-2,\ \frac{1}{0},\ 2,\ 1,\ \frac{2}{3}$.

(12) 1. 2·10721; 2. 2·09691; 3. 3·39794.

LXI.

(3) $(a+b)^2$.

(4) 1. $x=5$; 2. $x=17$; 3. $x=\frac{1}{2}\sqrt[4]{(534\mp150\sqrt{13})\times(-1)}$;
4. $x=5,\ y=7$; 5. $x=3,\ -1\frac{1}{3}$; 6. $x=2,\ \sqrt[5]{-35}$.

(5) $x=566,532$, &c.; $y=2,\ 9$, &c.

(6) 1782·888; 31·09557.

(7) $x=\dfrac{9\log 2}{3\log 3-4\log 2}$.

(8) £3103 17*s.* 1·44*d.*

(9) 10416 and 20832.

(10) $\dfrac{\underline{\lvert 11}}{\{\underline{\lvert 2}\}^3}$.

(11) $\dfrac{\underline{\lvert 36}}{\underline{\lvert 31}\ \underline{\lvert 5}}$; $\dfrac{\underline{\lvert 35}}{\underline{\lvert 31}\ \underline{\lvert 4}}$.

(12) $x=1·4743\ldots$.

LXII.

(1) 1. $x-y$; 2. m; 3. $xy\sqrt[3]{x^2}$; $\dfrac{4\sqrt[3]{3}}{5}$; $a^{\frac{2}{n^2-1}}$.

(2) 1. $110\sqrt{5}$; 2. 36; 3. $x+y$.

(3) 1. $x=\dfrac{ab}{a+b(m-n)}$; 2. $x=4,\ \frac{1}{2}(21\pm\sqrt{41})$; 3. $x=\frac{1}{2},\ y=\frac{1}{3}$;
4. $x=\left(\dfrac{a^2-b^2}{am-bn}\right)^2,\ y=\sqrt{\dfrac{a^2-b^2}{an-bm}}$; 5. $x=2,\ -3\frac{1}{15}$;
6. $x=\dfrac{ma}{n},\ \dfrac{mn}{a}$.

(4) 1880.

(5) Alg. i. p. 97; $\sqrt{10}-\sqrt{2}$; $\frac{1}{8}(9+\sqrt{3})$

(6) $3n-1$; 15050.

(7) 1. 2; 2. $\frac{5}{6}$.

(8) 1751.

(9) 63 or 23, and 37 or 77.

(10) $\dfrac{\underline{\lvert 50}}{\underline{\lvert 44}\ \underline{\lvert 6}}$; $\dfrac{\underline{\lvert 49}}{\underline{\lvert 43}\ \underline{\lvert 6}}$.

(11) 1. 1·69897; 2. $\bar{4}$·69897; 3. 2·2922561; 4. ·561141.

(13) 8 and 6.

LXIII.

) 40 and 28. (2) 48^2 and 36^2.

) 1. $4\{\sqrt[3]{25}+\sqrt[3]{20}+2\sqrt[3]{2}\}$; 2. $\sqrt{x^2+x}+x$; 3. $7-2\sqrt{3}$.

) ALG. I. P. 116; $\frac{7}{9}$. (5) 1. -7; 2. $\dfrac{5(3^n-2^n)}{2\cdot 3^n}$.

) 14105. (7) 456.

) $1+\dfrac{1}{2}x+\dfrac{1\cdot 3}{2\cdot 4}x^2+\dfrac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3+\dfrac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}x^4+\text{etc.}$

) (10) ALG. II. PP. 63, 73.

1. $x=\cdot 9492328$; 2. $x=3\cdot 773705$. (12) $1\cdot 186867$.

LXIV.

1. $9\frac{3}{13}$; 2. $7\frac{1}{17}$; 3. $17\frac{1}{7}$; 4. 10. (3) hx^2-2x+k.

$a^4-7a^2b^2+26ab^3-40b^4$. (5) 1. $\dfrac{a+b+c}{a-b-c}$; 2. $\dfrac{a^{2m}+2a^m+2}{a^m+2}$.

1. $6\left(6^{\frac{2}{3}}-5^{\frac{1}{3}}\cdot 6^{\frac{1}{3}}+5^{\frac{2}{3}}\right)$; 2. $1+\dfrac{2\sqrt{a}}{b}(\sqrt{a}+\sqrt{a+b})$; 3. $3-2\sqrt{-1}$.

1. $x=5,\ -9$; 2. $x=81$; 3. $x=\dfrac{a}{\sqrt{a+b}},\ y=\dfrac{b}{\sqrt{a+b}}$;

4. $x=a,\ y=z=b$; 5. $x=1\frac{1}{4}$;

6. $(2x-1)^2=(\sqrt{15-4x}-1)^2$, $x=1\frac{1}{2},\ -2\frac{1}{2},\ \dfrac{1\pm 2\sqrt{3}}{2}$.

4 sov., 59 sh., 55 sixp. (9) VI., B.

1. $\dfrac{n-1}{2}$; 2. 342; 3. $1\frac{1}{8}$.

ALG. II. P. 71. (12) 8, 12, 16.

LXV.

1. $x^{12}-a^2x^{10}-a^6x^6+a^{10}x^2+a^{12}$; 2. $x^4-x^2y^2+y^4$; $13y^4$.

Let $x=y+z$. $\therefore \dfrac{x^n-y^n}{x-y}=\dfrac{(z+y)^n-y^n}{z}$. (3) ALG. I. P. 89.

$c^{\frac{1}{3}}$. (5) 1. $x=6$; 2. $x=7,\ 4$; 3. $x=\dfrac{a}{2}(1\pm\sqrt{33}),\ \dfrac{a}{2}(1\pm\sqrt{97})$;

4. $x=-(a+b),\ -(2a+3b)$; 5. $x=5,\ -3$; $y=-3,\ 5$.

$\dfrac{(n+1)^3}{2(n-1)^2}$. (8) $\dfrac{|\underline{20}}{|\underline{9}\,|\underline{11}}$; 1. $\dfrac{|\underline{19}}{|\underline{9}\,|\underline{10}}$; 2. $\dfrac{|\underline{17}}{|\underline{9}\,|\underline{8}}$.

1. $(3a)^{\frac{4}{5}}\left\{1-\dfrac{4}{5}\cdot\dfrac{4x}{3a}-\dfrac{4\cdot 1}{5\cdot 10}\cdot\left(\dfrac{4x}{3a}\right)^2-\dfrac{4\cdot 1\cdot 6}{5\cdot 10\cdot 15}\cdot\left(\dfrac{4x}{3a}\right)^3-\right\}$;

2. $\dfrac{(r+1)(r+2)}{1\cdot 2}x^r$. (10) ALG. I. PP. 131, 132.

16, 12; 4, 3. (12) 12 and 5.

LXVI.

$x^{\frac{1}{3}}+x^{\frac{1}{6}}+1-x^{-\frac{1}{6}}$. (3) 1. (4) $\dfrac{a^2+b^2}{2a(2a^2-b^2)}$.

(5) 1. $x=4$; 2. $x=8$; 3. $x=5,\ y=8$. (6) £5.

(7) 1. 0; 2. $\frac{9}{2}(2+11\sqrt{3})$; 3. 25. (9) $r=\frac{1}{2}$.

(10) ALG. II. P. 64. (11) $\frac{1}{2x}+\frac{5}{6(x-2)}-\frac{4}{3(x+1)}$. (12) $3240x^7$.

LXVII.

(1) ALG. I. P. 104. (2) $x=5, 4;\ y=4, 5$. (3) 6, 7, 8, 9.

(6) 20. (7) $-\frac{1}{64}\sqrt{\frac{x^7}{2a^5}}$. (8) x^5-x^2+1.

(9) ALG. II. P. 68. (10) 2875733. (11) $x=\frac{\log. 2}{\log. 3};\ y=\frac{1}{x}$.

(12) 1. $y^2-4y+1=0$; 2. $x=1\pm\sqrt{2},\ \pm 1$.

LXVIII.

(1) 1. $x=2, -5$; 2. $x=1,\ -\frac{3\mp\sqrt{5}}{2}$;

3. $x=\pm 3,\ \mp\frac{\sqrt{2}}{2}$; $y=\pm 2,\ \pm\frac{5\sqrt{2}}{2}$. (2) 82.

(3) 1. $112\frac{1}{2}$; 2. $12\frac{1}{2}$; 3. $\frac{n}{4}\{a(n+1)-b(n-3)\}$.

(4) WOOD'S ALG. P. 138. (5) 8 and 3.

(7) ALG. I. P. 169; 10080. (8) 6; 7775; 1296.

(9) $7+\frac{1}{14+}\frac{1}{14+}\frac{1}{14+}$ etc.

(10) 1, 2, 3 at 3*s.* 6*d.*; 7, 4, 1 at 4*s.* 6*d.*; 12, 14, 16 at 5*s.* (11) 2925.

(13) 2. $1-\frac{5}{2}\cdot 2x+\frac{5\cdot 3}{2\cdot 4}\cdot(2x)^2-\frac{5\cdot 3\cdot 1}{2\cdot 4\cdot 6}\cdot(2x)^3-\frac{5\cdot 3\cdot 1\cdot 1}{2\cdot 4\cdot 6\cdot 8}\cdot(2x)^4$

$$-\frac{5\cdot 3\cdot 1\cdot 1\cdot 3}{2\cdot 4\cdot 6\cdot 8\cdot 10}(2x)^5-\text{etc.}$$

LXIX.

(1) 1. $3(a+3b)$; 2. 0. (2) $a^2-4b^2+12bc-9c^2$.

(3) mx^3+nx^2+nx+m. (4) $x-2$. (5) xy.

(7) $4+a-a^2$. (8) ALG. I. P. 26.

(9) 1. $x=\frac{ab}{a+b}$; 2. $x=4$. (10) 1. $5\sqrt{3}-2\sqrt{7}$; 2. 10.

(11) $\log_{.01} 1000=\bar{2}\cdot 5$. (12) 1. $\bar{4}\cdot 8750613$; 2. $1\cdot 4983106$.

(13) $\pm\sqrt{-3},\ 3-\sqrt{-1}$.

LXX.

(1) 1. $\left(\frac{b}{a}\right)^{\frac{3}{4}}$; 2. 3. (2) 1. $x=4,\ -2\frac{2}{3}$; 2. $x=-2$;

3. $x=\pm\frac{3}{2}$; $y=\pm\frac{1}{2}$. (3) 1. $77\frac{1}{2}$; 2. $1\frac{17}{648}$; 3. $\frac{23}{99}$.

(4) $3\frac{1}{9}$. (7) (8) ALG. II. PP. 63, 67, 68.
(9) 1. 1·7781513; 2. $\bar{2}$·4771213; 3. ·0211893. (10) 1·2435556.
(11) $\bar{2}$·480761. (12) (13) TODHUNTER'S ALG. PP. 36, 407.

LXXI.

(1) 1. $x=2, -\frac{1}{7}$; 2. $x=2, -1, -1\pm\sqrt{-3}$; $\dfrac{1\pm\sqrt{-3}}{2}$;
3. $x=1, -2$; 4. $x=5, -6$; $y=3, -4\frac{1}{3}$.
(2) $5a^2b\sqrt{3ab}$; $3\sqrt[3]{5}$; $a^{\frac{1}{r^2-s^2}}$; $\sqrt{7}+\sqrt{2}$; $2\sqrt{3}-\sqrt{-2}$.
(3) 1. $1\frac{1}{6}, 1\frac{1}{3}, 1\frac{1}{2}, 1\frac{2}{3}, 1\frac{5}{6}$; 2. $1\frac{1}{7}, 1\frac{1}{3}, 1\frac{3}{5}$. (4) 1. 0; 2. 81; 3. $\dfrac{3(3^n-2^n)}{2^n}$.
(6) $\frac{1}{2}, \frac{1}{18}$. (8) 2.
(9) 24; 6·666. (10) $x=\dfrac{2l}{b+c}$; $y=\dfrac{2m}{a+c}$; $z=\dfrac{2n}{a+b}$.
(11) £181 16s. $4\frac{4}{11}d$. (12) ALG. II. P. 50.
(13) $x^4-2x^3-9x^2+10x-2$.

LXXII.

(1) $2\frac{137}{975}$. (2) £4 2s. $0\frac{3}{8}d$.
(3) 1. 78·3643725; 2. ·99238725; 3. ·0684405. (4) $4\frac{5}{6}$.
(5) £295. (6) £20134 18s. $1\frac{1}{2}d$. (7) $x=-12$.
(8) 12. (9) 1. 5·894009; 2. ·4401446.
(11) VI. 3, A.; I. 47; II. 5. (12) 7·1365 ft. (13) 1678·792 inches.

LXXIII.

(1) 1. $x=20, -4$; 2. $x=5, 1\frac{1}{3}$; 3. $x=2\frac{1}{2}, \frac{1}{2}$; $y=\frac{1}{2}, 2\frac{1}{2}$.
(2) $2(23-6\sqrt{14})$; $3\sqrt{5}-2\sqrt{3}$. (3) 6.
(6) $\dfrac{\sqrt{3}}{2}, \dfrac{1}{\sqrt{2}}$. (7) (8) PL. TRIG. I. PP. 35, 36.
(9) 1. $x=n\pi, (2n\pm\frac{1}{4})\pi$; 2. $x=\sqrt{2}(\sqrt{3}+1)$.
(10) PL. TRIG. I. P. 87. (11) 2·8; multiply by $\frac{3}{2}$.
(12) PL. TRIG. I. P. 101. (13) $14\sqrt{2}$; $7(\sqrt{6}+\sqrt{2})$.

LXXIV.

(1) 1. $\dfrac{a}{16}-\dfrac{\sqrt{ab}}{4}+\dfrac{3\sqrt{ab^{-1}}}{16}+\dfrac{9b^{-1}}{64}$; 2. $(a+b+c)(b-c)$.
(2) 1. $5+2\sqrt{6}$; 2. 0. (3) 1. $x=7, 2$; 2. $x=\pm\frac{5}{6}$.
(6) 2, 14. (7) $75^g\ 84'\ 76''$. (8) PL. TRIG. I. P. 15.
(9) 439·824 ft. (12) $\sqrt{3}:1$.
(13) $500(\sqrt{6}-\sqrt{2})$; $1000(\sqrt{3}-1)$.

LXXV.

(1) 1. $\frac{7\sqrt{2}}{4}$ 2. $\frac{(7+\sqrt{5})\sqrt{2}}{2}$. (2) 1. $x=1, -\frac{2}{9}$; 2. $x=1\frac{1}{3}, 5\frac{3}{8}$.

(4) 1. $-366\frac{2}{3}$; 2. $518\frac{1}{3}$. (5) 1′ 25″.

(7) $\frac{\sqrt{3}}{2}$; 1. (8) (9) (10) (11) Pl. Trig. pp. 35, 55, 87, 92.

(12) $15(3+\sqrt{3})$. (13) 351.

LXXVI.

(2) 1. $\frac{4+3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{4}$; 2. $3\sqrt{7}-2\sqrt{3}$. (3) Al. I. p. 104.

(4) 1. $x=1\frac{1}{2}, -\frac{1}{2}$; 2. $x=a, y=b$; 3. $x=\pm 6, y=\pm 8, z=\pm 12$.

(5) $8d$. (6) 1. 0; 2. $\frac{211\sqrt{6}}{48}$ 3. $\frac{x^n-1}{(x-1)^2}+\frac{nx^n}{x-1}$

(7) (8) Pl. Trig. I. pp. 7, 9, 35, 50. (9) $\frac{\sqrt{3}}{2}$; $\frac{1}{2}$; $2+\sqrt{3}$.

(13) $A=116^\circ\ 33'\ 54''$, $B=26^\circ\ 33'\ 54''$.

LXXVII.

(1) 1. $6\sqrt{2}-4$; 2. $\sqrt[6]{4000}$. (2) 1. $10\frac{1}{2}, 40\frac{1}{2}$; 2. $22\frac{25}{32}, 62\frac{11}{32}$.

(3) 3, 5, 7. (4) $x=10, -2$; $y=5, -1$.

(5) 5 : 162. (7) $\frac{1}{\sqrt{2}}$; $\frac{1}{\sqrt{3}}$; $\frac{3\sqrt{13}}{13}$; $\frac{\sqrt{13}}{2}$.

(11) Pl. Trig. I. p. 94.

LXXVIII.

(8) $\sqrt{3}$ mi.; $1\frac{1}{2}$ mi. (9) 8 yds. 13 ft. 1153 in.

(10) 15268·176 lbs. (11) $21000\sqrt{3}$ c. f.

(12) 4·1887902 c. f. (13) VI. C.

LXXIX.

(1) Alg. I. p. 54. (2) $\frac{ax-by}{ax+by}$.

(3) 1. $x=\frac{ac}{b}$; 2. $x=0, a\left\{1\pm 2\sqrt{\frac{b}{c}}\right\}$.

(4) 1. ·0001343781; 2. 13116; $x=5·435128$.

(5) 10·1932 in. (6) 25·1328 yds.; 916·09 lbs.

(8) Pl. Trig. I. p. 101. (9) 108·6409 yds. (11) $10800\sqrt{}$

(12) (13) Tate's Geom. and Mens. pp. 37, 79; 379·8168.

LXXX.

2) 1. ·02031379; 2. 1·162144. (3) £370 1·214*s.*
6) 232·7014 yds. (7) Pl. Trig. i. p. 88; 26° 30′ 22″
8) xi. 16; vi. 4. (10) 36·155 yds. per minute.
1) 301·5936 c. f. (12) i. 38. (13) iv. 10, 11.

LXXXI.

1) 20. (2) 6, 8, 10, 12, 14.
3) 1. ·00007473765; 2. ·29287. (4) $210x^6$.
5) 1·2. (6) 44. (7) iii. 20.
8) 45°; $\frac{\sqrt{3}-1}{2\sqrt{2}}$. (9) $2-\sqrt{3}$. (11) $486+250\sqrt{2}$.
2) 121793·4. (13) 4021·248 sq. ft.; 16084·992 cub. ft.

LXXXII.

1) 1. $x=-7, -11$; 2. $x=\frac{1}{2}\{-1\pm\sqrt{13\pm4\sqrt{7}}\}$;
3. $x=\pm3, \pm\frac{5\sqrt{2}}{2}$; $y=\pm2, \pm\frac{\sqrt{2}}{2}$.
2) 1. ·1245208; 2. ·3722812. (3) 17. (4) 8·6247.
5) 881·06. (6) 8011·9; $12\frac{5}{8}$.
7) $\infty, -1, \sqrt{3}$. (10) 48° 7′ 30″.
2) 82° 24′ 39″, 22° 24′ 39″, 75° 10′ 42″. (13) $127·3\sqrt{57}$.

LXXXIII.

1) £11 2*s.* $4\frac{3}{4}$*d.* (2) £841.
3) Ar. p. 61.; ·0031. (4) $\frac{x-7}{x^2+2x-15}$.
5) 1. $x=11, y=7$; 2. $x=7, 5$; 3. $x=\pm3, y=\pm2$.
5) 9, 13, 17, 21, 25, 29. (7) £1014 13*s.* 3·86*d.*
8) 1. $\frac{9}{2(x-1)}-\frac{19}{x-2}+\frac{31}{2(x-3)}$; 2. $\frac{3}{1+x}-\frac{1}{2-3x}$.
0) 2 and 14. (11) $12\sqrt{3}$, 24.
2) 40·363 ft. (13) 42·774.

LXXXIV.

1) £15 0*s.* $7\frac{1}{2}$*d.* (2) $17\frac{8}{11}$ da.
3) 1. 1·230509; 2. ·4721987; 3. ·955753. (4) n^2.
5) $\frac{\sqrt{3}-1}{2\sqrt{2}}$; $\frac{\sqrt{3}+1}{2\sqrt{2}}$. (8) 13330.
) B = 18° 21′ 21″, C = 33° 34′ 39″. (10) $47\frac{3}{4}$.
) The isosceles. (12) Sides of smaller rectangle. $2\frac{1}{4}$ and 3; Area of complements $22\frac{1}{2}$. (13) $3+\sqrt{5} : 8$.

LXXXV.

(1) 1. $x=21, 2\frac{4}{5}$; 2. $x=5, -3\frac{3}{4}$; $y=20, -15$.
(2) ALG. II. P. 67.
(3) 1·70279.
(4) £1479 9s. $4\frac{3}{4}d$.
(8) $A=C=65° 22' 32·5''$.
(9) $\frac{\sqrt{5}-1}{4}, \frac{\sqrt{5}+1}{4}$.
(11) 204·2 ft.
(12) $\frac{200}{3\pi}$ in.
(13) I. 47; II. 12, 5, COR.

LXXXVI.

(1) 1971.
(2) $x=c-b, y=a-c, z=b-a$.
(3) £5000.
(4) ALG. II. P. 63.
(5) 1. ·0001403648; 2. 20·95046.
(6) $n=\frac{\log b-\log (b-ar)}{\log (1+r)}$; 17 yrs. nearly.
(8) 87·55287.
(9) $4\sqrt{3}$, 8.
(11) 1 : 2.
(12) 768π.
(13) 21π.

LXXXVII.

(1) 850.
(2) $\frac{1}{3}$ of its strength.
(3) ALG. I. P. 8.
(4) 1, 3, 5, 7, 9.
(5) $a^{-4}+2a^{-6}x^2+3a^{-8}x^4+4a^{-10}x^6+5a^{-12}x^8+6a^{-14}x^{10}+$ &c.; $12a^{-26}x^{22}$.
(6) 47·19 yrs.
(7) PL. TRIG. I. P. 74.
(8) 239° 16′ 38″.
(10) $\frac{100\sqrt{3}}{3}$.
(11) 73° 10′ 26″.
(12) $x=\sqrt{3}$.
(13) PL. TRIG. I. 23.

LXXXVIII.

(1) £1 1s. $9\frac{1}{3}d$.
(2) $x=\pm\frac{1}{3}, y=\pm 3, \pm\frac{1}{3}$.
(3) 2300.
(4) 1. 1777·777; 2. 7651.
(5) £3946 10s.
(9) 1·4149 mi.
(10) 138 yds.
(11) I. 26, 4.
(12) III. 27; I. 27.
(13) 14 ft.

LXXXIX.

(1) $\frac{1167}{37673}$.
(2) ·13, 1600000.
(3) $\dot{7}1428\dot{5}$; ALG. I. P. 155.
(4) $6x^6+3x^5y-15x^4y^2-12x^3y^3+51x^2y^4-45xy^5+12y^6$.
(5) $(a-b)^3$.
(7) $x=20$.
(9) ALG. II. P. 73.
(10) $1333\frac{1}{3}$ yds.
(11) VI. 4, 17; II. 2.

XC.

(1) 1. 13° 20′; 2. $14^g\ 81^{\backprime}\ 48^{\backprime\backprime}·14\dot{8}$; 3. $\frac{2\pi}{7}$.
(2) PL. TRIG. I. P. 52.
(3) $A=45°, B=30°$.
(4) 3. Put $A=n$ A, $B=(n-2)$ A, in $\sin A+\sin B=2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$.
(5) $A=31° 17' 27·3''$, $B=23° 8' 32·7''$.
(7) If $p=\perp r$, $bp+cp=b^2\sin C+c^2\sin B$.
(8) PL. TRIG. I. P. 102.
(9) 188·1162.
(10) 1884·96 sq. ft.
(11) $A=n\pi, (2n\pm\frac{1}{6})\pi$.
(13) PL. TRIG. I. P. 129.

XCI.

(1) 6. (2) $x+y$. (3) $x-1$.
(4) 1. $x=4$; 2. $x=4y=4$. (5) 5 yds. and 4 yds.
(6) 1. 22·16533; 2. 8·0562. (7) ALG. II. P. 50.
(8) $2n(n+1)+1$. (9) $\frac{1}{2(x-1)}-\frac{4}{x-2}+\frac{7}{2(x-3)}$.
(10) II. 9. (11) III. 31, 32.
(12) III. 22. (13) $x=n\pi$, $(2n\pm\frac{1}{8})\pi$.

XCII.

(1) $26\frac{1}{9}$ yds.; 19*s.* 7*d.* (2) ·0272.
(3) £127 18*s.* 3·9*d.*; 6 per cent. (4) 18750 oz.
(5) ALG. I. P. 87; $\frac{1}{4}$. (6) x^3-x^{-3}. (7) $\pm\left(3a-\frac{x}{2}\right)$.
(8) 1. $x=7$, $y=10$; 2. $x=4$, -1; 3. $x=2$, $y=3$, $z=-5$.
(9) 1. $\sqrt{5}-\sqrt{2}$; 2. $9\sqrt{3}-9\sqrt[3]{5}+3\sqrt{3}\cdot\sqrt[3]{25}-15+5\sqrt{3}\sqrt[3]{5}-5\sqrt[3]{25}$.
(10) ALG. II. P. 83. (11) 20.
(12) ALG. II. P. 112; 1867405. (13) $\frac{5\text{ AB}}{2}$.

XCIII.

(2) ALG. II. P. 65. (3) 1. 32310·07; 2. ·32702.
(4) ALG. II. PP. 72, 104. (5) 15 in. (6) $\sqrt{15}$.
(7) 108°, 120^g, $\frac{3\pi}{5}$; 72°, 80^g, $\frac{2\pi}{5}$; 36°, 40^g, $\frac{\pi}{5}$. (8) 629·83 yds.

XCIV.

(1) £50 8*s.* 9*d.* (2) £25 7*s.* 2*d.*
(3) 1. $x=5$; 2. $x=\frac{ac}{a+b}$, $y=\frac{bc}{a+b}$. (4) £64 16*s.*; £1 5*s.*
(6) I. 10; III. 8; II. 12, 13. (9) 28 ft. (11) $14\sqrt{3}+12\sqrt{14}$.
(12) $\frac{427\pi}{3}$. (13) 1164·93 c. f.; 744 sq. ft.

XCV.

(1) £1963541 13*s.* 4*d.* (2) 27.
(3) 1. $x=5$, $y=2$; 2. $x=\frac{1}{4}(1\mp\sqrt{2})$, $y=-\frac{1}{4}$. (4) 2·406015.
(5) £800; £1200. (6) 50 in.
(7) $1+2^2x+3^2x^2+4^2x^3+5^2x^4+6^2x^5+$ &c. (8) $x=2·366589$.
(9) PL. TRIG. P. 35; $\frac{2\sqrt{2}}{3}$, $\frac{1}{3}$, $2\sqrt{2}$. (11) 101° 32′ 12″.
(12) II. 12. (13) III. 20.

XCVI.

(1) ·0000532; 2·8. (2) 3 cwt. 2 qrs. 26·05 lbs.
(3) $\sqrt{122}:\sqrt{91}$. (4) 6 hr. 4′.
(5) 1. $x=1\frac{1}{2}$; 2. $x=75$. (6) 5.
(7) 1·000200120088. (8) $1+x+x^2+\frac{x^3}{2}+\frac{x^4}{3}+\frac{x^5}{12}+$ etc.
(9) 5; ALG. II. P. 57. (10) $\frac{\pi}{4}$, $\frac{\pi}{5}$, $\frac{2\pi}{3}$.
(12) 33·465 ft. (13) $5\sqrt[3]{4}$.

XCVII.

(1) 1 $\frac{17}{36}$; 2. $6\frac{1}{9}$; 3. $\frac{53}{112}$; 4. $\frac{17}{43}$; 5. $2\frac{8}{35}$. (2) 45. (3) $\frac{x}{4x^2-y^2}$.

(4) $x=\frac{1}{2}\left\{\sqrt[3]{2b-a}\pm\sqrt{\frac{a}{\sqrt[3]{2b-a}}}\right\}$, $y=\frac{1}{2}\left\{\sqrt[3]{2b-a}\mp\sqrt{\frac{a}{\sqrt[3]{2b-a}}}\right\}$.

(5) ALG. II. P. 11. (6) 16·7554, 40·1064. (8) ·79963, 39·989.

(10) B $=144^\circ\ 34'\ 43''$, C $=27^\circ\ 5'\ 42''$, $b=31802{\cdot}55$.

(11) $x=\left\{n+\frac{(-1)^n}{3}\right\}\pi$. (12) 160·85 ft. (13) $\frac{125\pi}{9}$.

XCVIII.

(1) £22983 15*s.* (2) £90981 5*s.* 6*d*

(3) 8 cwt. 1 qr. 21 lbs. (4) 1. $x=36$;

2. $x=\left(\frac{b^q}{a^n}\right)^{\frac{mp}{mp-nq}}$; $y=\left(\frac{a^m}{b^p}\right)^{\frac{nq}{mq-np}}$; 3. $x=\sqrt{\left\{a^2-\left(\frac{b-2a}{3\sqrt[3]{b}}\right)^2\right\}}$.

(5) 17. (7) $\left(\frac{P^{n-q}}{Q^{n-p}}\right)^{\frac{1}{p-q}}$.

(7) $n=1+\frac{\log\left(\frac{b}{a}\right)}{\log\left(\frac{s-a}{s-b}\right)}$. (9) PL. TRIG. I. P. 108.

(10) 4·257169 mi. (12) III. 31; I. 14.

(13) Rad. $=13{\cdot}361$ ft.; volume $=1447{\cdot}23$ ft.

XCIX.

(1) £55 6*s.* $10\frac{1}{2}$*d.* (2) ·0046; 46000; 13*s.* $10\frac{1}{2}$*d.*

(3) 1. $x=46$; 2. $x=\frac{ac}{a+b}$, $y=\frac{bc}{a+b}$. (4) ALG. I. PP. 115, 116, 121.

(5) 1. 1120768; 2. 1·815868. (7) 2295·086.

(8) $\frac{40}{\sqrt{\pi}}$. (9) 80·584. (10) Sum of $\perp$ rs $=\frac{2S}{a}$.

(11) I. 47; II. 5, COR. (12) XI. 10, 17.

(13) AD $=20\sqrt{6}$; Area $=100(6+\sqrt{6})$.

C.

(1) £929 12*s.* $8\frac{1}{4}$*d.* (2) 3·7356; ·01176; 76·9230; 1·2854.

(3) 3·5463. (4) 15·43125; ·4125. (5) $1\frac{1}{3}$ h.

(6) 1. $x=\pm5, \pm3$; $y=\pm3, \pm5$; 2. $x=2, 4$; $y=4, 2$.

(7) $\frac{n(n^2-1)}{3}$. (8) 1297·12.

(9) S A $=777{\cdot}47$; S B $=502{\cdot}16$; S C $=790{\cdot}114$.

(11) I. 34, 26. (12) III. 22; IV. 2.

(13) 150°; $8\sqrt{29+10\sqrt{3}}$.

CI.

(1) $5\frac{7}{120}$. (2) 8. (3) 237500. (4) -1.
(5) $x=4\frac{1}{2}$. (6) 55; $18\frac{1}{3}$; $8\frac{1}{3}$. (9) PL. TRIG. II. P. 28.

(10) 392·9252. (13) $\dfrac{3r^2\sqrt{3}}{\sqrt{3}\sin\theta+2\cos^2\dfrac{\theta}{2}}$.

CII.

(1) 48240. (2) ALG. II. P. 77.
(3) $4\frac{1}{2}$. (4) ALG. I. P. 49. (5) 1.

(6) 1. $\dfrac{5x^2}{2}$; 2. 1. (7) $3\frac{\dot{5}}{8}$.

(8) 1. $x=6$; 2. $x=16,\ 6\pm3\sqrt{13}$; $y=4,\ -6\pm3\sqrt{13}$.

(9) 2·01; x^2+2x-1. (10) $\dfrac{c^2-b^2}{2(a-c)}$. (11) 2.

(13) 1. $40\frac{5}{6}$; 2. $1\frac{1}{2}$. (14) $\frac{1}{216}$.

(15) WOOD'S ALG. P. 212; $x=y+\dfrac{y^2}{\underline{\lvert 2}}+\dfrac{y^3}{\underline{\lvert 3}}+\text{etc.}$

CIII.

(1) £23 15*s.* (2) $1\frac{1}{3}$. (3) 7*s.* 9*d.*
(4) $5\frac{5}{6}$. (5) $2\frac{413}{990}$. (7) WOOD'S ALG. P. 40.

(9) $r=\left(\dfrac{s_2}{s_1}\right)^{\frac{1}{n}}$; $a=\dfrac{{s_1}^2}{s_1+s_2}\left(\dfrac{s_1}{s_2}\right)^{\frac{n-1}{n}}$. (10) $n^2+4n-3+3^{n+1}$.

(11) $\dfrac{np}{mq}$. (12) $\bar{5}$·635483. (13) $\sqrt{10}$.

(14) 1. 1; 2. $\dfrac{1}{4}-\dfrac{2n+3}{2(n+2)(n+3)}$.

CIV.

(1) 1. ·40708; 2. ·625; 3. $\cdot1\dot{8}5714\dot{2}$. (2) 2296.

(3) 23·002. (4) 1. (5) $2a+\dfrac{17b}{3}-\dfrac{3c}{4}$.

(6) $x^6-2x^5y-x^4y^2+4x^3y^3-x^2y^4-2xy^5+y^6$.
(7) $3x-5y$. (8) 1. $x=12$; 2. $x=20,\ y=12$.

(9) Yes. (10) $\dfrac{a^2+b^2}{2}$.

(15) Velocity = 3·6569 mi. an hr. Direction W.N.W.

CV.

(1) 500. (2) £28000000. (4) $-\dfrac{1-\sqrt{2}}{2}$ and $-\dfrac{1+\sqrt{2}}{2}$.

(5) 1. $x=-8$; 2. $x=\pm\sqrt{\dfrac{a}{2b}},\ y=\pm\sqrt{\dfrac{b}{2a}}$.

(6) 600. (7) $-\dfrac{4039}{6^9}$. (11) $90^\circ\ 6'$.

(14) $\theta=\left\{n+(-1)^n\cdot\frac{5}{12}\right\}\pi,\ \left\{n+(-1)^n\cdot\frac{1}{12}\right\}\pi$. (15) I. 47.

CVI.

(1) $30\frac{5}{8}$. (2) £113 9*s.* $1\frac{1}{2}$*d.* (3) Alg. i. p. 109. (4) 1·98734.

(6) Coeff. of x^n in $(1+x)^n.(x+1)^n = 1 + n^2 + \left\{\frac{n(n-1)}{\lfloor 2}\right\}^2 +$ etc.

$$= \text{coeff. of } x^n \text{ in } (1+x)^{2n} = \frac{2n(2n-1)\cdots(n+1)}{\lfloor n} = \frac{\lfloor 2n}{\{\lfloor n\}^2}.$$

(7) 11. (8) 3214; 32140. (12) 6383·73.

(13) Pl. Trig. p. 130; 89° 54′ 4″. (15) i. 5, 37.

CVII.

(1) ·23*d.* (2) ·1581139; 237. (3) $\frac{x^3+x^2+x-3}{3x^4+3x^3+3x^2-17x-2}$.

(4) 1. $x=3$; 2. $x=8, 2$; $y=64, 4$. (5) 62 yrs.

(6) 1101101, 11011010, 101000111, 110110100.

(7) $5+\frac{1}{2}+\frac{1}{1}+\frac{1}{1}+\frac{1}{2}+\frac{1}{10}+$ &c.

(8) ·09999960000479992960011264.

(9) Angle at vertex $=65°\ 7'\ 45''$. (10) $2\sqrt{6}$.

(12) $A=116°\ 13'\ 20''$; $B=26°\ 20'\ 40$; $c=4325{\cdot}126$.

(14) Bisect exter^r. ∠s.

(15) Bisect ∠s of triangle formed by joining the centres.

CVIII.

(1) $x=4$, 1·6. (2) $x=2, y=-1$.

(3) $x=243, 32$; $y=64, 729$. (4) 77 : 14.

(5) ·646. (6) ·00001233072.

(7) $x=6{\cdot}576532$. (8) Alg. ii. p. 56; $-1\frac{3}{8}$.

(9) (10) Pl. Trig p. 61. (11) A segment of a circle, similar to B C.

(12) 4214. (13) i. 34; vi. 1, 4. (15) Con. Sec. p. 12.

CIX.

(1) ·503125. (2) $\frac{11}{14}$; 1240; ·00096875. (3) 15·7129; 5·453.

(4) 1. $x=4$; 2. $x=2, -5, \frac{1}{2}(-3\pm\sqrt{241})$. (5) ·1940783.

(6) Wood's Alg. p. 164. (8) $\frac{19\pi}{6}$. (9) 5·258.

(10) 135° 2′ 25″. (11) 145·32972 yds.

(12) Use the formulæ $\frac{a}{a+b+c} = \frac{\sin A}{\sin A + \sin B + \sin C}$; and
$\sin A + \sin B + \sin C = 4 \cos\frac{1}{2}A \cos\frac{1}{2}B \cos\frac{1}{2}C$.

(14) Pl. Trig. p. 127. (15) Draw planes bisecting 3 dihedral ∠s not having a common vertex.

CX.

(1) £60000. (2) 82·28 and 53·72. (4) $(x+y)(1-x^{-1}y^{-1})$.

(5) 1. $x=8, -2$; 2. $x=3, -9$; $y=5, 4$; $z=4, 5$.

(6) 2·0969100. (7) ·04; 675. (8) 6.

(9) ALG. I. P. 141. (10) $-\dfrac{5x^{-\frac{21}{2}}}{2^{11}}$.

(11) $\left(\dfrac{1+x}{1-x}\right)^n = \left(\dfrac{1-x}{1+x}\right)^{-n} = \left(1-\dfrac{2x}{1+x}\right)^{-n}$. (12) III. 36.

(13) $\left(n+\dfrac{(-1)^n}{6}\right)\pi$. (14) $2+\sqrt{2}$, $2(1+\sqrt{2})$.

(15) $\dfrac{a^2-b^2}{c^2}\sin C+\dfrac{b^2-c^2}{a^2}\sin A+\dfrac{c^2-a^2}{b^2}\sin B=\sin(A-B)+\sin(B-C)+\sin(C-A)$.

CXI.

(1) 56·641...oz. (2) £1750. (3) $\sqrt[3]{10}=2·1544347$.
(4) 1. 254·5363; 2. 17. (6) 4. (7) $y=0, 4, 2$.
(8) ALG. II. P. 83; 20 yrs. (10) 24470. (13) 0, 1, −2.
(14) $\cos^{-1}\left(\dfrac{\cot\alpha}{\cot\beta}\right)$ W. of N. (15) 30·096.

CXII.

(1) ·0021; ·04375. (2) 9*s.* 9¼*d.* = £·4885416̇.
(4) $x=98, 75, 52, 29, 6$; $y=6, 25, 44, 63, 82$. (5) ·50786.
(6) 4. (7) $\dfrac{\sqrt{5}-1}{4}$; $\dfrac{\sqrt{5}+1}{4}$.
(8) PL. TRIG. P. 95. (9) 34¼ nearly. (11) I. 8, 4.
(12) II. 12, 13. (13) (14) (15) CON. SEC. PP. 23, 28, 31.

CXIII.

(1) £11 2*s.* 4¾*d.* (2) 3·590625; 1·43625.
(3) £4 3*s.* 4*d.* (4) $\dfrac{1}{(x-a)^3}$. (5) $2(ac-bd)$.
(6) 1. $x=\pm 3, \mp\sqrt{-1}$; $y=\pm 1, \pm 3\sqrt{-1}$; 2. $x=1-\sqrt{3}, \pm 2$.
(7) 139·8287. (8) 21·673 yrs. (9) $-\frac{78}{25}$.
(10) III. 32. (11) VI. 2. (12) VI. 4, 16, 8.
(13) (14) (15) CON. SEC. PP. 77, 78, 116.

CXIV.

(1) £1800000. (2) ·096.
(3) £4300 10*s.* 7·2*d.* (4) $a=10$, $d=-2$.
(7) 1. $x=7, -1\frac{3}{4}$; 2. $x=16, 146·41$; $y=81, ·81$.
(9) 100 wks.; 24750 lbs. (11) ALG. II. P. 73.
(12) VI. 2. (13) (14) (15) CON. SEC. PP. 181, 168, 175.
(14) 168 ft. (15) 6·4; 20π.

CXV.

(1) £128 3*s.*; £598 0*s.* 8*d.*; £170 17*s.* 4*d.*; $\frac{4}{21}$. (2) $7\frac{1}{2}$.
(3) 55 : 53. (5) 9·936095.
(6) 13·33654. (8) $xy^{-1}+\frac{\sqrt{-1}}{2}-x^{-1}y$.
(9) 1. $x=15, y=12$; 2. $x=a\pm b, y=a\mp b$.
(10) $\text{Sin } 15^\circ=\frac{\sqrt{3}-1}{2\sqrt{2}}$. (13) Centre $(-2, 3)$; rad. $=4$.
(14) $\frac{27\sqrt{58}}{58}$. (15) Hind's Trig. p. 182.

CXVI.

(1) 365·249718. (2) £2147 10*s.* 3*d.*
(3) $\frac{(a+b^{\frac{1}{2}}-c^{\frac{1}{2}})\{a^2-(b^{\frac{1}{2}}-c^{\frac{1}{2}})^2\}}{a^4-2a^2(b+c)+(b-c)^2}$. (6) $x=\frac{3}{16}$.
(8) Pl. Trig. p. 52. (11) I. 32.
(12) III. 22; I. 13, 29. (13) Hind's Trig. p. 183. (14) 2·4.

CXVII.

(1) 3·3463. (2) £30. (3) ·00001; 8.
(4) $\bar{3}$·544068. (6) 1. $x=\frac{bc}{a}$; 2. $x=3, \frac{1}{3}, -2, -\frac{1}{2}$.
(7) 1. 1; 2. a^2-b^2. (8) 45°, 225°, 405°, 585°.
(10) 1207·13. (12) I. 27; III. 27, 31. (13) $\frac{1}{3}, \frac{1}{7}$.
(14) (15) Equation to common chord, $y-4x+15=0$; equation to line joining centres $4y-x-15=0$.

CXVIII.

(1) £158 8*s.* $7\frac{7}{8}$*d.*
(2) 6*s.* $8\frac{1}{4}$*d.*; 7 h. 51′ 36″; 3 fur. 32 po. 2 yds. 2 ft. 3 in.
(3) $\cdot 9\dot{6}\dot{2}$, $\cdot\dot{3}7\dot{0}$. (4) $(x^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}})-z^{\frac{1}{3}}(x^{\frac{1}{3}}+y^{\frac{1}{3}}-z^{\frac{1}{3}})$.
(5) $\frac{ax+by}{ax-by}$. (6) Apply the result of Quest. 4.
(7) 1. ·124546; 2. ·0001128889. (8) Alg. II. p. 104.
(11) 2·165 yds. (12) 993·8727 yds.
(13) (14) Hind's Trig. pp. 165. 199, 321. (15) Con. Sec. p. 175.

CXIX.

(1) 1. $\frac{15}{1792}$; 2. ·00625; 3. $\frac{31}{73}$. (2) £2. (3) ·0967.
(4) 1. x^4-5x^2+4; 2. $1+5x+15x^2+45x^3+$.
(5) $a^{\frac{2}{3}}b^{-\frac{1}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+a^{-\frac{1}{3}}b^{\frac{2}{3}}$.
(6) 1. $x=\pm 2a, \pm a\sqrt{-6}$; 2. $x=12, y=4, z=24$. (7) 64.
(8) 55. (9) 11, 20, 1100. (11) 27·4945.
(12) Tate's Geometry, p. 173.
(13) 20° 54′ 49″, 31° 43′ 2″, 37° 22′ 38″.
(14) Con. Sec. p. 163. (15) $\frac{a^2b^2}{a^2y_1^2+b^2x_1^2}$.

CXX.

(1) 24854 mi. 1504 yds. 1 ft. 4 in. (2) 8·305667 in.
(3) £6, £19. (4) 26·96525 ft.
(5) ·96696. (6) 36° 52′ 11″·63; 53° 7′ 48″·37; 90°.

(9) $\sin\theta=\sqrt{\dfrac{m^2-n^2}{1-n^2}}$; $\cos\theta=\sqrt{\dfrac{1-m^2}{1-n^2}}$; $\sin\phi=\dfrac{1}{m}\sqrt{\dfrac{m^2-n^2}{1-n^2}}$;

$\cos\phi=\dfrac{n}{m}\sqrt{\dfrac{1-m^2}{1-n^2}}$. (10) 107·2377 ft.

(11) $\dfrac{1}{c}\sqrt{\{(c+r+r')(c-r+r')(c+r-r')(-c+r+r')\}}$.

(12) IV. 5; I. 29, 34. (13) (14) (15) CON. SEC. PP. 13, 193.

CXXI.

(1) £10 15*s*. (2) When the first has travelled, 80 mi. and $133\frac{1}{3}$ mi.
(4) 1. $x=11, 5$; 2. $x=4, 1$; $y=1, 4$; $\sqrt{x}=6, -3$; $\sqrt{y}=-3, 6$.
3. $11x^2-52x+61-10\sqrt{11x^2-52x+61}+25=x^2-12x+36$;
$x=3, 2, \frac{1}{2}(3\pm\sqrt{33})$. (5) $x=4$.

(6) $\dfrac{\lfloor 20}{\lfloor 8\,\lfloor 12}\times\dfrac{\lfloor 6}{\lfloor 2\,\lfloor 4}\times 4$. (7) ·3010300;

(8) ·9942207. (9) $10^{\frac{4}{3}}=21{\cdot}54434$.
(11) 68° 22′ 58′, 55° 24′ 2″. (12) 178 ft. nearly.
(13) (14) CON. SEC. PP. 156, 186. (15) $(15a, 2a\sqrt{15})$.

CXXII.

(1) £6 5*s*.; 10 p. c. (2) £120090 15*s*.
(3) $1\frac{13}{32}$. (4) 18030; $\frac{7}{198}$.
(5) £468 11*s*. 3*d*. (6) 69·1522; $6\frac{7}{15}$.

(7) 1. $x=-13, -11$; 2. $x=\dfrac{\pm 10\sqrt{21}}{7}$; $y=\pm\dfrac{2\sqrt{21}}{7}$.

(8) 1, 4, 7; 25, 4, −17. (9) 1. $\dfrac{\lfloor 98}{\lfloor 10\,\lfloor 88}$; 2. $\dfrac{\lfloor 98}{\lfloor 12\,\lfloor 86}$.

(10) VI. 4, 1. (11) VI. 19. (12) 17°·18873385.

(15) ·422. (16) $\dfrac{a^2+c^2}{a-c}$.

(17) 269·4. (18) $750\sqrt{3}$.

CXXIII.

(1) £42 8*s*.; £5 12*s*. (2) £330. (3) 6800 : 7221.
(4) 3·4641016; ALG. II. P. 8. (5) $\frac{4}{7}$.
(7) $(x^2-y^2)(x^2-4y^2)$. (8) 1. 0; 2. 0.
(9) 1. $x=3$; 2. $x=a, y=b, z=-c$; 3. $x=2-\sqrt{3}, 2(\sqrt{3}-2)$.
(10) Rate, 20 mi.; distance, 140 mi. (11) $\dfrac{2n}{5}, \dfrac{3n}{10}, \dfrac{n}{5}, \dfrac{n}{10}$.

(12) I. 20. (13) I. 39.
(14) I. 47; VI. 8. (17) $n\pi$, $(2n \pm \frac{1}{15})\pi$.
(18) $A = 67° 48' 26''$ or $112° 11' 34''$; $C = 73° 31' 34''$ or $29° 8' 26''$;
$c = 181\cdot3257$ or $92\cdot0771$.

CXXIV.

(1) £70 18*s.* 8*d.* (2) $x = \frac{a^4 \pm 2a^2b^2 + 2b^4}{4(a^2 \pm b^2)}$, $y = \frac{ab}{2}\sqrt{\frac{2b^2 \pm a^2}{a^2 \pm b^2}}$
(4) $B = 57° 3' 30''$; $C = 52° 36' 30''$; $c = 806\cdot5965$.
(5) 13·72947 miles. (6) 169·4391 yds.
(7) 1194·856 yds. (8) 562600; 4·768 ft.
(9) $y = \frac{x}{\sqrt{3}}$; $x = 0$. (10) CON. SEC. P. 88.
(12) $\left(a\sqrt{\frac{a^2-2b^2}{a^2-b^2}}, \frac{b^2}{\sqrt{a^2-b^2}}\right)$. (13) $c^2 = a^2l^2 - b^2m^2$.
(14) An ellipse; centre, $\left(\frac{8}{15}, \frac{8}{15}\right)$; $a = \frac{2\sqrt{30}}{15}$; $b = \frac{2\sqrt{2}}{15}$; $\theta = 45°$.
(15) CALC. PP. 7, 11, 20. (16) $1 + x - \frac{2x^3}{\lfloor 3} - \frac{4x^4}{\lfloor 4} \dots$
(17) 1. $\sqrt{ax + x^2} + \frac{a}{2} \log \left\{\sqrt{x^2 + ax} + x + \frac{a}{2}\right\}$; 2. $x + \frac{3}{4} \log \frac{x-2}{x+2}$;
3. $\frac{e^{ax}(a \sin cx - c \cos cx)}{a^2 + c^2}$. (18) $\frac{a^2}{2}(\pi - 2)$.

CXXV.

(1) AR. PP. 91, 105; £1120. (2) $1\frac{2}{3}$. (3) £270.
(5) 1. $x = 4, -\frac{1}{2}$; 2. $x = 8, y = 2, \frac{1}{2}$. (6) 139.
(7) III. 36. (8) 92267. (9) $\frac{2000\sqrt{2}}{3}$.
(10) CON. SEC. PP. 17, 22. (11) $y - 2x - 3 = 0$; $y + \frac{x}{2} + \frac{1}{2} = 0$.
(12) Centre (3, 5); rad. = 7; $3y - 5x = 0$.
(13) (14) CON. SEC. PP. 167, 131, 145, 175, 178.
(16) $A = 120°$; $a = 109° 28' 16''$.
(17) 1. $-3x\sqrt{a^2 - x^2}$; 2. $\frac{2x(2 - x^2)}{\sqrt{x^2 - 1}\,(x^4 - x^2 + 1)}$;
3. $\frac{m \cos(m-1)x}{(\cos x)^{m+1}}$; 4. $\frac{1}{1 + x^2}$. (18) Height = rad. of circle × 3.
(19) 1. $\frac{2bx - a}{2a^2x^2} - \frac{b^2}{a^3} \log \frac{a + bx}{x}$; 2. $-\frac{1}{3}(1 - x^2)^{\frac{3}{2}}(x^2 + 2)$;
3. $\frac{1}{2}(x - \sin x \cos x)$. (20) CALC. P. 157.

CXXVI.

(1) $176\frac{41}{44}$ tons. (3) $x = 6$. (4) WOOD'S ALG. P. 368.
(7) 606·3228. (9) PL. TRIG. P. 108.

(10) 273·72 ft.; 158·03 ft. (11) TATE'S GEOM. P. 78.
(12) $166\frac{2}{3}$ c. ft. (13) (14) HIND'S TRIG. PP. 173, 177.
(17) $x=\dfrac{a(5+\sqrt{13})}{6}$ gives a max.; $x=\dfrac{a(5-\sqrt{13})}{6}$, a min.
(18) CALC. P. 193.

CXXVII.

(1) £2505 4s. $11\frac{1}{4}d$. (2) ALG. P. 63.
(3) 1. 21·32, 2. $5\frac{26}{33}$. (4) 1. ·23046; 2. $\frac{1}{12}$.
(5) 1. $x=\dfrac{c^2-ab}{a+b-2c}$; 2. $x=1, -\dfrac{3\mp\sqrt{5}}{2}$ (7) 9.
(8) ·0001343781. (9) £740 2s. 5d.
(10) $\dfrac{\sqrt{3}}{2}$; $\left(n+\dfrac{(-1)^n}{15}\right)\pi$. (11) III. 31.
(12) I. 29; III. 22. (16) CALC. P. 19.
(17) 1. $-\dfrac{x\sin\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}$; 2. $\dfrac{x\cos\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}$; 3. $\dfrac{x}{\sqrt{x^2+a^2}}$.
(18) Ratio of the sides $2n : m+n$.

CXXVIII.

(1) $x=-1, 1, 3, 5$. (3) b. (5) $\dfrac{(n-r+1)x}{ra}$; 5^8.
(7) 700. (8) 400 ft. nearly.
(9) PL. TRIG. I. P. 74; II. P. 7. (10) II. 12, 13.
(11) VI. 8. (12) VI. 8. (13) HIND'S TRIG. P. 179.
(14) 1. $y=\dfrac{a\pm 1}{a\mp 1}x$; 2. CON. SEC. P. 30.
(16) CALC. P. 10; 1. $\dfrac{1}{\sqrt{a^2+x^2}}$; 2. $\dfrac{2}{1+x^2}$.
(17) CALC. P. 68. (18) $h=2\sqrt[3]{\dfrac{3v}{\pi}}$.

CXXIX.

(1) 1. $\frac{3}{100}$; $\frac{231}{8000}$. (2) £4000. (3) £325 10s.
(4) $\sqrt{10}$. (5) ·1599022. (6) £1362 1s. 8·16d.
(7) 1. $x=-1, 3, -2$; 2. $x=-4, 3, -\frac{1}{3}$. (8) ALG. II. P. 60.
(10) 45°, 225°. (13) $\tan^{-1}(-3)$.
(14) $xx'+yy'-r^2=0$. (15) (16) CON. SEC. PP. 161, 137.
(17) $x=m$ gives a max.; $x=-m$, a min. (18) CALC. P. 37.

CXXX.

(1) 1. $x=3$; 2. $x=a, \dfrac{a+b}{2}$; $y=b, \dfrac{a+b}{2}$.
(2) 1. ·2776574; 2. ·3215509; 3. 1·5811388. (4) 7393; 8.
(7) 91° 52′ 43·6″. (8) 1514·396 and 4163·745 yds.
(9) $\dfrac{625\pi}{3}$; 1125π. (10) 3593·55 ft.; 85° 48′.

(11) 1. III. 21; 2. III. 31; I. 47. (12) III. 30.

(13) $x^2+y^2+xy-2a\sqrt{3}(x+y)+3a^2=0$. (14) Con. Sec. p. 163.

(15) $\left(-\dfrac{a}{99}, -\dfrac{70a}{99}\right)$. (17) $e\left(1-\dfrac{x^2}{\underline{|2}}+\dfrac{4x^4}{\underline{|4}}-\dfrac{31x^6}{\underline{|6}}\text{ etc.}\right)$

(18) Calc. p. 149.

CXXXI.

(1) $1\frac{3}{11}$. (3) 1000. (4) 600.

(5) $\bar{5}\cdot3979400$. (6) 16·6096; 10^5. (7) $\dfrac{x^2(x^2+a^2)}{a^4}$.

(9) $x=9, 1;\ y=3;\ z=1, 9$. (10) 5.

(12) 1122·243 yds. (14) Pl. Trig. II. p.

(15) $y-3x-2=0;\ y+\dfrac{x}{3}-1=0$.

(18) 1. $x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\dfrac{x^7}{7}+\dfrac{x^9}{9}-$ etc.; 2. $x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}+\dfrac{x^5}{5}-$etc.

CXXXII.

(1) $91\frac{1}{4}$. (2) £127 = 160 Nap.

(3) $(x^2-y^2)^2+z^2(x^2-y^2)+z^4$. (4) Alg. II. p. 104.

(5) $x=4, -2$. (6) 60°. (9) $50(243+125\sqrt{2})$.

(10) Tate's Geom. p. 89. (11) $\dfrac{10\pi}{3}$. (12) I. 34.

(13) Hind's Trig. p. 168. (14) Centre, $(\frac{1}{2}, 2)$; rad. $=3$; $y-4x=0$.

(16) $x+\dfrac{x^3}{\underline{|2}}+\dfrac{2x^3}{\underline{|3}}+\dfrac{9x^5}{\underline{|5}}+$. (18) $\dfrac{x'}{x^{\frac{1}{3}}}+\dfrac{y'}{y^{\frac{1}{3}}}=a^{\frac{2}{3}};\ a$.

CXXXIII.

(1) £5 12*s*. $8\frac{3}{4}$*d*. (2) 12*s*. 6*d*. = £·625.

(3) £67 9*s*. 0·816*d*. (4) 1. $x=10$; 2. $x=-7, -9$.

(5) Alg. I. p. 116; £6 8*s*. (6) 1. 3·266552; 2. ·5918233.

(7) $c^2=2ab\times\dfrac{1+\left(\dfrac{a-b}{a+b}\right)^2}{1-\left(\dfrac{a-b}{a+b}\right)^2}$; apply the formula $\log\dfrac{1+x}{1-x}$

$$=2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+\text{ etc.}\right).$$

(9) Height $=40\sqrt{6}$ ft.; distance $=40(\sqrt{2}+\sqrt{14})$ ft.

(10) 1085·13. (11) III. 32, 22; I. 29.

(12) III. 31, 3. (13) 130° 3′ 10″.

(14) Hind's Trig. p. 187. (15) Con. Sec. p. 172.

(16) $\rho=\frac{1}{36}$. (17) Calc. p. 159; $\dfrac{a^2}{2}$.

(18) $\text{Sin}^{-1}x+\dfrac{h}{(1-x^2)^{\frac{1}{2}}}+\dfrac{x}{(1-x^2)^{\frac{3}{2}}}\cdot\dfrac{h^2}{\underline{|2}}+\dfrac{1+2x^2}{(1-x^2)^{\frac{5}{2}}}\cdot\dfrac{h^3}{\underline{|3}}$

$$+\frac{3x(3+2x^2)}{(1-x^2)^{\frac{7}{2}}}\cdot\frac{h^4}{\underline{|4}}\text{ etc.}$$

CXXXIV.

(1) £9060. (2) 1200; 360 Eng. and Sc., 480 Ir.
(3) $x=1, -2$. (4) $x=9, 4$; $y=4, 9$.
(5) TODHUNTER'S ALG. P. 179. (6) 12 days. 780 miles.
(7) $1707\frac{1}{6}$. (8) 30·044 ft.
(12) III. 18, 36; I. 15. (13) $\left(-\frac{15}{17}, \frac{20}{17}\right)$; $\frac{72\sqrt{5}}{85}$.
(14) (15) CON. SEC. PP. 185, 191. (16) $3x^2+2x+1$.
(17) $\frac{1}{2}\left(\frac{\pi}{2}+a\right)$ gives a max.; $\frac{1}{2}\left(\frac{3\pi}{2}+a\right)$, a min. (18) $r^2d\theta$.

CXXXV.

(1) 6·16; 5·2164. (2) £900; 5*d*. (6) $x=1, 1, 1, -7$.
(7) $141x^6$. (11) $R=\frac{b}{2\sin B}$.
(12) 339·1585. (13) HIND'S TRIG. P. 171.
(18) 1. $\frac{1}{a}\tan^{-1}\frac{x}{a}$; 2. $\log\tan\frac{1}{2}\left(\frac{\pi}{2}+x\right)$; 3. $\log\tan\frac{x}{2}$.

CXXXVI.

(1) $213\frac{1}{3}$. (2) 15. (3) £1900 and £9 10*s*.
(4) 1. $x=4, y=9, z=16$; 2. $x=\frac{1}{2}\{1-\sqrt{3}\mp\sqrt{2(4+\sqrt{3})}\}$,
$\frac{1}{2}\{1+\sqrt{3}\pm\sqrt{2(4-\sqrt{3})}\}$; 3. $x=6$.
(5) 10 hrs. and 6 hrs. (6) 20. (7) 31864.
(8) $x^3-qx^2+prx-r^2=0$. (9) $x=3, 1, -2\pm\sqrt{-3}$.
(10) 15°. (11) 108° 15′ 57″, 34° 44′ 3″, 5529·175, 110·89.
(12) 787·84 yds. (13) TOD. CON. SEC. P. 156.
(14) $\cos\theta=-\frac{e+e'}{1+ee'}$; $r=a(1+ee')$. (15) CON. SEC. P. 315.
(16) CALC. P. 311; 1250π. (17) $\frac{(av-bu)\sin\theta}{(u^2-2uv\cos\theta+v^2)^{\frac{3}{2}}}$.
(18) $1+x+\frac{x^2}{2}-\frac{x^3}{6}-\frac{7x^4}{24}\ldots$

CXXXVII.

(1) 1. $x=-2$; 2. $x=\frac{a-b}{2}$. (3) 13.
(4) 1. ·0406276; 2. 1·162144. (5) 45°, 60°, 75°.
(6) 6236·549. (7) 1095·47. (8) 7·831 hrs.
(9) 119·38, $h=2·409$. (10) III. 32; VI. 4.
(11) The line bisecting the sides. (12) $32\frac{4}{5}$.
(13) (14) (15) CON. SEC. PP. 315, 191. (16) CALC. P. 231.
(17) TOD. INT. CALC. P. 3, 22.
(18) 1. $-\frac{1}{2}\left\{\frac{\sin(a+b)x}{a+b}-\frac{\sin(a-b)x}{a-b}\right\}$;
2. $x^2\sin x+2(x\cos x-\sin x)$;
3. $x+\frac{1}{a-b}\left\{a^2\log(x-a)-b^2\log(x-b)\right\}$.

CXXXVIII.

(1) 8·0259. (2) 40. (3) £3 17*s.* $10\frac{1}{2}$*d.*

(4) $\frac{1}{abc}$. (5) 1. $x=4, y=3, z=5$; 2. $x=\frac{5}{2}, -\frac{3}{2}$; $z=3, -5$.

(6) 150 miles. (7) 3 ft. (8) 7·0735 in. (9) 38·775 c.f.

(10) 72 ft. (11) 1 mi., ·923497 mi. (12) II. 11.

(13) $\frac{a(a \pm b)}{\sqrt{a^2 \mp ab + b^2}}$. (14) CALC. P. 136.

(15) $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$. (17) $-\frac{5}{3}$. (18) CALC. P. 161.

CXXXIX.

(1) £26 12*s.* (2) 14 yds. 1 ft. (3) £56 8*s.* 4*d.*

(4) 1. $x=1\frac{1}{3}$; 2. $x=12, y=20, z=30$;

3. $y=\frac{1\pm 3}{2}, \frac{1\pm\sqrt{-47}}{2}, \frac{1\pm 3\sqrt{5}}{2}, \frac{1\pm\sqrt{-11}}{2}$; $x=\sqrt{y\pm 7}$.

(6) $p^2=3q$. (7) 2·714417; $x=2·8934$.

(8) 336. (9) 9 or 16.

(10) 6·336. (11) I. 30; VI. 4.

(12) $PA=24\sqrt{5}$, $PB=8\sqrt{10}$, $PC=12\sqrt{5}$, $PD=24\sqrt{10}$.

(15) 1. $\frac{2a}{x^2-a^2}$; 2. $-\operatorname{cosec}^2 x$;

3. $(\tan x)^{\cot^{-1}x}\left\{2 \operatorname{cosec} 2x \cot^{-1}x - \frac{\log \tan x}{1+x^2}\right\}$;

4. $-\sin x \sin(\cos x) \sin \cos(\cos x)$.

(16) $x=a$ gives a min., $x=\frac{a}{5}$ a max.

(17) $\rho=\frac{a}{\sqrt{2}}$; centre $\left(\frac{3a}{4}, \frac{3a}{4}\right)$. (18) 1. $x+\log\frac{x-3}{x-2}$;

2. $2\tan^{-1}x-\frac{1}{2}\tan^{-1}\frac{x}{2}$; 3. $\log\sqrt{x+2\sqrt{x^2+4}}+\frac{1}{2}\tan^{-1}\frac{x}{2}$;

4. $\frac{x^4}{4}\left\{(\log x)^2-\frac{1}{2}\log x+\frac{1}{8}\right\}$.

CXL.

(2) 99. (3) 2·0945. (4) WOOD'S ALG. P. 382.

(6) $a=\frac{s \sin A}{2\cos\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C}$. (7) 124·3 ft.

(8) 1·7919 miles. (9) $\angle DBA=85° 13' 24''$.

(10) 695·9625 sq. ft.; 709·3142 c. f. (11) III. 36; VI. 13.

(13) Rad. of cylinder $=\frac{hr}{2(h-r)}$: h being the height, r the rad. of base of cone; h must be $>2r$. (14) $\rho=\frac{(a^2-e^2x^2)^{\frac{3}{2}}}{ab}=\frac{(r_1r_2)^{\frac{3}{2}}}{ab}$.

(15) 1. $x+\frac{a^2\log(x-a)}{(a-b)(a-c)}+\frac{b^2\log(x-b)}{(a-b)(b-c)}+\frac{c^2\log(x-c)}{(a-c)(b-c)}$;

2. $\frac{2}{25}\log\frac{x-3}{x+2}-\frac{3}{5(x-3)}$. (16) CALC. P. 311.

(18) 1. n even, $\int_0^{\frac{\pi}{2}}(\sin x)^n dx=\frac{(n-1)(n-3)(n-5)\ldots\ldots 1}{n(n-2)(n-4)\ldots\ldots 2}\cdot\frac{\pi}{2}$;

n, odd, $\int_0^{\frac{\pi}{2}}(\sin x)^n dx=\frac{(n-2)(n-4)(n-6)\ldots\ldots 2}{(n-1)(n-3)(n-5)\ldots\ldots 3}$;

2. $\int_0^{\infty}e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$.

CXLI.

(1) 1. ·1875; 2. ·948; ·928. (2) $20505\frac{89}{144}$.
(3) £1500. (4) £2 15*s*. (5) $3\frac{9}{14}$.
(6) 28·3613...; 33·3613.... (7) 61. (8) 15.
(10) WOOD'S ALG. P. 365. (11) I. 39.
(13) HIND'S TRIG. P. 190. (15) (16) CON. SEC. PP. 155, 158, 159.
(17) 1. $\frac{\pi a^2}{3}$; 2. πa^2. (18) $y'-b=-\frac{b}{a}(x'-a)$.

CXLII.

(1) 9*s*. 4*d*. (2) $7\frac{1}{6}$. (3) $\frac{9}{11}$.
(4) 1. $\cdot 652\dot{4}$; 2. $\frac{17}{300000}$. (5) Length $=86\cdot 4$ in.
(6) 1. $x=16, y=35$; 2. $x=5, 7\frac{2}{9}$; 3. $x=1\frac{3}{5}$; $y=5$; $z=-6$.
(7) 720. (8) 3 days.
(13) $\frac{2}{1+x^2}$; $\frac{1}{1-x^4}$. (14) $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+$ etc.
(15) $\frac{200\sqrt{3}}{3}$. (17) $\rho=4a\sqrt{2}$. (18) $\frac{4a^2}{5}$.

CXLIII.

(1) ALG. II. P. 80. (2) £432, £360, £294.
(3) 203975; 1·2909944.
(4) 1. $x=28\frac{16}{37}$; 2. $x=32$; $y=15$; $z=-7\frac{1}{2}$. (5) 126.
(6) $3+\frac{1}{3}+\frac{1}{6}+\frac{1}{3}+\frac{1}{6}+$ etc. (7) (8) PL. TRIG. 85, 126.
(10) £903 11*s*. $10\frac{1}{2}d$. (11) $\frac{\pi a^3\sqrt{3}}{2}$.
(12) 11, 5, 2. (16) CALC. P. 141.
(17) 1. $\frac{q}{p+q}x^{\frac{p+q}{q}}$; 2. $-\frac{1}{2}\log(a^2-x^2)$; 3. $\frac{1}{a}\sec^{-1}\frac{x}{a}$.
(18) CALC. P. 157.

CXLIV.

(1) 2 ft. × 1 ft. and 2 ft. 6 in. × 1 ft. 4 in.
(2) 1. $x=\frac{a}{b}, \frac{b}{a}$; 2. $x=6, 4$; $y=4, 6$; $z=5$. (7) $\frac{35\sqrt{6}}{24}$.

(9) $93^\circ\ 49'$. (10) Pl. Trig. II. p. 7.
(11) $22^\circ\ 19'\ 54''$; $49^\circ\ 27'\ 30''$; $108^\circ\ 12'\ 36''$. (12) 114 ft.
(13) $B=59^\circ\ 15'\ 57''$ or $120^\circ\ 44'\ 3''$; $C=131^\circ\ 29'\ 46''$ or $24^\circ\ 37'\ 28''$.
(14) $99x-27y-79=0$; $21x+77y-1=0$.
(15) Con. Sec. p. 186. (16) $x=1$ gives a min.
(17) 1. $2\,(-1)^{n-1}.\dfrac{\lfloor n-3}{x^{n-2}}$;

2. $e^x\{x^3+3nx^2+3n\,(n-1)\,x+n\,(n-1)\,(n-2)\}$.

(18) 1. $\dfrac{1}{2}\sin^{-1}\left(\dfrac{x^2}{a^2}\right)$; 2. $\log\sqrt{\dfrac{(x+3)^3}{x+1}}$; 3. $\dfrac{1}{2}\log\tan\left(\dfrac{\pi}{4}+\theta\right)$;

4. $\log\,(\theta+\sin\theta)$.

CXLV.

(1) $6{\cdot}\dot{2}\dot{6}d$. (2) £50 13*s.* $10\frac{2}{3}d$. (3) $\cdot 08970717\dot{5}9\dot{2}$.
(4) 24 wks. (5) $x=3,\ \frac{5}{3}$. (6) $B^2C=3AC^2=9ABD$.
(8) $\pm\frac{1}{2}$. (9) ·258819. (10) 5426·9; 1122·3.
(11) $1250\,(2-\sqrt{3})$ yds. (12) 45360.
(13) (14) Calc. p. 1, 6, 45.
(15) $x-\dfrac{x^3}{\lfloor 3}+\dfrac{x^5}{\lfloor 5}-$etc.; Calc. p. 134.
(18) 1. $\log\sqrt{\dfrac{a+x}{a-x}}$; 2. $\dfrac{1}{5\,(a^2-x^2)^{\frac{5}{2}}}$; 3. $\dfrac{2}{3}\tan^{-1}\dfrac{\tan x}{2}-\dfrac{x}{3}$;

4. $\dfrac{1}{\sqrt{2}}\log\left\{\dfrac{x}{2+x+\sqrt{4+4x+2x^2}}\right\}$.

CXLVI.

(1) 200. (4) 1. $x=\dfrac{43\pm 7\sqrt{37}}{2},\ \dfrac{11\mp\sqrt{21}}{2}$; 2. $x=1{\cdot}769\ldots$

(5) 1. 574; 2. $\dfrac{3^n-(-1)^n}{2^4.3^{n-2}}\pm\dfrac{n}{4.3^{n-1}}$. (7) 111·1838.

(9) 175·9 ft. (10) $\sqrt{3}$ yds. (11) I. Def. 15. (12) VI. 4. 1.
(13) $63^\circ\ 38'\ 22{\cdot}5''$; 60°. (14) Con. Sec. p. 140.
(16) $3y+5x+15=0$. (18) Calc. pp. 296, 304.

CXLVII.

(1) £1600. (2) 1. ·4442054; 2. 4856·0865.
(4) $x=\frac{1}{3},-\frac{1}{8}$; $y=\frac{1}{4},-\frac{1}{7}$; $z=\frac{1}{6},\frac{1}{28}$. (5) $r=\pm 2,\ a=3,-9$.
(6) 156. (9) 5773·081 yds. (10) Hexagon.
(11) $\cos^{-1}\frac{1}{3}$. (12) XI. 19.
(15) Tod. Dif. Calc. p. 325. (16) πa.
(17) 1. $\dfrac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left\{\sqrt{\dfrac{a-b}{a+b}}\,.\,\tan\dfrac{x}{2}\right\}$; 2. $\log\left\{x+\sqrt{x^2+a^2}\right\}$;

3. $\sin\theta-\theta\cos\theta$. (18) Calc. p. 310.

CXLVIII.

$\frac{2}{7}$; $\frac{3}{14}$. (2) £8125000. (3) £154678 12*s.* $7\frac{1}{2}$*d.*

$x+1$. (5) 85. (7) 1·44354.

$x=2, 2, 2, -3$. (9) $A=91^\circ\ 5'\ 57''$, $B=27^\circ\ 1'\ 3''$, $c=3623$.

£2 9*s.* 4*d.* (11) 29179 ft. and 17507·4 ft.

3·0586. (13) CALC. P. 66.

Ratio of sides 2 : 1. (15) $x^4+\frac{4}{3}x^6+\frac{6}{5}x^8+$ etc.

1. $\dfrac{a^{m+x}}{\log a}$; 2. $\dfrac{1}{a}\log\dfrac{x}{\sqrt{a^2+x^2}+a}$; 3. $\dfrac{1}{3}(1+x^2)^{\frac{1}{2}}(x^2-2)$.

CALC. P. 155.

CXLIX.

$\frac{3}{4}$; 1609·31... (2) 75 wks. (3) 8000.

13. (5) 37, 39, 41 miles, nearly.

$-7 . 2^5 . 3^7$. (8) $x=11, 11, -1, -3$.

76·545 mi. (10) 6 and 10 feet.

$5\sqrt{455}$. (12) 1570·8.

1. The lines $y-\dfrac{2x}{3}+\dfrac{1}{3}=0$ and $y+\dfrac{5x}{7}-\dfrac{2}{7}=0$; 2. The point $(3, -5)$.

1. $x^x(1+\log x)$; $x^x\{(1+\log x)^2+x^{-1}\}$;
2. $e^{\tan x}.(1+x\sec^2 x)$; $e^{\tan x}\sec^2 x\{2+x(\sec^2 x+\tan x)\}$;
3. $\dfrac{1}{1+x^2}$; $-\dfrac{2x}{(1+x^2)^2}$.

CALC. P. 243. (18) TOD. DIF. CAL. P. 344; $\dfrac{a^2}{b}$.

CL.

b^2. (4) 1. $x=1\pm\sqrt{-1}$; $-1\pm\sqrt{3}$; 2. $x=\pm 3, \mp 5\frac{1}{2}$; $y=\pm 1, \mp 16$.

$\frac{5}{1}, \frac{51}{10}, \frac{515}{101}$. (6) WOOD'S ALG. P. 369.

$\dfrac{12}{\pi}\times\dfrac{a}{r}$. (8) $(6n+1)\dfrac{\pi}{6}$.

1. $\dfrac{2(1-x^2)}{(1+x^2)^2}$; 2. $e^{\sin x}.\cos x$; 3. $\dfrac{\log_a e}{\sin^{-1}x\sqrt{1-x^2}}$.

$x=4$, gives a min., $x=3\frac{1}{3}$, a max (12) $h=r$.

1. $\log a+\dfrac{x}{a}-\dfrac{1}{2}.\dfrac{x^3}{3a^3}+\dfrac{1\cdot 3}{2\cdot 4}.\dfrac{x^5}{5a^5}$ etc.;

2. $2^n\left\{1+\dfrac{nx^2}{\underline{|2}}+n(3n-2).\dfrac{x^4}{\underline{|4}}+\ldots\text{etc.}\right\}$.

CALC. P. 136. (16) 1. $\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$; 2. $\dfrac{1}{2a^2}\tan^{-1}\left(\dfrac{x^2}{a^2}\right)$;

3. $x^3\sin x+3x^2\cos x-6x\sin x-6\cos x$.

$ab=2c^2$.

MIXED MATHEMATICS.

I.

(1) EL. MEC. P. 19.

(4) 126° 52′ 11″, 143° 7′ 49″, 90°.

(6) 65·27 and 44·647 lbs.

(7) EL. MEC. P. 200; 1 : 2.

(8) 1. $\frac{Pg}{P+Q}$; 2. $\frac{P\sqrt{P^2+Q^2}}{(P+Q)^2}\cdot g$.

(9) Velocity $=\sqrt{v^2-4g^2}$; $\tan\phi=\frac{1}{v}\sqrt{\frac{v^2-16g^2}{3}}$;

space $=\frac{v\sqrt{3}}{4g}(v-\sqrt{v^2-16g^2})$.

(10) 1. 45°; 2. $\tan^{-1}\frac{1}{\sqrt{2}}$.

(12) $w=69$ oz., s.g. $=6·9$.

II.

(2) $2\sqrt{5}$; inclined 63° 26′ to the force 5.

(4) R = 80, coincides with force 40.

(6) $\text{Cos}^{-1}\frac{-Q}{P}$.

(7) EL. MEC. P. 185.

(9) $\tan\alpha=\frac{2h\pm\sqrt{4h(h-b)-a^2}}{a}$; the pt. being (a, b).

(10) 20·39 ft.

(11) 7·2 in. edge.

III.

(1) (2) EL. MEC. PP. 7, 29, 31.

(3) $2\frac{1}{12}$ ft. from wt. 9 lbs.

(4) $T=\frac{W\sqrt{3}}{4}$.

(5) $24\frac{1}{2}$ lbs. nearly.

(6) 30°; $t=10·392''$.

(7) ·838 mi.; $t=10·062''$.

(8) $\frac{b^2}{4a}$.

(9) 10° 30′; $1\frac{1}{2}$ miles.

(10) EL. HYD. P. 97.

(11) 17·55.

(12) $\frac{r}{2}$.

IV.

(3) $R=\sqrt{3}$; $\angle=90°$.

(4) Pres. on B, vertical, = 81·65 lbs.; Pres. on A = Pres. on C = 70·29 lbs. inclined $-7\frac{1}{2}°$ to horizontal.

(5) $7\frac{3}{7}$ ft. from given pt.

(6) EL. MEC. P. 36.

(7) $\sqrt{\frac{2R\tan i}{g}}$.

(9) $29\frac{1}{6}$ hrs.

(10) $\frac{2l}{3}$ from fixed end.

(11) EL. HYD. P. 123.

(12) 10 ft. nearly.

V.

L. MEC. P. 15. (3) $\frac{1}{2}P(\sqrt{6}-\sqrt{2})$. (4) $80\sqrt{3}$.
$=9\sqrt{2}$, acting parallel to AC; $CE=\frac{3}{4}CD$.
ECH. PHIL. P. 23. (7) $\frac{7}{13}$.
L. MEC. P. 165. (9) $v : v_1 :: 5 : 3$.
$=120750$ ft.; $PS=40250$ ft.
. HYD. P. 49. (12) 14000 grs.

VI.

·76 lbs.; 131° 24′. (4) 135 lbs.
. MEC. P. 88. (6) 13·06.
$\frac{1}{2}°$ to horizon. (9) $\frac{(2-3e)v}{5}$.
$4\frac{26}{27}$ ft. (11) EL. HYD. P. 123. (12) 45·059 in.

VII.

L. MEC. P. 22. (2) $R=10\sqrt{4+\sqrt{3}}$. (3) 9 ft.
° and 45°. (5) $\sqrt{3} : 1$.
$=\sqrt{29+20\sqrt{2}}$, inclination to CA, $\tan^{-1}\frac{1+\sqrt{2}}{2}$.
ft. a second. (9) MECH. PHIL. P. 22.
4400; $8\frac{8}{55}$. (11) EL. HYD. P. 71. (12) $11\frac{1}{2}$ lbs.

VIII.

L. MEC. P. 114. (2) $\tan^{-1}\frac{(\mu+\mu_1)a}{2a+(\mu_1-\mu)c}$; c = dist. of *c.g.* from plane, $2a$ = dist. between upper and lower feet.
ft. from greater sphere.
DHUNTER'S STAT. P. 200; 11 : 18. (5) MECH. PHIL. P. 130.
cwt. (8) 144·9 ft. (9) EL. MEC. P. 204.
$=41\cdot92$ ft.; $t=2\cdot79''$. (11) 16 oz. (12) 12 in.

IX.

L. MEC. P. 70. (2) $\frac{2^2}{2^n-1}\{2^{n-1}(n-2)+1\}$.
RAC. MEC. P. 100. (4) $W=\frac{wr\tan^{-1}\mu}{b-r\tan^{-1}\mu}$.
os ϵ : Cos ϵ' :: 1 : 2. (6) EL. MEC. P. 194.
$=320$; $t=10''$.
$=\tan^{-1}(2\pm\sqrt{3})$; and $\tan^{-1}\left\{2\pm\sqrt{3-\frac{8gh}{v^2}}\right\}$.
0) EL. MEC. PP. 234, 236; 88·0634 in.
$320-\frac{9w}{17}\Big)$ lbs.; w = wt. of air in bell at first.

X.

L. MEC. P. 40. (2) $\alpha=\tan^{-1}\frac{\mu W+\mu_1 W_1}{W_1+W}$.
$\frac{(3r^2+2rr_1+r_1^2)}{4(r^2+rr_1+r_1^2)}$. (4) $5\frac{14}{15}$ lbs.

(5) R=12 lbs.; 8 in. from greater force. (6) 644 ft.
(7) 57960 ft. (9) $\frac{13}{15}$.
(10) 29 : 1, nearly. (11) 1·005. (12) $3000\sqrt{2}a$.

XI.

(1) EL. MEC. P. 85. (3) $\frac{w}{3}$ and $\frac{2w}{3}$.
(4) $10\frac{10}{11}$ in. from end without weight. (6) 849·28.
(7) (8) MEC.H PHIL. PP. 251, 256.
(9) 1. 292·075 ft.; 2. 267·925 ft.; 3. 1324·6 ft.
(10) 15·316″. (12) 1548 lbs.

XII.

(1) TODHUNTER'S STATICS, P. 93. (2) $\frac{5h}{18}$ from base.
(3) Dist. from centre of greater sphere $=\frac{dr^3}{r_1^3+r^3}$; d=dist. between centres. (4) 6.
(5) (6) MECH. PHIL. P. 155.
(7) Vy. of A $=\frac{v\sqrt{3}}{2}$; $\angle 90°$; Vy. of B $=\frac{v}{2}$, $\angle 0°$.
(10) $\sigma : 2\sigma+\sigma_1$. (11) EL. HYD. P. 37. (12) $\frac{p+q}{mp+nq}$.

XIII.

(1) EL. MEC. P. 77. (4) $\frac{w}{4}\cdot\frac{h^2+4r^2}{h^2+r^2}$; $\frac{3w}{4}\cdot\frac{h^2}{h^2+r^2}$.
(5) 2·5 in. (6) 10 lbs. (7) EL. MEC. P. 191.
(8) 500 ft. (9) 10·372″. (12) 5819·18 lbs.

XIV.

(1) EL. MEC. P. 124. (2) $\frac{1}{5}$. (3) 8.
(4) 1. $\frac{a}{8}$; 2. $\frac{3\sqrt{2}-2}{18}a$. (5) $5\frac{13}{19}$ ft. (6) $\sqrt{gh}$.
(7) $\frac{gt^2}{2}\left\{\frac{v\sqrt{2}-gt}{v-gt\sqrt{2}}\right\}^2$ (8) EL. MEC. P. 228.
(9) 1956 ft. (10) 36·88 lbs.
(11) EL. HYD. P. 44. (12) 1. Depth $=\frac{2h}{5}$; 2. Depth $=r(1-\cos\theta)$; θ being found from equation $\theta-\frac{1}{2}\sin 2\theta=\frac{2\pi}{5}$.

XV.

(1) EL. MEC. P. 56.
(2) $W_1 : W_2 :: \sin\beta : \sin\alpha$; $R_1 : R_2 :: \tan\beta : \tan\alpha$.
(3) Distance of point of support from greater wt. $=\tan^{-1}\frac{3+\sqrt{3}}{6}$.

(5) 1 : 6. (6) (7) El. Mec. pp. 133, 148.
(8) $v_1=100$; $v_2=500$; $g=32 \cdot 2$.
(9) $\frac{h}{3}$ from vertical plane through summit of given plane.
(10) $5\sqrt{6}$. (11) El. Hyd. p. 49. (12) $38\frac{89}{292}$ oz.

XVI.

(1) El. Mec. p. 126. (2) Four times st. line joining mid. pts. of diagonals. (3) $x=\frac{P-R}{Q-R}\cdot l$; $w=\frac{2(QR-P^2)}{2P-(Q+R)}$.
(4) $2-\sqrt{3}$. (5) El. Mec. p. 92. (6) 12 ft.
(7) El. Mec. p. 238. (9) $s=2672$ ft., $t=1' \, 31''$.
(10) Mech. Phil. p. 243; 47·543.
(11) $60^\circ \, 2'$. (12) 7 ft.

XVII.

(1) El. Mec. pp. 127, 130.
(2) R = 2·1283556 cwt.; P = ·72794 cwt. (3) 1·0546 lbs.
(8) $\frac{3}{4}\sqrt{gl}$. (9) 37·18 ft.; 269 ft. (10) 70·4.
(11) El. Hyd. p. 122. (12) ·6.

XVIII.

(2) 45°; $56\sqrt{2}$. (4) El. Mec. p. 103.
(5) $2\sqrt{5}$ lbs.; inclination to A B $63^\circ \, 26'$; pt. of application $\frac{5}{3}$ of side from C, in D C produced.
(6) 12 lbs. (7) El. Mec. p. 194.
(8) 3·1″. (9) El. Mec. p. 152.
(10) Vel. of A $=-\frac{5a}{3}$; of B $=\frac{7a}{3}$.
(11) 368·5 grs. (12) 5·78 grs.

XIX.

(1) El. Mec. p. 99. (2) 20 oz.
(3) $\operatorname{Tan}\theta=\frac{1}{2}\cdot\frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$. (4) Inclination of B C to horizon $=\tan^{-1}\frac{bw_2+aw_1\cos\alpha}{w_1\sin\alpha}$, where a and b are the distances of the centres of gravity from B.
(5) (7) (9) El. Mec. pp. 103, 173, 236; $88^\circ \, 16'$; 33·06 lbs.
(10) 11. (11) (12) El. Hyd. pp. 4, 25, 145.

XX.

(1) Todhunter's Stat. p. 179. (2) $94\frac{4}{7}$ lbs.
(3) Dist. of c.g. from right angle $=1 \cdot 972a$. (6) 2 P and P.
(7) (8) El. Mec. pp. 165, 199; P : Q :: 5 : 3.
(9) $s=\frac{v^2}{2g\sin\alpha}$; 2. $s=\frac{v^2}{2g(\sin\alpha+\mu\cos\alpha)}$.
(10) El. Mec. p. 238; SP = 64·4 ft., inclined to horizon at an $\angle\, 2\alpha-90^\circ$.
(11) El. Hyd. p. 37. (12) 20 in.

XXI.

(1) 50, $50\sqrt{3}$. (3) $33\frac{1}{3}$ lbs. (4) $n+1$.

(5) Dist. fr. centre of larger circle $=\dfrac{dr_1^2}{r^2-r_1^2}$.

(6) (7) El. Mec. pp. 177, 220. (8) 70° 58′.

(9) Mech. Phil. p. 257. (11) Depth $=\dfrac{3h}{7}$. (12) 53·875 c.i.

XXII.

(1) El. Mech. p. 121.

(2) Four times the line joining the mid. pts. of A B and C D.

(3) 5·633 in. (4) $\text{Tan}^{-1}\dfrac{\sqrt{3}}{2}$ from greater wt.

(5) El. Mec. p. 112. (6) After ·5″; 13·315″.

(7) 15698 ft.; 1000·52 ft. (10) $v\sqrt{\dfrac{wl}{ge}}$. (11) El. Hyd. p. 78.

XXIII.

(1) (2) El. Mec. pp. 58, 132. (3) $\mu=\dfrac{\mathrm{H}r}{\sqrt{\mathrm{W}\rho^2-\mathrm{H}^2(r^2-\rho^2)}}$.

(4) Mech. Phil. p. 142. (5) $5\sqrt{2}$ ft. fr. vertex.

(6) $14\frac{2}{3}$ in. fr. wt. 4 lbs. (7) Mech. Phil. p. 22.

(8) 38. (9) 3741 ft. (10) 4·8 lbs., nearly.

(11) 2 : 1. (12) $\frac{875}{1296}\sqrt{3}a^3$ oz.

XXIV.

(1) El. Mec. p. 117. (2) $50(2-\sqrt{3})$ lbs.

(4) 1. 170 lbs.; 2. 238 lbs. (5) 30°.

(6) ·3659. (7) 81·5 ft.

(9) $\tan^{-1}\frac{5}{8}$. (10) Equal.

(11) 9·938 lbs. (12) El. Hyd. p. 44.

XXV.

(1) (2) (3) Mech. Phil. pp. 58, 147. (5) El. Mec. p. 91.

(6) 99·38 ft. (7) 2·78 lbs. (9) 272·5 ft.

(10) El. Hyd. pp. 18, 21. (11) $\frac{1}{2}w$. (12) $\frac{91}{101}$.

XXVI.

(6) 3220. (7) $t=\dfrac{\sin(\beta-\alpha)}{g}\{v\sec\alpha-u\sec\beta\}$.

(8) $r=9{\cdot}75$ in.; $t=$·499″. (10) El. Hyd. p. 125.

(11) 18. (12) 1 : 817.

XXVII.

(4) Todhunter's Stat. p. 200.

(7) Velocity $=32{\cdot}2\sqrt{21}$: $\phi=\pm\tan^{-1}\dfrac{\sqrt{3}}{5}$; $s=80{\cdot}5\sqrt{3}$, or $322\sqrt{3}$.

(8) $58\frac{6}{11}$ miles. (10) (11) El. Hyd. p. 77. (12) 17 ft.

XXVIII.

(1) $\frac{1}{3}(h_1 - h_2)$ from base.
(2) (3) El. Mec. pp. 90, 91.
(4) $\mathrm{Sin}^{-1}\left(\frac{\tan(\alpha-\phi)}{\cos\phi}\right)$.
(5) W $\sin(\alpha+\phi)$; $\tan\phi$ being coef. of fraction.
(6) 2 lbs.
(7) 39·1393 in.
(9) Mech. Phil. p. 243.
(11) El. Hyd. p. 93.
(12) ·1314 in.

XXIX.

(2) 7 in. from force 2 lbs.
(3) $\epsilon = -14^\circ\ 2'$; $P = 27{\cdot}5$ lbs.
(4) El. Mec. p. 129.
(5) $7\frac{3}{8}$ ft.
(6) 4 in.
(7) 20·125 ft.
(8) El. Mech. p. 208; 39284 ft.
(9) $3''$ and $57''$.
(10) $15''$.
(11) El. Hyd. p. 93.
(12) 4·5.

XXX.

(2) 258·81 and 965·925 lbs.
(3) $\frac{r}{3}(3+\sqrt{6})$.
(4) El. Mec. p. 65.
(7) 15528 ft.
(8) $v = \sqrt{2g(\sqrt{l^2+h^2}-h)}$, $2h$ = height, $2l$ = length of block; $v = \sqrt{2g\mu a}$.
(10) Equal.
(11) 33 : 29.

XXXI.

(1) 11·4 in. from end.
(3) $r\sqrt{3}$.
(4) Mech. Phil. p. 147.
(7) $28{\cdot}78''$.
(8) 11·65 ft.
(9) $7^\circ\ 43''$.
(12) El. Hyd. p. 91.

XXXII.

(1) 5 in. fr wt. 8 oz.
(2) $\frac{2h}{9}$ from base.
(3) El. Mec. p. 138.
(4) Todhunter's Stat. p. 28.
(6) $\mathrm{Tan}^{-1}\frac{2}{3}$.
(8) 322 ft.
(9) El. Mec. p. 188.
(10) Mech. Phil. p. 158.
(11) El. Hyd. p. 12.
(12) $\frac{40aw}{b}$.

XXXIII.

(2) 9·3164.
(3) ·18334 in.
(8) 35 lbs.
(9) 707·1 ft.; $h = 3882$ ft.
(10) El. Mec. p. 224.
(11) El. Hyd. p. 56.
(12) 5·6 lbs.

XXXIV.

(1) El. Mec. p. 112.
(2) 112·5 lbs.
(3) El. Mec. p. 141.
(4) $\frac{W\sqrt{3}}{4}$.
(5) $\frac{1}{8}$.
(6) Mech. Phil. p. 250.
(7) 2450 ft.
(9) 32·2 ft.; 4·8 oz.
(11) 20.

XXXV.

(1) MECH. PHIL. P. 155.

(2) $R=10$; distance fr. 8 lbs. $= 8{\cdot}0217$ ft.; $\tan\theta=\dfrac{24+7\sqrt{3}}{37}$.

(3) $\frac{1}{3}\{(OC-OA)^2+(OB-OD)^2+2(OC-OA)(OB-OD)\cos\theta\}$.

(4) $2\frac{5}{8}$ ft. from end. (5) $R=-P$; pt. (-3).

(6) $23{\cdot}67$; $T=\frac{1}{17}$ lb. (7) $\cdot00906$ in.

(8) $18917{\cdot}5$; 231840. (11) $432{\cdot}95$ lbs. (12) $3141{\cdot}6$ oz.

XXXVI.

(2) $2(2+\sqrt{3})$ ft. (3) $89{\cdot}6$, $67{\cdot}2$, $53{\cdot}76$ lbs.

(4) $\dfrac{4a}{7}$. (5) $\dfrac{W\sqrt{2}}{6}$. (6) 21548522 units.

(7) 409600 units. (8) 3938 ft.

(9) $268\frac{1}{3}$ ft. (10) $116{\cdot}23$.

(11) Distance from vertex $=\dfrac{r\tan\theta}{(4+\tan^2\theta)\sin\theta}$.

XXXVII.

(2) 5 in. from end; 6 lbs. (3) $\dfrac{11h}{8(3+\sqrt{3})}$ from base.

(6) $\frac{75}{161}$. (7) EL. MEC. P. 181. (9) 2 : 3.

(10) $v=583{\cdot}12$ ft.; $h=353{\cdot}69$ ft. (11) 6.

(12) MECH. PHIL. P. 281; Theoretical discharge, $\cdot35$ c. f.; Coef. of Vy. $\cdot89$.

XXXVIII.

(1) EL. MEC. P. 53. (3) $4r\cot\alpha$; $2r\cot\alpha$.

(5) 32 oz. (7) $292{\cdot}4$. (8) $4{\cdot}1''$.

(9) $t=\dfrac{3v}{4g}$; $h=\dfrac{15v^2}{32g}$. (10) $313{\cdot}42$.

(11) $13{\cdot}2$ mi. an hr. (12) $8{\cdot}6107$ oz.

XXXIX.

(1) TODHUNTER'S STATICS, P. 146. (2) $\dfrac{5\sqrt{3}}{3}$.

(3) W rests at a pt. which divides the string into two parts,

$$a\left\{1\pm\frac{c\sin\alpha}{\sqrt{a^2-c^2\cos^2\alpha}}\right\};\ T=\frac{W}{2}\cdot\frac{\sqrt{a^2-c^2\cos^2\alpha}}{a\sin\alpha}.$$

(4) $1{\cdot}95$ cwt., and $3{\cdot}55$ cwt. (8) $19{\cdot}05$ oz.

(9) $\dfrac{8gmn}{25}$ ft. (11) 21 : 1, nearly.

XL.

(2) 2 or 5 lbs. (4) $2\frac{1}{4}$ ft. from 30 lbs.

(5) 4 ft. from P. (8) $5''$ and $35''$.

(11) $589{\cdot}05$ lbs. (12) EL. HYD. P. 73.

XLI.

(1) 1. 3230 lbs.; 2. 3350 lbs. (2) $\tan^{-1}(\frac{4}{3})$.
(3) 1584·3 lbs. (5) Mech. Phil. p. 92.
(6) $\tan\theta = \dfrac{a-\mu\mu_1 b}{\mu(a+b)}$. (8) 19·6 lbs.
(9) (10) Mech. Phil. p. 255; $3\frac{1}{4}$.

XLII.

(1) El. Mec. p. 78. (2) Through the centre.
(3) Inclination to horizon $=\cos^{-1}\dfrac{a+\sqrt{32a^2+r^2}}{8r}$, r = rad. of sph., a = dist. of c. g. of beam from lower end.
(4) Dist. from rt. $\angle = \sqrt{2}$. (6) 3·26 ft., nearly.
(7) 18·11″; 60; $v=17\cdot8$ ft. (8) Mech. Phil. p. 212.
(9) 187. (10) $s=21\frac{7}{15}$ ft.; $T=\dfrac{4P}{3}$.
(11) 248510 lbs. (12) 2592 lbs.

XLIII.

(2) El. Mec. p. 129. (3) 1 lb. (7) 2″ and 8″.
(8) $67\frac{1}{12}$ ft. (9) $4\cdot56t$.
(10) $u'=\dfrac{u}{\sqrt{2}}$, $\theta=90°$; $v'=\dfrac{u}{\sqrt{2}}$, $\phi=0°$. (11) $\dfrac{9m}{4}$, m = mass of plate.

XLIV.

(1) Todhunter's Statics, p. 36. (2) 45°; 1.
(5) $\dfrac{2a}{7}$ from centre, a = side. (6) (7) El. Mec. pp. 134, 236
(8) R = 38·82 ft.; $h=5\cdot2023$ ft., $t=\cdot804''$. (9) 179·67 ft.
(10) $v=\sqrt{\dfrac{gr}{\mu}}$. (11) El. Hyd. p. 11. (12) 10751 lbs.

XLV.

(1) Todhunter's Statics, p. 36. (3) $\dfrac{a\sqrt{3}}{20}$. (5) 40h. 47·5 m.
(6) $v=20$ ft.; direction inclined at $\angle$ of 36° 52′ to ship's course.
(8) 78. (10) $u=\cdot043593$; $k=\dfrac{4\sqrt{3}}{9}$

APPENDIX.

ANSWERS.

I. (*a.*)

(1) 94760000 miles.
(2) 1. 8 mi. 4 fur. 10 po. 4 yds. 1 ft. 1 in.; 2. 363 lbs. 11 oz. 8 dwts. 6 grs.
(3) 2 qrs. 1 lb. (4) 11 lbs. 7 oz. 16 dwts.
(5) £188 10*s.* (6) £524 13*s.* 9*d.* (7) 18 days.
(8) 5 dwts. $3\frac{513}{1869}$ grs. (9) 1. $\frac{13}{15}$; 2. $2\frac{31}{42}$; 3. 2.
(10) 1. ·02016; 2. 286·5; 3. 100·032. (11) $52\frac{1}{2}$ sq. ft. (12) $11\frac{7}{85}a$.

II. (*a.*)

(2) $187870\frac{9}{14}$. (3) $1215\frac{15}{19}$.
(4) 1. 207 c. yds. 17 c. ft. 409 c. in.; 2. £449 6*s.* $7\frac{1}{2}$*d.*
(5) $8\frac{1}{4}$ ft. (6) £129 15*s.* 10*d.* (7) $3\frac{57}{80}$ days.
(9) 1. $\frac{71}{1568}$; 2. 3; 3. $\frac{119}{160}$.
(10) 1. 1·6; 2. ·5136; 3. 8*s.* 9*d.* (11) $26\frac{2}{3}$.
(12) £55. (13) £41 11*s.* $5\frac{3}{5}$*d.*

III. (*a.*)

(1) 306475278. (2) 1364. (3) £104 5*s.*
(4) 10*s.* $4\frac{3}{4}$*d.* (5) 11. (6) 4217199.
(7) 1. £53 2*s.* $0\frac{7}{8}$*d.*; 2. 4698. (8) $7\frac{1}{5}$ days.
(9) 1. $\frac{11}{40}$; 2. $1\frac{1}{8}$; 3. $\frac{1}{196}$.
(10) 1. ·00002; 628000; 2. 1·7395; 3. ·821875.
(11) £12 $13\frac{3}{14}$*s.* (12) 140,170,190. (13) $15\frac{685}{1408}$.

IV. (*a.*)

(1) 563714964. (2) £16 5*s.* and £4 15*s.*
(3) 1. £194 11*s.* 8*d.*; 2. 1030850 lbs. (4) 5000; $9\frac{3}{4}$*d.*
(5) £2 3*s.* 4*d.* (6) 23 lbs. 3 oz. 12 dwts. (7) 15*s.*
(8) 8 cwt. 14 lbs. (9) 1. $1\frac{3}{8}$; 2. $\frac{3}{7}$; 3. £14 1*s.* $10\frac{6}{7}$*d.*
(10) 1. 58·7378; 2. ·10875; 3. $\cdot 03\dot{5}7142\dot{8}$. (11) 8*s.* 4*d.*
(12) $\frac{53}{63}$. (13) 16.

V. (*a*.)

1) 99 tons 9 cwt. 11 lbs. 6 oz. (2) £12621 7*s*. 6*d*.
3) 2 tons 13 cwt. (4) £3 10*s*. 11½*d*. gain. (5) £6884 5*s*.
6) £2594 15*s*. 5½*d*. (7) £5 19*s*. $4\frac{8}{19}$*d*. (8) £21 7*s*. 1½*d*.
9) 1. 13125; 2. $\frac{7}{10}$; 3. $\frac{33}{2642}$; 4. $51\frac{3}{4}$.
0) 1. 6·300009; 2. ·167433; 3. ·00910776. (11) 112½ ft.
2) $2\frac{2}{7}$. (13) $\frac{23}{24}$.

VI. (*a*.)

1) 1 ton 18 cwt. 1 qr. 11 lbs.; 335 tons 10 cwt. 3 qrs. 21 lbs.
2) 53. (3) 1846. (4) £224, £240, £350.
5) 28. (6) £23 12*s*. 6*d*. (7) £18 13*s*. $6\frac{12}{365}$*d*.
8) $4\frac{7}{12}$; £304 6*s*. 8*d*. (9) 1. $1\frac{29}{40}$; 2. 2*s*. 7*d*.; ·5740.
0) £1507 10*s*. (11) $1\frac{3}{4}$.
2) 1. $\frac{9}{400}$; $1\frac{86}{2475}$; 2. 1200. (13) £1 15*s*. 3·008*d*.

VII. (*a*.)

1) 1. £41344 7*s*. 10½*d*.; 19845309; 2. £110 12*s*. 7½*d*.
2) 7040. (3) 15.
4) 1. $\frac{7}{38}$; 2. 3; 3. 33·384; 4. ·390625. (5) $3\frac{3}{4}$.
6) 530·8416; 12·0101. (7) $2\frac{1}{4}$; $x^3-6x^2+11x-6$.
8) 1. $\dfrac{5x}{6}-\dfrac{x^3}{6}$; 2. $x^3+y^3+z^3-3xyz$; 3. $2x^3-(3x+1)$. (9) $x-b$.
0) 1. $x=10$; 2. $x=4$; 3. $x=168$, $y=56$. (11) 4000.
2) 2. 4, 7, 10. (13) 1. 4·641587; 2. ·000002211518.
4) 487·2983 c. f.; 623·6095 c. f.

VIII. (*a*.)

1) 1. 4 tons 14 cwt. 1 qr. 22 lbs. $10\frac{2}{9}$ oz.; 2. £87 10*s*.
2) 2. (3) £17 3*s*. 4*d*. (4) £51 11*s*. 4*d*.
5) 1. $\frac{2}{1859}$, $\frac{1}{52}$; 2. $\frac{1}{5}$, ·2; 3. £12 10*s*. 7½*d*. (6) £705, £1175.
7) 1. ·0001352; 2. 3·872983.
8) 1. 49; 0; 2. $\dfrac{x-1}{x}$; 3. $x^4-(a^2x^2-2ab^2x+b^4)$; 4. $x^3-3x^2y+6xy^2 -y^3$; 5. $4x^{\frac{1}{3}}y^{\frac{1}{3}}$. (9) 1. $x-20$; 2. $(x+a)(x-a)(x^2+ax+a^2)$.
0) 1. $x=\dfrac{2b+1}{a}$; 2. $x=20$; 3. $x=8$, $y=5$. (11) 30 days.
2) $x=2$, $\frac{1}{4}$, $\dfrac{9\pm\sqrt{-31}}{8}$. (13) 1. 1·5; 2. 2·695543; 3. £1798 2*s*.

IX. (*a*.)

1) 1. £18812 10*s*.; 2. £1 5*s*. 2½*d*.; 3. 23 lbs. 9 oz. 6 dwts.
2) £3 10*s*. 9¾*d*. (3) 3 cwt. 3 qrs. 21 lbs.
4) 1. 61 ac. $5\frac{1}{3}$ po.; 2. 92300; 3. 30*d*. (5) £951 10*s*. 7*d*.
6) ·0705. (7) −190.

(8) 1. $x^3-10x^2+33x-36$; 2. $2x^2-3x+4$; 3. $\frac{x-2}{x+2}$.
(9) 1. $x=8$; 2. $x=3$; 3. $x=-1\cdot2$; 4. $x=46\frac{2}{3}$, $y=55$.
(10) 1. 5 h. cr. 17 sixp.; 2. 18 and 24. (11) 6 and 8 yards.
(12) 10. (13) 1. 507·61; 2. £1515 12*s.*

X. (*a.*)

(1) 1. 104; 2. 14080. (2) 20 days. (3) $7\frac{1}{5}$ tons.
(4) £1527 3*s.* 9*d.* (5) 1. $6\frac{84}{121}$, 9; 2. $(5\frac{1}{2})^2 \times 40 \times 4 = 4840$; 3. $\frac{5}{11}$.
(6) 1. ·07546; 2. ·008; 3. £11 2*s.*, £1 15*s.*
(7) 1. ·000176; 2. 8·426. (8) 2. 36.
(9) 1. $\frac{5a}{6}$; 2. $a^6 - 8a^5b + 15a^4b^2 - 25a^3b^3 + 21a^2b^4 - 14ab^5 + 8b^6$;
3. $(a^4-x^4)^2 = a^8 - 2a^4x^4 + x^8$; 4. $x^3 + 3x^2y + 9xy^2 + 27y^3$.
(10) 1. $x-2y$. (11) 1. $x=5$; 2. $x=21$; 3. $x=20\frac{4}{5}$, $y=18\frac{8}{15}$.
(12) 260. (13) 12.
(14) 1. 4·643453, $\bar{5}$·643453; 2. 1·041393; 3. 3·918071; 4. ·02160564.
(15) 1. 314·16 ft.; 2. 1 : 9; 3. $\frac{\sqrt{3}}{2}$; 1.

XI. (*a.*)

(1) 1. £23 11*s.* $5\frac{1}{2}$*d.*; 2. 640. (2) $5\frac{2}{5}$. (3) $1\frac{1}{8}$.
(4) 1·609... (5) 1. 143; 2. $5\frac{1}{5}$; 3. $\cdot035 = \frac{7}{200}$.
(6) £2 1*s.* 5*d.* (7) 1. ·0000000169; 2. 9·1109.
(8) 1. $x+x^2+x^3$ or $x+\frac{x}{2}+\frac{x}{4}$.
(9) 1. $1+\frac{23a}{12}+\frac{11a^2}{12}$; 2. $(a-x)^6$; 3. $2x^2-(3ax+a^2)$; 4. $8x^{\frac{3}{2}}+27y^{\frac{3}{2}}$.
(10) 1. $\frac{x^2-3y^2}{3x^2+y^2}$. (11) 1. $x=48$; 2. $x=14$; 3. $x=\frac{2a}{5}$, $y=-3b$.
(12) 24. (13) 1. $x=5, -\frac{5}{2}$; 2. $x=\frac{a}{b}, -\frac{b}{a}$; $y=\frac{b}{a}, -\frac{a}{b}$.
(14) 1. $\sqrt{10}=3\cdot162277$; 2. 00000000033378; 3. ·000547.
(15) 408·75 sq. ft.

XII. (*a.*)

(1) 1. 50688000; 2. 7*s.* 10*d.*; 3. £87 0*s.* 9*d.*
(2) 1. $27\frac{13}{20}$, $1\frac{3}{4}$; 2. 4*s.* 6*d.* (3) 1. £9 1*s.* 8*d.*; 2. 3·7; 3. 269700.
(4) £2536 10*s.* (5) 7070.
(6) 1. $x^3-y^3-z^3-2xyz+xy^2+3x^2y-x^2z-xz^2-y^2z-yz^2$;
2. $-(3+x+4x^2)$; 3. $\frac{a-b}{a+b}$; 4. -96.
(7) 1. $x=8$; 2. $x=3$; 3. $x=\frac{25}{36}$; 4. $x=9\frac{1}{7}$, $y=-4\frac{4}{7}$.
(8) *A*, 22*s.*; *B*, 26*s.* (9) $2\frac{5}{8}$ miles, $3\frac{1}{2}$ miles. (10) $(2a^{\frac{1}{2}}-b^{-\frac{1}{3}})^2$.
(11) 1. $\sqrt{x}=1, -1\frac{1}{2}$; 2. $x=\frac{2}{5}$, $y=\frac{2}{3}$, $z=2$. (12) 20.
(13) 1. ·5409683; 2. 6·29. (14) 24 yards.
(15) 1. 7·6942 yds.; 2. 31 ac. 3 ro. 15·2728 po.

XIII. (*a.*)

1) 1. 60048000; 2. £37 5*s*. $10\frac{1}{2}d.$; 3. 2 oz. 3 dwts. 7 grs. (2) 200.
3) 1. 28*d*.; 2. ·75; 3. 817000; 4. 54 po. (4) £75 13*s*. 9*d*. (5) 567.
6) 1. $(a^2-b^2)(x^2+y^2)-2(a^2+b^2)xy$; 2. $x+c$; 3. $\frac{x-3}{x^2-3}$, -1.
7) 1. $x=3$; 2. $x=\frac{a(b+1)}{a-2}$, $x=y=3$. (8) 17*s*.
9) 8 minutes. (11) 36. (12) $183\frac{1}{3}$.
3) 1. 225·7257; 2. 12.; 3. 2·3333135... (14) 70° 29′ 43″.
5) 4·0453 inches.

XIV. (*a.*)

1) 1. 37; 2. 3*s*. $5\frac{26}{33}d.$ (2) 88. (3) £13 3*s*. $6\frac{1}{2}d.$
4) £2039 1*s*. 3*d*. (5) 1. $\frac{7}{16}$; 2. $\frac{3}{5}$; 3. 1·8019; 4. ·0074.
6) 1. $\frac{4}{27}$; 2. 4*s*. $10\frac{1}{2}d.$ (7) 1. ·0009; 2. 22·004.
8) 1. 33. (9) 1. $3a+8b+c$; 2. $\frac{2a}{a^2-b^2}$; 3. $\frac{x-2}{x-3}$.
0) 1. $(a-b)^5$; 2. $x^2+2xy-3y^2$; 3. $(x+y)(x^3+xy+y^2)$.
1) 1. $x=2$; 2. $x=4$; 3. $x=5$, $y=10$. (12) 1470.
3) 1. $x=\sqrt{\frac{b_1c-bc_1}{ab_1-a_1b}}$, $y=\sqrt{\frac{a_1c-ac_1}{a_1b-ab_1}}$; 2. $x=5$, -6.
4) 1. 1·77; 2. ·03465025; 3. 2·56. (15) 42·32 ft.

XV. (*a.*)

1) 55. (2) 4*d*. (3) 1050 lbs.
4) £9. (5) £2760 18*s*. 9*d*. (6) 2. $4\frac{1}{3}$; 3. £6 10*s*.
7) 2. ·005628; 3. 240; 4. ·0004; 10·001. (8) 1. 2; 2. $\frac{1}{2}$.
9) 1. $\frac{5(b+c)}{6}$; 2. 0; 3. $12x^2+23xy+10y^2$; 4. 2.
0) $(x-2)(x-3)(2x+1)(2x-1)$.
1) 1. $x=\frac{ab+bc-ac}{bc+ac-ab}$; 2. $x=42$; 3. $x=15$, $y=10$.
2) £450, £270. (13) 1. 5; 2. ·12328065; 3. 5·54634.
4) 8. (15) 11·8 inches.

XVI. (*a.*)

1) 1. 7200000; 2. 3*s*. 11*d*.; 3. £37 2*s*.
2) 2 lbs. 10 oz. 11 dwts. 12 grs. (3) 22 cwt. 2 qrs.
4) 1. ·000999; 2. £1 9*s*.; 3. ·00298828125; 4. 3007.
5) £33 14*s*. $4\frac{1}{2}d.$
6) 1. a^4-x^{12}; 2. x^4+y^4; 3. $a^2+2a-3=(a+3)(a-1)$; (a^2+5a+6) $=(a+3)(a+2)$.
7) 1. $x=\frac{1}{30}$; 2. $x=3$; 3. $x=10$, $y=12$, $z=15$; 4. $x=1\frac{1}{2}$.
8) 1. 15*s*. 6*d*., £2 6*s*. 6*d*., £4 13*s*.; 2. 78 yards.
9) 1. $x=3$, $-\frac{5}{3}$; 2. $x=\frac{4}{9}$, $y=0$. (10) 1904 yards.

(11) 0. (12) 1. 21·5878; 2. ·000045498.
(13) 1. 2377·292 ft.; 2. 12 ac. 2 ro. 7·6832 po.
(14) 1507·352 ft. (15) $\frac{29^3}{12^3}\pi$ c. ft.

XVII. (*a.*)

(1) 1. £4302 13*s.* 9½*d.*; 2. 4; 3. 3*s.* 9*d.* (2) 8.
(3) 18 yards. (4) £49 10*s.*
(5) 1. $6\frac{16}{63}$; 2. ·904; 3. £5; 4. ·18; 5. 141·05. (6) 1. 14; 2. $7\frac{1}{3}$.
(7) 1. $a+5b-4c$; 2. $\frac{5y}{3}$; 3. $x^5-16x^3y^2+20x^2y^3-25xy^4+12y^5$;
4. $a^3-4a^2b+3ab^2-4b^3$.
(8) 1. $(x+y)^2$; 2. $\frac{2x+3y}{4x^2+3y^2}$.
(9) 1. $x=8$; 2. $x=3\frac{105}{191}$; 3. $x=18, y=24$. (10) 850.
(11) 1. $a-\frac{2b}{3}, a-\frac{b}{3}, a+\frac{b}{3}, a+\frac{2b}{3}$; 2. 414; 3. 5, 10, 20, 40; or −15, 30, −60, 120. (12) 1. 3·162277; 2. 6·402364; 3. 1536·582.
(13) $2\sqrt{3}$. (14) 1. $\frac{\sqrt{3}+1}{2\sqrt{2}}$; 2. $(2n+\frac{1}{3})\pi$.
(15) 1. 71° 46′ 1″; 2. 57° 9′ 23″, 47° 50′ 37″.

XVIII. (*a.*)

(1) 1. 52; 2. 39. (2) £512.
(3) 1. £18; 2. £14 15*s.* $2\frac{18}{271}$*d.*
(4) 1. $\frac{8}{11}$; 2. 14·4321; 3. $\frac{2}{3}$; 4. ·2953125.
(5) 1. 13000000; 2. $\frac{23}{49}$, ·2846. (6) 3.
(7) 1. $2x$; 2. $\frac{a^2+b^2}{a^2-2ab-b^2}$; 3. $\frac{x^2-1}{3x^2+1}$; $a-3a^{\frac{2}{3}}x^{\frac{1}{3}}+3a^{\frac{1}{3}}x^{\frac{2}{3}}-x$.
(8) 1. $x=1$; 2. $x=4$; 3. $x=4, y=2$; 4. $x=3, 1\frac{10}{11}$;
5. $x=\frac{1}{2}, -1$; $y=-1, \frac{1}{2}$; 5. 22.
(9) 1. ·49485; 2. 8286·97; 3. 11 years. (10) 20·10624 ft.
(12) 86° 30′ 57″.
(13) Fulcrum distant 6 in. from wt. 10 lbs.; pressure $=2\sqrt{91}$ lbs.
(15) 5·449″.

XIX. (*a.*)

(1) 1. £1432 14*s.* 9*d.*; 2. £21 3*s.* 5*d.* (2) 6. (3) 144 fr. = 117*s.*
(4) 1. $x=\frac{3}{16}$; 2. $x=1, y=2, z=3$; 3. Put the equation in the form $x^4+2x^2+1=x^2-2\sqrt{2}x+2$; $x=\frac{1}{2}\left\{\pm1\pm\sqrt{-(3\pm4\sqrt{2})}\right\}$;
4. $x=3, -\frac{3}{2}, \pm\sqrt{-3}$. (5) £1200.
(6) 1. ·3220239; 2. 152388600000000. (7) 11*s.* 9½*d.*
(8) 32 sq. ft. (11) $\sqrt{2}(1+\sqrt{2})$ ft. and $2(1+\sqrt{2})$ ft.
(12) 1. $x^2+y^2-2r(x+y)+r^2=0$. (13) $x^2+y^2=a^2+b^2$.
(14) $\left(\frac{a^2}{\sqrt{a^2+b^2}}, \frac{b^2}{\sqrt{a^2+b^2}}\right)$. (16) $y=\pm a$.

) 1. $\frac{2(2a+x)}{\sqrt{a+x}}$; 2. $\log\left(\frac{x^2-1}{x^2-3}\right)^{\frac{1}{8}}$; 3. $\tan x + \frac{1}{3}\tan^3 x$; 4. $x - \tan\frac{x}{2}$.

) $\frac{4\pi a^3}{5}$.

XX. (*a*.)

) 75. (2) £2450; £2205.

) 4. ·7240773; 5. £2848 2*s*. $9\frac{1}{2}d$.

) $\frac{A^2}{x^2}=\frac{B^2}{y^2}=\frac{C^2}{z^2}$, $\therefore A^2+B^2+C^2=\frac{A^2}{x^2}(x^2+y^2+z^2)$; and

$$\frac{A^2+B^2+C^2}{x^2+y^2+z^2}=\frac{A^2}{x^2}=\frac{A^2}{x^2}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)=\frac{A^2}{a^2}+\frac{B^2}{b^2}+\frac{C^2}{c^2}.$$

) 1. $x=\frac{b}{c^2}, -\frac{c^2}{b}$; 2. $x=1, y=3, z=5$; 3. $x=5, 5, 5, -1$.

) ALG. II. P. 104. (8) I. 29; III. 32; VI. 4, 16, C.

) 17°·1887... (12) 2. 1·3438 miles. (13) 32° 48′ 46″.

) 2·4. (16) 1. CALC. P. 66; 2. $\tan\theta=\frac{b\sin\alpha}{a-b\cos\alpha}$.

) 1. $\text{Log}\,(x+\sqrt{x^2+a^2})$; 2. $\frac{1}{2}\{\sin^{-1}x - x\sqrt{1-x^2}\}$;

3. $\frac{1}{2}\left\{\log\left(\tan\frac{x}{2}\right)-\frac{\cos x}{\sin^2 x}\right\}$. (18) CALC. P. 154.

XXI. (*a*.)

1. £455 4*s*. 6*d*.; 2. 6*s*. 4*d*. (2) 500. (3) ·42.

2. $\frac{1}{2}$. (5) 1. $x=6$; 2. $x=3, -\frac{1}{3}$; $x=-·3, y=·1$.

Put the equation in the form $4(x^3+1)+5(x^2-1)+x(x+1)=0$;

$x=-1, \frac{-1\pm\sqrt{5}}{4}$. (6) $\frac{3}{7}$.

1. (8) 30.

1. ·519572; 2. £4 16*s*. 1*d*. (10) VI. 2.

417·4325 yds. (14) 156 ft. (15) $9·6\pi$.

1. $\frac{y-x\tan\alpha-c}{\sqrt{1+\tan^2\alpha}}=\pm\frac{y-x\tan\alpha'-c'}{\sqrt{1+\tan^2\alpha}}$; 2. 45°.

$u=16, 12$; $x=1, 3$.

1. $\text{Log}\,\frac{x}{1-x}$; 2. $\log\frac{1+\sqrt{x}}{1-\sqrt{x}}$; 3. $\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$;

4. $a^x\left\{\frac{x^2}{\log a}-\frac{2x}{(\log a)^2}+\frac{2}{(\log a)^3}\right\}$; 2. $\left(\frac{3\pi}{4}+2\right)a^2$.

XXII. (*a*.)

8 ft. (2) 1. 3·01; 2. 1. (3) £793, £549, £305; $15\frac{1}{4}$ per cent.

1. $10^{\frac{3}{4}}=5·62\ldots$; 2. ·001939.

1. $\frac{x(x-a)-b^2}{x(x+a)-b^2}$; 2. $\frac{a^3-3ab^2}{a^2-b^2}$.

(6) 1. $\sqrt{x}=2, -1$; 2. $x=\pm 5, \mp 4\sqrt{-1}$; $y=\pm 4, \mp 5\sqrt{-1}$;
3. Put the equation in the form $(x+x^{-1})^2-\frac{35}{6}(x+x^{-1})=-\frac{25}{3}$; $x=3, \frac{1}{3}, 2, \frac{1}{2}$; 4. $(x-2)(x^2-1)=0$; $x=2, 1, -1$.
(7) 2. -20. (8) 2. III. 20, 21. (9) 2. 32·567 yds.

(10) 1. $\frac{1}{\sqrt{2}}, \sqrt{2}$; 4. ·000290888.

(11) 1. $\sin A+\sin B+\sin C=4\cos\frac{1}{2}A\cos\frac{1}{2}B\cos\frac{1}{2}C=\frac{4ss}{abc}$;
2. AC = 3224·7, AB = 4379·6, C = 90°.

(12) 259·41 ft. (13) $\frac{200}{3\pi}$.

(15) 1. Centre (3, 4); radius = 5; $3x+4y=0$.

(16) 1. CALC. P. 10; 2. $\frac{du}{dx}=\frac{a^3}{(a^2-x^2)^{\frac{3}{2}}}$; $\frac{d^2u}{dx^2}=\frac{3a^3x}{(a^2-x^2)^{\frac{5}{2}}}$; $h=\frac{4r}{3}$.

(17) CALC. P. 201. (18) $5\pi^2a^3$.

XXIII. (a.)

(1) 1. 3s. $3\frac{1}{2}d.$; 2. 175 oz. Troy = 192 oz. Avoirdupois = 84000 grs.
(2) £2077 3s. 6d. (3) 42.
(4) 1. $x=-3$; 2. $x=3, y=1, w=9, z=5$; 3. $x=-8, -12$;
4. $x=\pm 1, \pm\frac{11\sqrt{-5}}{5}$; $y=\pm 2, \pm\frac{4\sqrt{-5}}{5}$.
(5) 85, 72, 59. (6) $(\frac{5}{3})^7-7(\frac{5}{3})^9x+28(\frac{5}{3})^{11}x^2-$&c.; $5005(\frac{5}{3})^{25}x^9$.
(7) 1. 10645·55; 2. $x=·56$. (8) VI. 3.
(9) 2. $-(1+\sqrt{2})=-2·414213$. (10) 6° 52′ 14″.
(11) 67·597964 yds. (12) 27 in.

(16) 1. $a\sin^{-1}\frac{x}{a}-\sqrt{a^2-x^2}$; 2. $\frac{1}{4a^2}\log\frac{x^2+a^2}{x^2-a^2}$; 3. $-(x+\cot x)$.

(17) $\frac{3\pi}{8}$. (18) $\frac{2\pi a^2b}{3}$.

XXIV. (a.)

(1) 1. £2 5s. $7\frac{1}{2}d.$; 2. 500·08; 3. 2·06. (2) 2400; ·5.
(3) 3. 10·3911; 4. £16106 1·2s. (4) 3. $x=1, 1, 1, -2$.
(5) 11. (7) 1. ALG. II. PP. 131, 133; 2. $\frac{11}{16}, \frac{3}{32}$.
(8) 1. II. 12, 13; 2. 2 : 3.
(10) 2. 9·715047; 3. 10·149407. (11) 91° 45′ 34″.
(12) 1·3713 and 2·8713 hours. (13) 9 inches.
(14) 2. 104° 44′ 3″. (15) 1. $x-2y-1=0$, and $y+3y-1=0$.

(16) 1. CALC. P. 25; 2. $m\operatorname{cosec} mx$; 3. $x=\frac{dr_1^{\frac{3}{2}}}{r_1^{\frac{3}{2}}+r_2^{\frac{3}{2}}}$.

(17) 1. $\frac{2a}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}}$; 2. $\frac{e^{ax}(a\cos bx+b\sin bx)}{a^2+b^2}$;
3. $\frac{3a^2}{2}\operatorname{vers}^{-1}\frac{x}{a}-\frac{3a+x}{2}\sqrt{2ax-x^2}$.

(18) CALC. P. 296; $\frac{\pi a^2}{12}$.

XXV. (*a*.)

(1) 1. £1886 5*s*.; 2. £784 4*s*. $4\frac{1}{2}d$. (2) ·9983. (3) £101170

(4) 1. $x=\dfrac{b\sqrt{a}}{\sqrt{2b-a}}$; 2. $x=8$, $y=8\frac{1}{2}$; 3. Put the equation in the form

$x^3+\dfrac{1}{2^3}-2(x+\frac{1}{2})=0$; $x=-\frac{1}{2}, \dfrac{1\pm\sqrt{29}}{4}$;

4. $x=\dfrac{a(a+b)^{\frac{2}{3}}}{(a+b)^{\frac{2}{3}}+(a-b)^{\frac{2}{3}}}$, $y=\dfrac{(a^2-b^2)^{\frac{4}{3}}}{(a+b)^{\frac{2}{3}}+(a-b)^{\frac{2}{3}}}$.

(5) 3*s*. 6*d*. (6) 1. ·05732135; 2. £334 13*s*. 4*d*.
(7) 3600. (8) 2. I. 32, VI. 19. (10) 4·3344.
(11) 5·42916 miles. (12) ·53477 sq. ft. (13) 80° 32′ 16″.
(14) 1. $\tan^{-1}\frac{7}{24}$; 2. $\frac{1}{4}$. (15) Sides $=a\sqrt{2}, b\sqrt{2}$.

(16) CALC. P. 193; $\rho=\dfrac{(x^2+y^2)^{\frac{3}{2}}}{2xy}$.

(17) 1. $\operatorname{Log}\sqrt{\dfrac{x^2+1}{x^2+2}}$; 2. $\log\dfrac{x}{1+\sqrt{1+x^2}}$; 3. $\dfrac{1}{\sqrt{2}}\log\left\{\tan\left(\dfrac{\pi}{8}-\dfrac{\theta}{2}\right)\right\}$;
4. $\dfrac{e^{4x}}{4}(x-\frac{1}{4})$. (18) CALC. P. 315.

XXVI. (*a*.)

(1) 1. ·0948, remainder ·00001296; 2. 10·02. (2) 16 oz., £123.
(3) 420 miles. (4) 1. 8, 7; 2. 16·010412; 3. 5.

(5) 1. $x=\dfrac{ab(c+d)-cd(a+b)}{ab-cd}$; 2. $x=1+\sqrt{3}, 1-\sqrt{3}, 1, -1$:
3. $(x+1)(2x^2+x-1)=0$, $x=-1, -1, \frac{1}{2}$;
4. $x^3+x^2-4(x^2-1)=0$, or $(x+1)(x^2-4x+4)=0$; $x=-1, 2, 2$.

(6) 1. $a^{\frac{5}{2}}-a^2b^{\frac{1}{3}}+a^{\frac{3}{2}}b^{\frac{2}{3}}-ab+a^{\frac{1}{2}}b^{\frac{4}{3}}-b^{\frac{5}{3}}$; 2. $1-\sqrt{3}-\sqrt{5}$.
(7) 1. ALG. II. P. 77; 2. 360·2, 17·4.
(9) 1. III. 31; 2. III. 36, VI. 16, 6, III. 22.

(10) 4. $\operatorname{Sin}\theta=\dfrac{b\sqrt{c^2-d^2}}{\sqrt{b^2c^2-a^2d^2}}$; $\cos\phi=\dfrac{ac-bd}{bc-ad}$.

(11) 3. C $=90°$, A $=53°\ 7'\ 49''$. (12) 8·7551 miles.
(13) $375\sqrt{3}$ c. ft.; $\tan^{-1}4\sqrt{3}=81°\ 47'\ 41''$.
(15) $x^2+y^2-16y+39=0$; $3y+2x-24=0$.

(16) 1. $x+\dfrac{x^3}{2.3}+\dfrac{1.3x^5}{2.4.5}$, &c.; 2. $x=4$; $y=\pm(x+2)$.

(17) $\frac{1}{25}\left\{3\tan^{-1}x+\log\dfrac{(x^2+1)^2}{(x-2)^4}\right\}-\dfrac{1}{5(x-2)}$. (18) CALC. P. 310.

XXVII. (*a*.)

(1) 1. £3440 18*s*. $10\frac{1}{2}d$.; 2. £186 6*s*. 11*d*.; 3. ·9714285.
(2) Amount left in £1 after paying rates and income tax $=£\frac{65}{100}=13s.$;
∴ amount left in £1 after paying rates $=\dfrac{40\times 13}{39}s.=13s.\ 4d.$;
rates 6*s*. 8*d*. in the £.

(3) 2·23. (4) 1. $\frac{4k-13}{7}$; 2. ·4678026.

(5) 1. $x=7$; 2. $x=\frac{3}{4}$; 3. $x=-2, -3$; 4. $x=3, \frac{3\sqrt[4]{2}}{2}$; $y=1, \sqrt[4]{2}$.

(6) 4 and 9. (8) 1. I. 43, 36; 2. $3 : 7+4\sqrt{3}$.

(9) 2. $\theta=11° \ 47' \ 20{\cdot}75''$, $78° \ 12' \ 39{\cdot}25''$; 3. ·7784701.

(10) 3·952. (11) 213·13 yards. (12) $\frac{7\pi}{12}$ sq. ft.

(13) $r=3$. (15) CALC. P. 19. (16) $6\frac{3}{4}$.

(17) $\text{v.} = \pi \int_0^a (a^2-x^2)\,dx = \frac{2\pi x^3}{3}$.

(18) 1. $\frac{x^3}{3} - a\log x$; 2. $\log(x-1)$; 3. $\frac{a^x}{(\log a)^2}\{(x+1)\log a - 1\}$;
4. $\frac{1}{2}(x-\frac{1}{2}\sin 2x)$.

XXVIII. (*a.*)

(1) 1. 1. (2) 48 oz. of gold; 16 oz. of silver.

(3) 7. (4) 3. ·2866868; 4. 10·5669.

(5) $e^{3x} - pe^{2x} + qe^x - r = 0$.

(6) 1. $x=3, 1$; 2. $x=\pm\left(\frac{1}{a}+\frac{1}{b}\right)$; $y=\pm\left(\frac{1}{b}-\frac{1}{a}\right)$;
3. $x^3-12x+16=0$; $x=-4, 2, 2$; $y=-1, 5, 5$.

(7) 1. WOOD'S ALG. P. 192; 2. 56.

(8) 1. ALG. II. P. 89; 2. 3, $\frac{22}{7}$, $\frac{355}{113}$. (9) 3. VI. 2, 4, 19.

(10) 2. 2·0944. (12) Height of tree $=256{\cdot}9416$ ft.

(15) 2. 45°. (16) CALC. PP. 8, 22, 193, 200.

(17) 1. $\frac{1}{2}\{x(1+x^2)^{\frac{1}{2}} - \log(x+1+x^2)^{\frac{1}{2}}\}$; 2. $\frac{1}{3}(2\tan x + \sin x \sec^3 x)$.

(18) $6a$.

XXIX. (*a.*)

(1) 1. 13*s.* $3\frac{1}{2}$*d.*; 2. 3*s.* $5\frac{1}{2}$*d.*; 3. 59·52. (2) £504. (3) 4

(4) 1. $x=-\frac{1}{6}, -\frac{1}{2}$; 2. $x=\pm 2$, $y=\pm 1$; 3. $x=\pm\frac{1}{2}, \pm 9\frac{1}{2}$; $y=\pm 7, \mp$

(5) 280 yds. and 140 yds.

(6) 10 labourers, 14 skilled men. (7) ·3535519.

(8) 3. (9) 5·828.

(10) I. 29, 6, III. 21, VI. 4. (11) 2. 11·848 . . .

(12) 145 yds. 2 ft. 10 in.; 2. 107530 sq. yds. (13) 163·67 sq. in.

(14) $y-x-1=0$. (15) $\frac{8a}{3}$. (16) $\pm\frac{3\sqrt{3}}{16}$.

(17) $2y+x-3a=0$.

(18) 1. (i) $\log\left(\frac{x-2}{x-1}\right)$; (ii) $2\{x^{\frac{1}{2}} - \log(1+x)^{\frac{1}{2}}\}$;
(iii) $\frac{a^{3x}}{9(\log a)^2}(3x\log a - 1)$; 2. π; 3. $\frac{\pi a^2}{2}$.

XXX. (*a.*)

(1) 1. ·324; 2. 3000. (2) £1500. (3) 2. 311·6439; 3. 1·0005.
(4) $(x^2-a^2)(x^2-b^2)(x^2-c^2)$.
(5) 1. $x=-1, 4, \frac{1}{4}$; 2. $x=\pm\frac{a\sqrt{3}}{2}$; $x=2, 3, 5$.
(7) 2. 210. (8) 1. I. 8, 4; 2. III. 22; 3. $6\sqrt{6}$; VI. 20.
(10) 2. 120·875. (11) 11·25 miles an hour; direction 8° 8′ S. of E.
(12) 1. Arc = 218·6 ft.; area = 10930 sq. ft.; 2. $50\sqrt{2}$.
(15) 1. CALC. P. 51;

$$2.\ \sin^{-1}x+\frac{h}{\sqrt{1-x^2}}+\frac{h^2x}{2\sqrt{(1-x^2)^3}}+\frac{h^3(1+2x^2)}{2\cdot 3\sqrt{(1-x^2)^5}}+\&c.$$

(17) $\frac{a}{2(a^2-x^2)}$. (18) $2\pi r^2$.

ROYAL MARINES PAPERS.

(1) 1 cwt. 1 qr. 23 lbs. $9\frac{3}{5}$ oz. (2) 240000 lbs.
(3) $13\frac{1}{7}$ ft. (4) £595 15·05*s.* (5) £135 11*s.* $10\frac{22}{59}d$.
(6) £44 17*s.* $8\frac{124}{205}d$. (8) 51·778816; 2518. (9) $\frac{37}{60}$.
(10) 11·108...; 15·3011. (11) -117.
(12) (*a.*) $144(a^4+b^4)-337a^2b^2$; (*b.*) $a^2+2ab-b^2$;
(*c.*) $(a-b+c)(a-b-c)$; (*d.*) $(a-b-c)^2$.
(13) (*a.*) $\frac{3(x+y)}{a+2b}$; (*b.*) $-\frac{x^2+y^2}{2xy}$.
(14) (*a.*) $x=18$; (*b.*) $x=\frac{bc}{b^2+n-abm}$; (*c.*) $x=11, y=8$.
(15) £300. (20) (*a.*) $x=16, 9$; (*b.*) $x=1, 1\pm 2\sqrt{15}$.
(21) $3\log 3-6$; (*a.*) 1·707628; (*b.*) ·0363424.
(22) $\frac{4\pi a^2b}{3}$; 41888 c. in.
(28) $\text{Cos}^{-1}\left(\frac{l-b}{b}\right)^{\frac{1}{3}}$; b = distance of centre of gravity from the end resting on the wall. (29) 200 ft. ($g=32$).

STAFF COLLEGE, ENTRANCE.

(1) £48 3*s.* $1\frac{1}{2}d$. (2) 16*s.* $2\frac{1}{4}d$. (3) 6 days.
(4) £9 12*s.* 6*d.* (5) 1. (6) 2550.
(7) $4\frac{7}{8}$. (8) £4 1*s.* 3·1*d.*

(9) 1. 57·004; 2. 4·1231; Rem^r = ·00004639.

(10) 1. $\frac{a}{2} - \frac{b}{12} - 8c$; 2. $(1 + x)^2 - (2x^2 + 3x^3)^2 = 1 + 2x + x^2 - 4x^4 - 12x - 9x^6$; 3. $a + b + c - d$.

(12) 1. $x = 7\frac{26}{121}$; 2. $x = 2, y = -1$. (13) 60 hours.

(18) 1. $x = 4$, $\sqrt[3]{49}$; 2. $x = \pm 2, y = \pm 4, z = \pm 6$.

(19) $1 + 2x^2 - x^3 + \frac{8x^4}{3} - \frac{5x^5}{2} \ldots$

(20) 1. 125; 2. ·0000001064768; 3. 19·88159.

(22) PA = 598·664; PB = 720·0018. (23) 318·12 ft.

(24) 12·926 in.

(25) Tension = pressure on each tack = 5·7735 lbs.

(27) $v = \sqrt{2ga} + \frac{a}{n}$. (29) $f = \frac{28g}{95}$.

(30) Depth of orifice = $\frac{1}{2}$ height of cylinder.

STAFF COLLEGE, FINAL.

(2) A = 40° 41′ 47″; B = 49° 18′ 13″; AB = 55·59.
(3) Side = 161·8 ft.; angles = 36°, 72°, 72°. (4) 90° 56′.
(5) 27° 23′ 50″. (6) 57·402 miles. (7) 95·636 yds.
(8) $40\sqrt{3}$ yds. (10) $61\frac{1}{11}$. (11) 120°.
(15) 417·9 lbs. (16) 273·81 lbs. (18) 208 ft.
(19) 46·6. (20) $625\sqrt{13}$ lbs. (24) 432″.
(25) 3938 ft. (27) 240π lbs. (29) 427 c. in.
(30) 32·725 lbs.

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